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Neutron-proton transfer reactions leading to $T=1$ particle-hole states of ^{56}Co

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Abstract. Zero range DWBA analysis for two-particle (neutron and proton) transfer reactions is carried out, using simple shell model structure wave functions for ^{54}Fe, ^{56}Co and ^{58}Ni, with ^{56}Ni inert core. In this structure calculation, a microscopic set of two-body interaction matrix elements derived from the non-local separable potential of Tabakin are employed. These matrix elements include in the perturbation theory two corrections (i) the second-order Börn term and (ii) the appropriate core excitations. Unlike the situation in many two-particle transfer reactions, the fragmentation of the reaction strengths to the excited states with respect to the lowest states of same spin and parity in the above transfer processes is satisfactorily borne out from this analysis.

Keywords. Two-particle transfer reactions; shell model wave functions; zero range DWBA; spectroscopic amplitudes; form factors; angular distributions; total cross-sections.

1. Introduction

Two-particle transfer reactions involving transfer of a neutron and a proton, leading to the low-lying states of ^{56}Co, have been studied by Miller and Kavanagh (1967), Hjorth (1967) and by Belote *et al* (1968). In $^{54}Fe(^3He, p)^{56}Co$ experiment with 11·5 MeV ^3He particles, only the numbers of counts at 15° Lab. angle are reported by Bertsch (1967). Also, the angular distributions for eight of the states of ^{56}Co (up to 3·577 MeV excitation energy) which are considerably populated in the above reaction (with $E_{^3He}=12$ MeV) have been measured by Belote *et al* (1968). The same authors have made a preliminary DWBA analysis for the purpose of extracting L-values for the 1·453 MeV ($L=0$), 1·72 MeV ($L=55\%$ and $L=2$, 45%), 1·929 MeV ($L=2$), 3·511 MeV ($L=0$) and 3·577 MeV ($L=0$) states. Their analysis indicates that, except for the 1·929 MeV ($L=2$) state, the remaining four states appear to have a $2p$-$2h$ character. The DWBA analysis of this experiment, has been carried out by Bertsch (1967). He used the Kuo-Brown interaction matrix elements (Kuo and Brown 1968; Kuo 1967) and the single particle (s.p.) energies of Vervier (1966) to generate the required shell model $T=1$ particle-hole states of ^{56}Co with respect to ^{56}Ni core. The calculated differential cross-sections at 15° have been compared with the corresponding observed numbers of counts. Unfortunately, such a comparison has been mismatched except for the lowest three states and for the 1·73 MeV 1^+ state, because of the missing spin-parity assignments for the other states. A systematic DWBA analysis for the low lying states of ^{56}Co has not been performed so far, perhaps due to the nonavailability of their spectroscopic data. Only very recently, complete spectroscopic data for all the levels up to 1·73 MeV, of ^{56}Co have been reported by Kampp and Buhl (1978). This, in fact, is one of the reasons which prompted us to carry out the DWBA analysis for ^{54}Fe (^{3}He, p) ^{56}Co reaction.

For the ^{58}Ni(d, α)^{56}Co ($E_d = 14{\cdot}9$ MeV) reaction the angular distributions and absolute differential cross-sections at 25° for zero angular momentum transfer ($L=0$), 35° for $L=2$, 50° for $L=4$, and at 35° for $L=6$ have been measured by Hjorth (1967). The zero-range DWBA analysis has also been performed by the same author. In his analysis, the ground state of ^{58}Ni with respect to ^{56}Ni core has been assumed to be

$$|^{58}\text{Ni (g.s.)}\rangle = 0{\cdot}764^* |(p_{3/2})^2 \; J=0\rangle + 0{\cdot}606^* |(f_{5/2})^2 \; J=0\rangle;$$

while the low lying states of ^{56}Co with respect to ^{56}Ni core have been borrowed from the work of Wells (1965), which are described, in terms of mixed proton hole (confined to $1f_{7/2}$) neutron particle (in $2p_{3/2}$ or $1f_{5/2}$) configurations. This DWBA analysis of Hjorth (1967) has been confined only to the lowest two (0·16 MeV 3^+ and 0·58 MeV 5^+) excited states. Similar analysis for other states has not been performed, again due to the lack of their spectroscopic data.

In this paper we present a complete zero range DWBA analysis for both ^{54}Fe (^{3}He, p) ^{56}Co ($E_{^3\text{He}} = 11{\cdot}5$ MeV) and ^{58}Ni (d, α) ^{56}Co ($E_d = 14{\cdot}9$ MeV) reactions. It differs from the earlier analyses in the following respects: Firstly, for both the reactions, we use the same wave functions for the low lying states of ^{56}Co generated in terms of $T=1$ proton hole neutron particle excitations with the microscopic set of Tabakin interaction m.e. Further, the ground state of ^{58}Ni used here is obtained in terms of mixed two neutron shell model configurations with the interactions m.e. of Cohen *et al* (1967). In addition, we analyse all the reaction processes leading to the various states of ^{56}Co up to 1·73 MeV excitation energy. In this sense the present investigation is complete and in a way supplements the earlier investigations.

2. Method and calculation

Only the spectroscopic amplitudes appearing in the analysis contain the nuclear structure information. The partial spectroscopic amplitude $S_J(j_1\, j_2)$ is defined (apart from a phase and a multiplication factor) as an overlap between the state ($|\psi_{Im}\rangle$) of the heavy nucleus and the state obtained by coupling the wave function ($|j_1\, j_2\, JM\rangle$) of the transferred pair of nucleons to the state ($|\psi_{I_1 m_1}\rangle$) of the lighter nucleus. Explicitly

$$S_J(j_1\, j_2) = \langle \psi_{Im}\,(A+2)\,|\,((j_1\, j_2)\, J \otimes \psi_{I_1})^I_m\rangle. \quad (1)$$

The structure wave functions for ^{54}Fe, ^{58}Ni and ^{56}Co are generated in the framework of simple shell model with ^{56}Ni core. Thus the ground state of ^{54}Fe is simply given by the two proton holes in $1f_{7/2}$ coupled to total angular momentum (J) zero, while that of ^{58}Ni is given in terms of mixed two-neutron configurations occupying $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ s.p. orbitals. Similarly, the low lying states of ^{56}Co are described

*These mixing coefficients are supposed to be inferred from (d, p) and (d, t) reactions on ^{58}Ni [see Fulmer *et al* (1964) and Fulmer and Daehnick (1965)].

in terms of the mixed proton hole (in $1f_{7/2}$) neutron particle (in any of $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ levels) configurations in contrast to those used by Hjorth (1967). In calculating the shell model wave functions, we have used a phenomenological set of interaction m.e. of Cohen *et al* (1967) for ^{58}Ni and a microscopic set of interaction matrix elements derived from the non-local separable potential of Tabakin (1964) for ^{56}Co. For both the nuclei, the single-particle energies employed are taken from the observed ^{57}Ni spectrum. The microscopic interaction matrix elements include two corrections within the perturbation theory: (i) the second order Börn term which was shown to be important by Kerman and Pal (1967) and (ii) the core polarisation corrections with $3p$-$1h$ intermediate states. The plane wave intermediate states and an angle averaged Pauli operator are used in the calculation of the second order Börn term. This correction term, unlike the bare term, is no longer diagonal in the centre of mass radial node quantum number (N). Therefore, to make the two-body relative matrix elements independent of N, an averaging procedure similar to that of Clement and Baranger (1968) is followed. In the estimation of the core polarisation corrections, six hole states ($1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$) and eight particle states ($2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $1d_{3/2}$) are considered. The excitations from the $1f_{7/2}$ hole state to the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ particle states are excluded. The details of the calculation of the two-body effective matrix elements are similar to those used in $2s$-$1d$ region by Clement and Baranger (1968).

The shell model wave functions for ^{54}Fe, ^{56}Co and ^{58}Ni with respect to ^{56}Ni core, can be written as

$$|\psi_{I=0}(^{54}\text{Fe})\rangle = |(f_{7/2})^{-2}\, I=0\rangle; \tag{2}$$

$$|\psi_{Im}(^{56}\text{Co})\rangle = \sum_{j_n} a^I_{j_n}\, |((f_{7/2})^{-1}_p\, j_n)^I_m\rangle; \tag{3}$$

and

$$|\psi_{I=0}(^{58}\text{Ni})\rangle = \sum_{j_n} b_{j_n}\, (j_n)^2\, I=0\rangle; \tag{4}$$

where the expansion coefficients $a^I_{j_n}$, b_{j_n} are determined from the configuration mixing calculations. Using equations (2) to (4) in the expression (1) for $S_J(j_1, j_2)$ yields:

$$S_J(f_{7/2}\, j_n) = a^J_{j_n}; \tag{5}$$

for ^{54}Fe (^{3}He, p) ^{56}Co reaction and

$$S_J(f_{7/2}\, j_n) = [(2J+1)/8(2j_n+1)]^{1/2}\, a^J_{j_n}\, b_{j_n}, \tag{6}$$

for ^{58}Ni(d, α) ^{56}Co reaction. The DWBA expressions for the form factors, angular distributions and total cross-sections involve (e.g see Broglia *et al* (1972) or Baer *et al* (1973)) in addition to $S_J(j_1\, j_2)$, the bound state wave functions of the transferred nucleons and the distorted wave functions of the projectile and the outgoing particles.

The single-nucleon bound state wave functions are obtained from the Woods-Saxon well with a radius of 1·25 $A^{1/3}$ fm; A being the mass number of the heavy nucleus, and a diffuseness of 0·65 fm. For the case of a proton, a Coulomb potential of a uniform charge distribution of appropriate radius ($r_0 A^{1/3}$) and total charge is also included. The spin-orbit coupling has been taken to be 25 times the Thomas value and the well-depth is adjusted so that the calculated eigenvalue is approximately half of the two nucleon separation energy. The bound state wave functions together with $S_J(j_1 j_2)$ are used to calculate form factors following the Bayman-Kallio (1967) procedure. The distorted waves for the projectile and the outgoing particles are generated through the optical model potential written as

$$V(r) = - V_0[1 + \exp(r - R)/a_0)]^{-1} - i\,[W'(1 + \exp(r - R')/a_0')^{-1} + 4\,W'' \exp((r - R'')/a_0'')(1 + (\exp(r - R'')/a_0''))^{-2}] + V_c. \quad (7)$$

Here, $R = r_0 A^{1/3}$, $R' = r_0' A^{1/3}$, $R'' = r_0'' A^{1/3}$, A being the mass of the heavier particle in the scattering channel and V_c represents Coulomb potential due to a uniform charged sphere of radius $r_c A^{1/3}$ fm. The optical model parameters used in the present analysis, are taken from the earlier studies and are listed in table 1. With this the differential ($\sigma(\theta)$) and the total (σ) cross-sections are calculated using the computer code DWUCK IV of Kunz (undated). An overall normalisation $D_0^2 = 22 \times 10^4$ MeV^{2}fm^3, in conformity with the earlier analyses (e.g. Ball *et al* (1971) Rapaport *et al* (1972)), is employed for the calculated $\sigma(\theta)$ which are then compared with the corresponding observed absolute differential cross-sections.

3. Results, discussion and conclusion

The calculated spectrum of ^{56}Co and some of the theoretical values of $\sigma(\theta)$ for ^{54}Fe (^{3}He, p) ^{56}Co and ^{58}Ni (d, α) ^{56}Co reactions are presented in figure 1. The same figure also shows the respective experimental results for comparison. In this figure are shown the observed numbers of counts at $\theta = 15°$ (Miller and Kavanagh 1967) and the calculated $\sigma(\theta = 15°)$ multiplied by the normalisation D_0^2. It is evident from the figure that the analysis satisfactorily reproduces the observed data for all but 1^+ state popu-

Table 1. Optical-model parameters used in the DWBA analysis, taken from the atomic data and nuclear data tables compiled by Perey and Perey (1976).

Reaction and bombarding energy (MeV)		V (MeV)	r_0 (fm)	a_0 (fm)	W' (MeV)	W'' (MeV)	$r_0' = r_0''$ (fm)	$a_0' = a_0''$ (fm)	r_c (fm)
^{54}Fe (^{3}He, p) ^{56}Co	^{3}He	167·7	1·07	0·82	16·88	0·0	1·694	0·602	1·40
$E_{^3\text{He}} = 11·5$	p	58·0	1·23	0·35	0·0	8·5	1·23	1·20	1·23
^{58}Ni (d, α) ^{56}Co d	p_1^*	98·6	1·105	0·709	0·0	16·49	1·17	0·831	1·30
$E_d = 14·9$	P_2	84·0	0·89	0·85	0·0	11·0	1·30	0·85	0·89
	α^*	46·3	1·574	0·586	15·4	0·0	1·574	0·586	1·30

*See also Hjorth (1967).

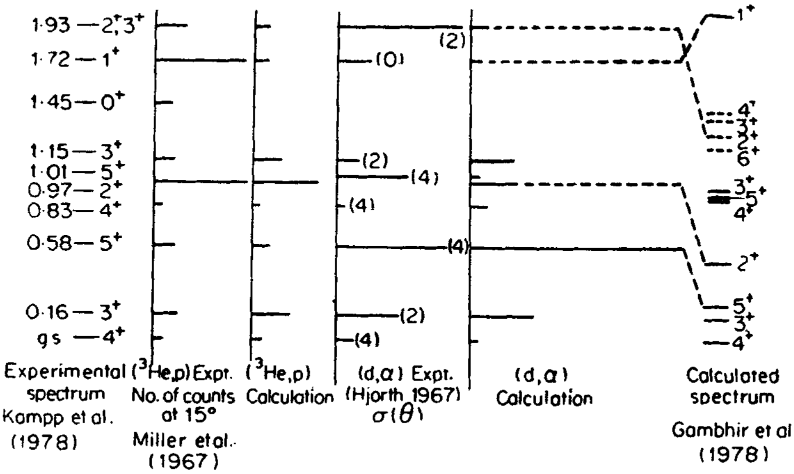

Figure 1. The '(*d*, *α*) Expt.' shows the measured (Hjorth (1967)) $\sigma(\theta)$ at $\theta=25°$ for $L=0$, $\theta=35°$ for $L=2$, $\theta=50°$ for $L=4$, for ^{58}Ni(*d*, *α*) ^{56}Co. The '(*d*, *α*) calculation' gives the corresponding calculated $\sigma(\theta)$ multiplied by D_0^2 $(=22\times10^4\ \mathrm{MeV}^2\ \mathrm{fm}^3)$. In '(^{3}He, *p*) calculation' for $4^+_{1,2}$, $2^+_{1,2}$ states, the average $\sigma(\theta)=\frac{1}{2}\,[\sigma(\theta)_{S=1}+\sigma(\theta)_{S=0}]$ at $\theta=15°$ are given.

lated in ^{54}Fe (^{3}He, *p*) ^{56}Co reaction. Goode and Zemick (1969) in fact have shown that 1·73 MeV 1^+ state is predominantly a $2p$-$2h$ state and therefore the present simple *p-h* picture used in this analysis is not adequate. In the same figure the measured $\sigma(\theta)$, at $\theta=25°$ for $L=0$, $\theta=35°$ for $L=2$, $\theta=50°$ for $L=4$ of Hjorth (1967) and the corresponding calculated $\sigma(\theta)$ multiplied by D_0^2, obtained with the optical model parameter set P_2 of table 1, for ^{58}Ni (*d*, *α*) ^{56}Co are compared. The figure reveals that the observed trends are well reproduced. The analysis overestimates $\sigma(\theta=50°)$ for 5_1^+ state and underestimates the same for 5_2^+ state. The probable reason for this is a negligible configuration mixing (5_1^+ turns out to be 98·9% $((f_{7/2})^{-1}\,p_{3/2})$ state) and hence very small cancellation for 5_1^+ state and accordingly a little coherence for 5_2^+ state, can occur. A slight increase/decrease in the mixing coefficient can reproduce the observed data for the $5_1^+/5_2^+$ state. The calculated $\sigma(\theta)$ with the set P_1 for the optical model parameters given in table 1, are roughly 1·4 times larger than those obtained with the set P_2. These observations indicate that the results obtained with the parameters of the set P_1 will worsen the quality of the agreement and hence not shown in the figure 1. The experimental (Kampp and Buhl (1978)) and the calculated spectrum for ^{56}Co, included in the same figure, agrees reasonably well except for 2_1^+ and 5_1^+ states, which are pushed down by about 0·5 MeV and 0·35 MeV respectively. The 1^+ state is slightly higher (see Gambhir and Parthasarathy 1978). This quality of agreement obtained without any adjustable parameter, can indeed be considered satisfactory.

In figure 2, the calculated angular distributions for

$$^{54}\mathrm{Fe}\ (^3\mathrm{He}, p)\ ^{56}\mathrm{Co}_{J^\pi}\ (J^\pi=5_1^+,\ 2_1^+,\ 1^+,\ 2_2^+)$$

are given along with the available (Belote *et al* 1968) experimental results. For the $2^+_{1,2}$ states, as both $S=0$ and 1 transfers are permitted, the average $\sigma(\theta)=\frac{1}{2}\,[\sigma(\theta)_{S=1}+\sigma(\theta)_{S=0}]$ is shown. The calculated curves are normalised separately, such that in

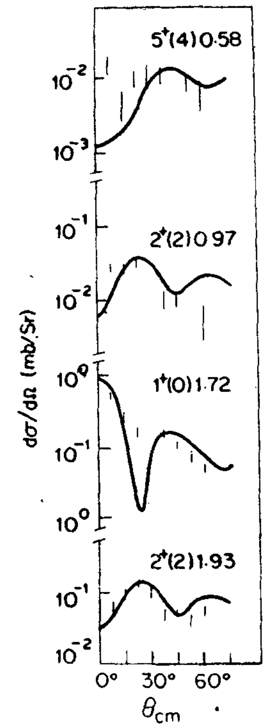

Figure 2. Angular distributions obtained in the zero range DWBA for ^{54}Fe (^{3}He, p) ^{56}Co$_{J^\pi}$ ($J^\pi = 5_1^+, 2_1^+, 1^+, 2_2^+$). The experimental data are of Belote *et al* (1968) and the theoretical curves are normalised so as to reproduce the first observed maximum (see text).

each distribution, the first observed maximum is reproduced. For the 5_1^+ state, as there is no well-defined observed maximum, the normalisation is arbitrarily chosen to reproduce the observed $\sigma(\theta)$ at 37·5°. The experimental trends are reproduced. It is to be noted that the calculated $\sigma(\theta)$ after multiplying by D_0^2, are considerably higher than the corresponding observed $\sigma(\theta)$.

The angular distributions for ^{58}Ni(d, α) ^{56}Co reaction are plotted in figure 3, along with the experimental data taken from Hjorth (1967). The L-transfer values are given in the parantheses. The theoretical curves shown are for the optical parameter set P_2 and are normalised so as to reproduce the respective observed $\sigma(\theta)$ at $\theta = 25°$ for $L=0$, $\theta=35°$ for $L=2$, $\theta=50°$ for $L=4$. This normalisation factor is close to unity (ranges between 1·3 to 0·7) except for the 1^+ state where its value is $\simeq 7$. The shape and the positions as well as the numbers of minima and maxima of the angular distributions for ^{58}Ni (d, α) ^{56}Co are well reproduced. The results with both P_1 and P_2 sets of parameters show similar trends and differ by a factor of 1·4 in magnitude as mentioned earlier. The curves of the P_1 set for 1^+ and 2_2^+ states show large oscillations.

In conclusion, we wish to state that simple shell model description with realistic effective interaction reproduces reasonably well, in the zero range DWBA analysis, the systematics observed in ^{54}Fe(^{3}He, p)^{56}Co and ^{58}Ni(d, α)^{56}Co reactions.

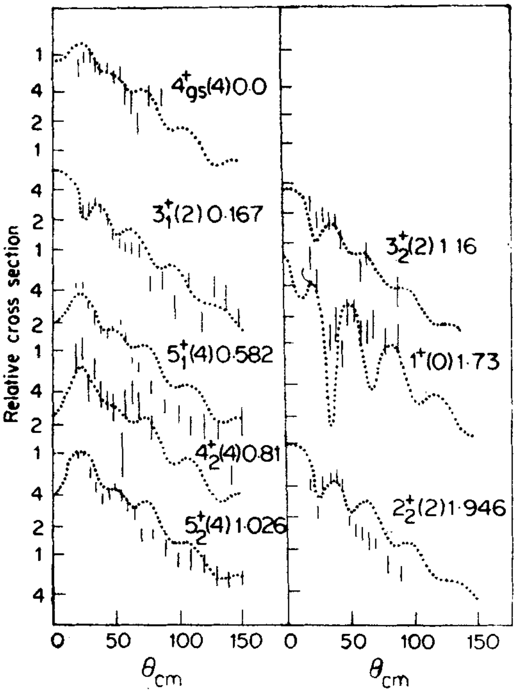

Figure 3. Angular distributions obtained in the zero-range DWBA for ^{58}Ni (d, α) ^{56}Co. The observed data is of Hjorth (1967). The theoretical curves for the optical parameter set P_2, are normalised so as to reproduce the observed $\sigma(\theta)$ at $\theta = 25°$ for $L=0$, $\theta = 35°$ for $L=2$, $\theta = 50°$ for $L=4$.

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