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Back-bending in ^{182,184,186}osmium

M R GUNYE and ASHOK KUMAR
Theoretical Reactor Physics Section,
Bhabha Atomic Research Centre, Bombay 400 085

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Abstract. The high spin yrast states upto $J = 22^+$ in ^{182,184,186}Os are studied in a microscopic variational approach with number-conserved projected states. The energy spectra, quadrupole, moments and B ($E2$) values are calculated by employing the Hamiltonian with quadrupole plus pairing interactions. The results of the calculations are in fair agreement with the available experimental data.

Keywords. Nuclear structure; Osmium isotopes; energy spectra; quadrupole moments; B ($E2$) values.

1. Introduction

The nuclei of tungsten, osmium and platinum form an important region of transition from the deformed rare-earth nuclei to the spherical ^{208}Pb nucleus and for this reason are of special interest in testing the predictions of different nuclear models. The nuclear structure studies, both experimental and theoretical, in osmium and platinum nuclei are being pursued with great interest in recent years. The back-bending phenomenon observed in some deformed rare-earth nuclei gave a stimulus to investigate high spin states in heavy nuclei (Sorensen 1973; Johnson and Szymanski 1973). The energy spectra of the yrast states upto the spin $J \leqslant 22$ in ^{182,184,186}Os are now well established (Davidson *et al* 1973; Warner *et al* 1973; Neskakis *et al* 1976; Spanhoff *et al* 1978). All the three nuclei ^{182,184,186}Os exhibit a back-bending at some critical angular momentum $J_c \approx 12$. The anomalous back-bending behaviour in the rotational band is interpreted as a manifestation of the coriolis interaction between the collective and intrinsic motion of nucleons. There are two alternative proposals to explain the anomalous behaviour in terms of the coriolis force. Mottelson and Valatin (1960) attribute the anomalous behaviour to the coriolis antipairing phase transition from the superfluid to the normal nuclear state. The alternative proposal by Stephens and Simon (1972) states that certain individual nucleons may respond to coriolis force prior to the phase transition proposed by Mottelson and Valatin (1960). This rotation alignment proposal of Stephens and Simon (1972) attributes back-bending to the coriolis decoupling of a nucleon pair in orbital with high angular momentum j from the rotating core and subsequent alignment with the core angular momentum. The coriolis force is strong if the Fermi surface is close to the substates of high-j orbitals with small projection Ω on the symmetry axis and under such circumstances the decoupled band can drop below the completely paired normal band at higher spins, thus causing back-bending. It thus seems that the single particle

orbitals with low Ω and high j near the Fermi surface play an important role in the anomalous effect occurring at some critical high angular momentum.

It is of interest to gain an insight into the intrinsic structure of the high spin states from a microscopic theory. Some attempts are made in this direction to understand the spectroscopy of high spin states in the framework of many-body variational formalism with good angular momentum (Faessler *et al* 1974; Warke and Gunye 1975 and 1976). Apart from the complication of angular momentum projection, there is yet another complication due to the number projection (Grummer *et al* 1975; Gunye and Warke 1979) from the pair-correlated variational state. It is found (Gunye and Warke 1979) that the number conservation in each projected angular momentum state improves the quality of the agreement between the theoretical and experimental results.

In this paper, we report the results of the microscopic calculations on the yrast states of the three osmium nuclei ^{182,184,186}Os. The calculations are performed in a variational formulation with angular momentum projection by conserving the nucleon numbers in each projected state. The details of calculation are given in § 2. The results are discussed in § 3 and the conclusions presented in § 4.

2. Details of calculation

A realistic nuclear structure calculation in a microscopic many-body formalism requires a dynamical treatment of a large number of nucleons in a large configuration space. The computational difficulties involved in such a realistic calculation can be reduced by employing a simpler many-body Hamiltonian. In this paper, we use the quadrupole plus pairing interaction Hamiltonian,

$$H = \Sigma\, \epsilon_\alpha\, a^+_\alpha\, a_\alpha - \tfrac{1}{2}\,\chi\, \Sigma\, q^\mu_{\alpha\gamma} q^\mu_{\delta\beta} a^+_\alpha a^+_\beta a_\delta\, a_\gamma$$

$$-\tfrac{1}{2}\, G\, \Sigma(-)^{j_\alpha - m_\alpha + j_\gamma - m_\gamma}\, a^+_\alpha\, a^+_{\bar{\alpha}}\, a_\gamma\, a_{\bar{\gamma}}. \qquad (1)$$

Where q^μ is the quadrupole operator and χ and G are the strengths of quadrupole and pairing interactions respectively. The subscript α in (1) denotes all the quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ necessary for the specification of a single particle state with energy ϵ_α. The parameters of this Hamiltonian in (1) are determined by Kumar and Baranger (1968) from the study of equilibrium deformations of heavy nuclei and we use their values in the present calculations. The intrinsic variational wavefunction is assumed to be axially symmetric in view of the fact that the nuclei under investigation are found (Kumar and Baranger 1968) to prefer axially symmetric equilibrium deformations.

The trial wavefunction is taken to be the good angular momentum state

$$\Psi^J_M\,(\beta,\ \Delta_p,\ \Delta_n,\ \lambda_p,\ \lambda_n)$$

projected from the intrinsic BCS state

$$\Phi_0\,(\beta,\ \Delta_p,\ \Delta_n,\ \lambda_p,\ \lambda_n).$$

The deformation β, the pairing gaps Δ_p and Δ_n and the chemical potentials λ_p and λ_n are the variational parameters for each angular momentum state J. The suffixes p and n refer to proton and neutron respectively. The nuclear energy E^J in the state J is calculated by minimising the expectation value of the Hamiltonian in (1) in the projected state Ψ^J_M. The energy minimum is found by varying the parameters β, Δ_p, Δ_n, λ_p and λ_n of the projected state. For each set of the values of β, Δ_p and Δ_n, the chemical potentials λ_p and λ_n are varied so as to yield the correct number Z of protons and N of neutrons for each angular momentum state J. The calculated numbers of nucleons are very sensitively dependent on λ_p and λ_n and it is necessary to incorporate very fine variations of λ_p and λ_n in the variational procedure so as to obtain the correct number in each angular momentum state. In the present calculations, we have achieved an accuracy upto the fourth decimal place in the nucleon numbers computed in each J state.

The values of the strength parameters χ and G are estimated by Kumar and Baranger (1968) in a truncated configuration space of two major shells N=4 and 5 for protons and N=5 and 6 for neutrons by assuming an inert core with 40 protons and 70 neutrons. The assumption of the inert core necessitates the modification of the nucleon charges and excitation energies. As is the standard practice (Kumar and Baranger 1968), we replace bare nucleon charges by effective charges to simulate the effects of core-polarisation and configuration truncation. The simplest way to incorporate the effect of the neglected core on the projected energies is by renormalising the calculated energy spectrum. We achieve it by introducing a parameter, namely the moment of inertia I_{core} of the core. The moment of inertia of the nucleus is assumed to be the sum of the moment of inertia I_{core} of the core and I_{calc} of the outer nucleons. The energy E^J_{calc}, computed by considering only the outer valence nucleons can be expressed as

$$E^J_{\text{calc}} = J(J+1)\frac{\hbar^2}{2\,I_{\text{calc}}}, \tag{2}$$

and similarly, the corrected or the renormalised energy can be expressed as

$$E^J_{\text{norm}} = J(J+1)\frac{\hbar^2}{2(I_{\text{core}} + I_{\text{calc}})} \tag{3}$$

Since the calculated energy E^J_{calc} deviates from the pure rotational pattern, the moment of inertia I_{calc} varies with the angular momentum J. The moment of inertia I_{core} however, may not vary with J and can be assumed to be constant, at least for a set of states. The present calculations in the three osmium nuclei indicate that I_{core} is nearly constant for all values of J.

3. Results and discussion

The calculated energy spectrum in each of the three nuclei ^{182,184,186}Os is renormalised by choosing the parameter I_{core} so as to obtain an overall agreement with the experi-

mental energy spectrum, rather than reproducing the excitation energy of a particular state. It is gratifying to note that the experimental energy spectra of the yrast states upto $J \leqslant 22$ in the three nuclei are reproduced well by employing nearly a constant value $I_{\text{corc}} = (11{\cdot}5 \pm 0{\cdot}5)\hbar^2/\text{MeV}$ in the renormalisation calculation. The renormalised energy and the variational parameters β, $\triangle_p$ and $\triangle_n$ corresponding to the minimum of energy for each angular momentum state J are shown in tables 1 to 3 for ^{182}Os,

Table 1. The deformation β, the pairing gaps $\triangle_p$ and $\triangle_n$ the energy E_{norm} obtained from the renormalisation procedure, the experimental energy E_{expt} the quadruple moment $Q(J)$ and the $B(E2; J \to J-2)$ value for each angular momentum state J in ^{182}Os.

J	β	$\triangle_p$ (MeV)	$\triangle_n$ (MeV)	E_{norm} (MeV)	E_{expt} (MeV)	$-Q(J)$ (eb)	$B(E2; J \to J-2)$ (e^2b^2)
0	0·24	0·79	0·84	0·00	0·00	—	—
2	0·24	0·79	0·84	0·12	0·13	1·44	0·51
4	0·26	0·81	0·77	0·37	0·40	1·96	0·84
6	0·26	0·81	0·77	0·75	0·79	2·15	0·93
8	0·26	0·81	0·54	1·20	1·28	2·26	0·99
10	0·26	0·81	0·31	1·71	1·81	2·34	1·02
12	0·24	0·79	0·04	2·21	2·34	2·25	0·92
14	0·24	0·55	0·04	2·75	2·84	2·27	0·92
16	0·24	0·31	0·00	3·34	3·32	2·29	0·93
18	0·24	0·08	0·00	3·97	3·85	2·29	0·93
20	0·24	0·08	0·00	4·65	4·48	2·29	0·93

Table 2. The deformation β, the pairing gaps $\triangle_p$ and $\triangle_n$, the energy E_{norm} obtained from the renormalisation procedure, the experimental energy E_{expt}, the quadrupol moment $Q(J)$ and the $B(E2; J \to J-2)$ value for each angular momentum state J in ^{184}Os. The numbers in bracket indicate the corresponding experimental values.

J	β	$\triangle_p$ (MeV)	$\triangle_n$ (MeV)	E_{norm} (MeV)	E_{expt} (MeV)	$-Q(J)$ (eb)	$B(E2; J \to J-2)$ (e^2b^2)
0	0·22	0·76	0·89	0·00	0·00	—	—
2	0·22	0·76	0·89	0·13	0·12	1·38	0·47
						$(2{\cdot}4 \pm 1{\cdot}1)^a$	$(0{\cdot}64 \pm 0{\cdot}12)^a$
4	0·24	0·77	0·81	0·41	0·38	1·90	0·79
6	0·24	0·77	0·81	0·82	0·77	2·08	0·87
8	0·24	0·54	0·57	1·31	1·27	2·18	0·91
10	0·24	0·54	0·32	1·84	1·87	2·25	0·94
12	0·24	0·34	0·04	2·38	2·55	2·30	0·96
14	0·22	0·08	0·00	3·02	3·26	2·23	0·89
16	0·22	0·08	0·00	3·62	3·79	2·24	0·90
18	0·22	0·08	0·00	4·32	4·34	2·24	0·90
20	0·22	0·08	0·00	5·09	5·00	2·23	0·90
22	0·22	0·08	0·00	5·93	5·74	2·23	0·90

[a]Lane and Saladin 1972.

Table 3. The deformation β, the pairing gaps Δ_p and Δ_n, the energy E_{norm} obtained from the renormalisation procedure, the experimental energy E_{expt}, the quadrupole moment $Q(J)$ and the $B(E2; J\to J-2)$ value for each angular momentum state J in ^{186}Os. The numbers in bracket indicate the corresponding experimental values.

J	β	Δ_p (MeV)	Δ_n (MeV)	E_{norm} (MeV)	E_{expt} (MeV)	$-Q(J)$ (eb)	$B(E2; J\to J-2)$ (e^2b^2)
0	0·20	0·76	0·97	0·00	0·00	—	—
2	0·20	0·76	0·97	0·14	0·14	1·40	0·48
						(1·47±0·54)[a]	(0·57±0·08)[a]
4	0·22	0·74	0·91	0·43	0·43	1·78	0·70
							(0·80±0·12)[b]
6	0·22	0·74	0·64	0·87	0·87	1·96	0·78
8	0·22	0·52	0·64	1·38	1·42	2·06	0·82
10	0·22	0·30	0·36	1·99	2·06	2·12	0·85
12	0·22	0·07	0·36	2·59	2·78	2·17	0·87
14	0·22	0·07	0·04	3·27	3·44	2·18	0·88
16	0·22	0·07	0·00	3·97	3·93	2·24	0·90
18	0·22	0·07	0·00	4·69	4·49	2·24	0·90
20	0·22	0·07	0·00	5·47	—	2·23	0·91

[a]Lane and Saladin 1972
[b]Milner *et al* 1971

^{184}Os and ^{186}Os respectively. It is seen from these tables that the renormalised energies are in good agreement with the corresponding experimental energies, the maximum deviation being about 200 keV. In order to visualise the agreement between the calculated and experimental energy spectra at a glance, we have displayed the energy spectra of ^{182,184,186}Os in figure 1. The anomalous back-bending effect is observed experimentally in all the three osmium isotopes under investigation. The back-bending behaviour is conventionally illustrated by the familiar S-shaped plot of moment of inertia I as a function of the square of the rotational frequency ω. It should be stressed here that the calculated energies should be in very precise agreement with the experimental energies so as to reproduce the characteristic experimental S-shaped curve. Since the microscopic calculations with the simple quadrupole plus pairing force model cannot yield such precise agreement, one should expect to see only the trend of I vs ω^2 curve. The theoretical and experimental S-shaped curves in ^{182}Os are shown in figure 2 for comparison.

It is seen from the results shown in table 1 for ^{182}Os that the deformation β is 0.24 for all the yrast states except those with $4\leqslant J\leqslant 10$ for which the deformation is 0.26. It is seen that the deformation in the osmium isotopes under investigation decreases with the increase in neutron number. Thus, in the case of ^{184}Os shown in table 2 the deformation is 0.22 for the yrast states with $J=0$, 2 and $J\geqslant 14$ whereas it is 0.24 for the yrast states with $4\leqslant J\leqslant 12$. In the case of ^{186}Os shown in table 3, the deformation is 0.20 for $J=0$ and 2 states and then increases to 0.22 for the higher yrast states with $J\geqslant 4$.

The pairing gap Δ_p for protons in the case of ^{182}Os (table 1) remains nearly constant for all states with $J\leqslant 12$ and then reduces smoothly with increasing J values. In the

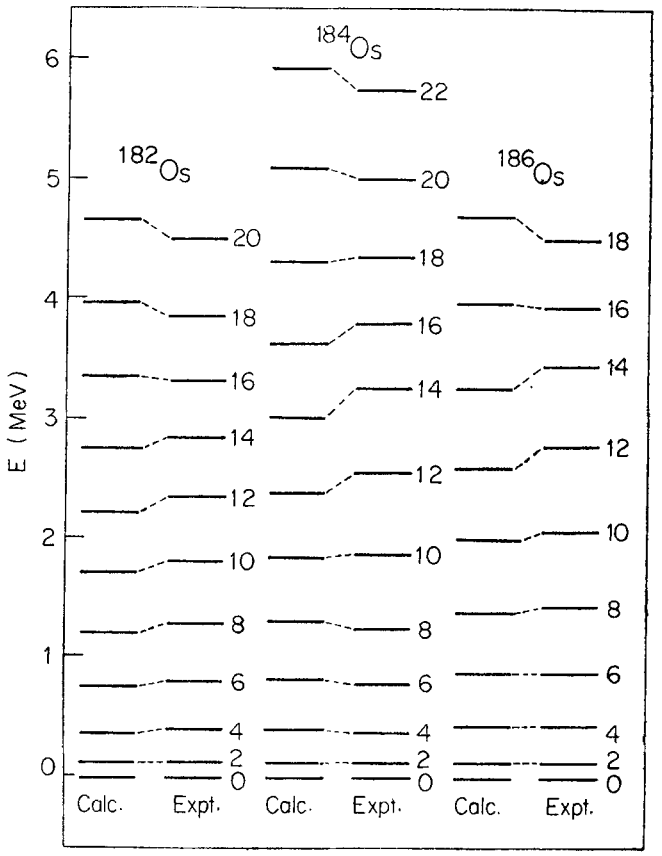

Figure 1. The calculated and the experimental energy spectra of ^{182,184,186}Os.

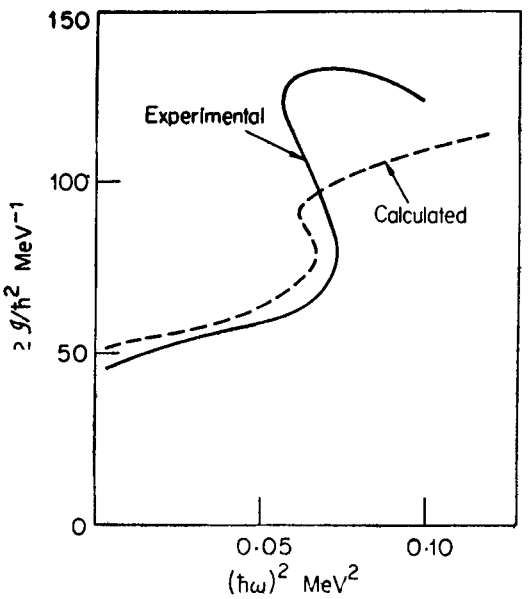

Figure 2. Moment of inertia as a function of the square of the rotational frequency plotted in the case of ^{182}Os.

case of ^{184}Os and ^{186}Os shown in tables 2 and 3 respectively, one finds that the gap $\triangle_p$ remains constant for the yrast states with $J \leqslant 6$ and then falls smoothly for higher J values. It should be mentioned here that the pairing gap $\triangle_p$ in all the three nuclei, though decreasing with increasing J, does not become zero but maintains a small non-zero value for higher yrast states. This behaviour in the variation of $\triangle_p$ is rather different from that observed in the variation of $\triangle_n$, the pairing gap for neutrons. It is seen from tables 1 to 3 that $\triangle_n$ in all the three isotopes ^{182,184,186}Os remains fairly constant for the first few yrast states and then falls smoothly to zero at $J=16$, remaining zero for all the higher yrast states. The experimentally observed back-bending in these nuclei may thus be due to the substantial reduction in both the pairing gaps $\triangle_p$ and $\triangle_n$ for the yrast states with $J \geqslant 14$. The unpaired band becomes energetically lower than the paired ground band for the high spin states with $J \geqslant 14$.

The back-bending in a particular nucleus would also depend in detail on the structure of single-particle orbitals near the Fermi surface of neutrons and protons. It is thus worthwhile to discuss the single-particle states at the Fermi surface of nucleons. This may give some insight into the phenomenological approach based on decoupling of bands (Stephens and Simon 1972) where the coriolis effects in high-j single particle orbits are assumed to play an important role. The proton and neutron orbitals near the Fermi surface of the three osmium nuclei are displayed in figures 3 and 4 respectively. It is seen from figure 3 that, in the range of deformations relevant for these nuclei, the high-j states near the proton Fermi surface are $h_{11/2}$ with $\Omega = 9/2$

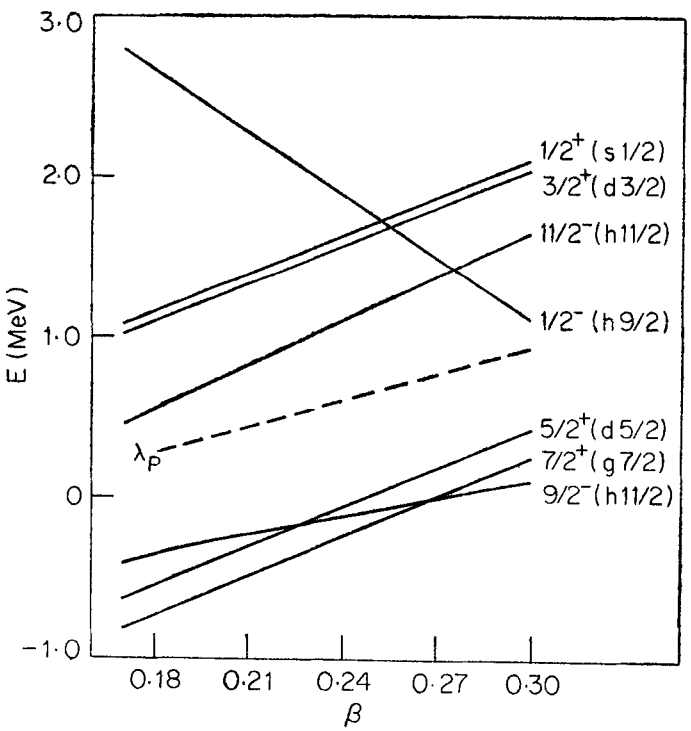

Figure 3. Energies of the deformed orbitals near the Fermi level λ_p for protons (dotted curve) in osmium nuclei are shown as a function of deformation β. The Ω^π value and the predominant basis state with the largest amplitude in the wavefunction of the orbital are shown on the right.

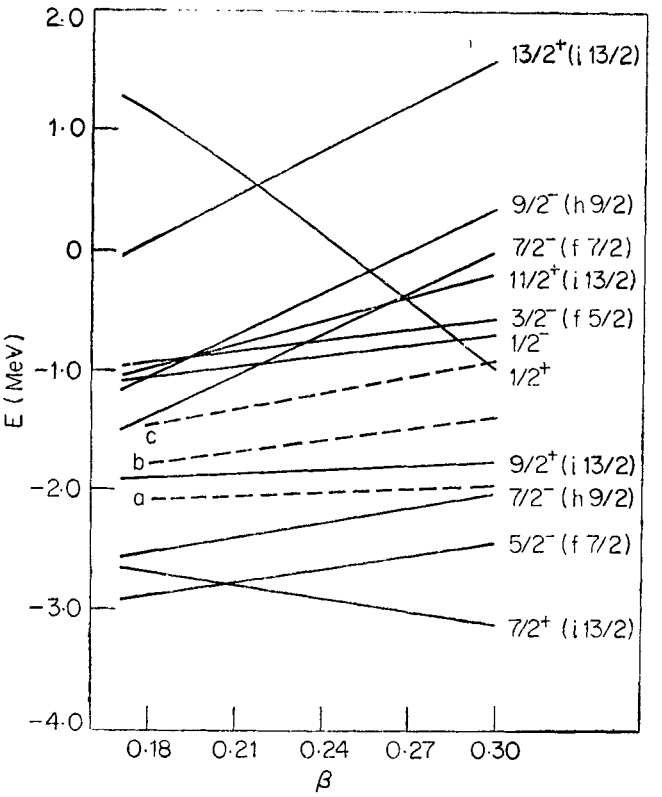

Figure 4. Energies of the deformed orbitals near the Fermi level for neutrons in osmium nuclei are shown as a function of deformation β. The Ω^{π} value and the predominant basis state with the largest amplitude in the wavefunction of the orbital are shown on the right. The dotted curves a, b and c indicate the Fermi levels for neutrons in ^{182,184,186}Os respectively.

and 11/2 substates and $h_{9/2}$ with $\Omega=1/2$ substate, though the latter is rather far away. For higher deformations, however, this $\Omega=\frac{1}{2}$ substate from $h_{9/2}$ orbital is quite close to the Fermi surface. Since the coriolis force is strong for low-Ω substates of high-j orbitals near the Fermi surface, the decoupling and subsequent realignment of a pair of protons in $\Omega=\frac{1}{2}$ ($h_{9/2}$) orbit may also contribute to the observed back-bending in ^{182,184,186}Os. The situation regarding the neutron Fermi surface of these osmium nuclei can be seen from figure 4. The only high-j states near the neutron Fermi surace are the $\Omega=9/2$ substate from predominantly $i_{13/2}$ orbit and $\Omega=7/2$ substate from predominantly $h_{9/2}$ orbit. Since the coriolis force is weak for high Ω substates, the observed back-bending at comparatively low critical angular momentum $j_c \simeq 12$ cannot be understood in terms of the decoupled neutron pair from the $i_{13/2}$ orbit. It may be noted here that many rare-earth nuclei exhibiting back-bending have the low $\Omega(\Omega \leqslant 5/2)$ substate of the $i_{13/2}$ orbital near the neutron Fermi surface.

The quadrupole moment $Q(J)$ and the $B(E2; J \rightarrow J-2)$ values are calculated by employing the number-conserved projected wavefunctions for each angular momentum state. The effects of core-polarisation are simulated by ascribing effective charges (Kumar and Baranger 1968) to protons and neutrons. The computed values of $Q(J)$ and $B(E2)$ are displayed in tables 1 to 3 for ^{182,184,186}Os respectively. The corresponding experimental values, however, are not available except for a few states

in ^{184,186}Os (Milner *et al* 1971; Lane and Saladin 1972). The calculated values agree well with these available experimental values.

In order to understand the connection between the Böhr-Mottelson collective model and the microscopic approach followed in this paper, we have calculated the intrinsic quadrupole moment Q_0 from $Q(J)$ values as well as from $B(E2)$ values using the following relations from the collective model:

$$Q(J) = -Q_0 \frac{J}{2J+3},$$

$$B(E2, J \to J-2) = \frac{15}{32\pi} Q_0^2 \frac{J(J-1)}{(2J-1)(2J+1)}.$$

It is found that in all the three nuclei ^{182,184,186}Os, the $Q_0(Q)$ value extracted from the quadrupole moment $Q(J)$ agrees well with the corresponding value $Q_0(E2)$ obtained from $B(E2)$ value. In general, $Q_0(Q)$ is very slightly less than the corresponding $Q_0(E2)$ value, the maximum difference being about 0.1 eb. The present calculations indicate a systematic trend in the behaviour of both Q_0 values as deformation changes for different angular momentum states. The average value of the intrinsic quadrupole moment Q_0 is found to be about 5.0 eb with slight variations with the deformations for different states in the three nuclei.

4. Conclusions

The microscopic formalism of variation with number-conserved projected states is applied to study the yrast states of ^{182,184,186}Os. The energy spectra, quadrupole moments and $B(E2)$ values are calculated by employing the Hamiltonian with quadrupole plus pairing interactions. The deformation β, pairing gaps $\triangle_p$ and $\triangle_n$ and the chemical potentials λ_p and λ_n are varied to obtain the energy minimum and to conserve the number of nucleons in each angular momentum state. The effect of core polarisation is simulated by ascribing effective charges to the nucleons and by introducing the moment of inertia of the core to renormalise the energy spectra. The experimental energy spectra of the yrast states in the three osmium isotopes under consideration are reproduced well by using nearly a constant value for the moment of inertia of the core. The present calculations indicate that the observed back-bending in the three nuclei is probably due to the near-vanishing of both the pairing gaps $\triangle_p$ and $\triangle_n$ for the yrast states with $J \geqslant 14$ so as to make the unpaired band energetically lower than the paired ground band for these high spin states. The decoupling and subsequent realignment of a pair of protons from the $\Omega = \frac{1}{2}(h_{9/2})$ orbital in the vicinity of the Fermi surface would also contribute to the observed back-bending in the three osmium nuclei.

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