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Quantum chromodynamics corrections to polarised deep-inelastic electron-nucleon scattering

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Abstract. Quantum chromodynamics corrections to order α_S (the running coupling constant), using the quark-parton approach are calculated for the spin-dependent structure functions in deep-inelastic polarised electron-nucleon scattering. Consequences of these corrections for the Bjorken sum rule and the asymmetry in the case of longitudinally polarised (with respect to the beam) nucleons is discussed which could provide possible tests of quantum chromodynamics. Comparison of our results with the moments of the flavour non-singlet contribution to the structure functions obtained using operator product expansion is also given.

Keywords. Quantum chromodynamics; parton model; deep inelastic theory; strong interactions; quark-parton approach; electron-nucleon scattering.

1. Introduction

Quantum chromodynamics (QCD) has recently emerged as a viable candidate for an underlying field theory of strong interactions (Politzer 1974; Marciano and Pagels 1978, provide excellent reviews). Since it is an asymptotically free theory, it can be reliably taken to provide the leading behaviour in problems where space-like momenta ($q^2 = - Q^2 < 0$) are involved (Gross and Wilczek 1973; Politzer 1973). In particular, to the lowest order, in the effective coupling constant $\alpha_s(Q^2)$, it gives scaling (Bjorken 1969) in deep-inelastic lepton-hadron processes and provides a justification for the naive quark-parton model (Feynman 1972). To higher orders in α_s, it predicts logarithmic violations of Bjorken scaling which can be computed perturbatively. These corrections, to order α_s, have been calculated by many authors (Altarelli *et al* 1978 and 1979 and references therein; Bardeen *et al* 1978; Altarelli and Parisi 1977; Floratos *et al* 1977) for the spin-averaged structure functions which enter lepto-production. The experimentally observed scaling violations (Anderson *et al* 1977; Barish *et al* 1978) seem to be in accord with QCD expectations. It is thus necessary to consider other processes and effects which could provide further tests of QCD. To this end we have calculated the order α_s corrections to polarised electron-nucleon scattering where (though not adequate enough for phenomenology) some data are available at present (Alguard *et al* 1978).

Basically, two approaches have been used to calculate order α_s effects. One is the operator product expansion (OPE) method which, however, only gives the moments of the structure functions. The other approach is an extension of the quark-parton model to include order α_s contributions. We will follow the quark-parton approach as this to our mind is simpler and physically more appealing. Moreover, this approach directly gives the desired structure functions themselves and hence their moments as well.

Both the approaches have given equivalent results for the spin-averaged structure functions as can be seen from the references cited above. However in calculating the spin-dependent scaled structure functions G_1 and G_2 (see § 2) the quark-parton approach suffers a limitation because one works in the limit of zero parton mass and zero parton transverse momentum. In this limit the initial parton (quark or gluon) in the nucleon would have its spin aligned with its momentum, that is have a definite helicity. In the infinite momentum frame (given by the electron-nucleon centre of mass) the energetic nucleon is viewed as made of these partons. Since one neglects masses and transverse momentum, these partons would give a nucleon with definite helicity in the leading order. For such a nucleon the hadronic tensor is given in terms of only one structure function G_1. Consequently, in the quark-parton approach, as for the naive quark-parton model, to order α_s also one is able to obtain only G_1 with G_2 effectively zero. Of course this is not so for the OPE method where one obtains corrections to the moments of both G_1 and G_2 (Ahmed and Ross 1976; Kodaira *et al* 1978). However for phenomenology the lack of knowledge of G_2 is not serious as its contribution to the asymmetry in the case of longitudinally polarised protons (with respect to the beam), in general, is down by a factor $1/Q^2$ compared to that of G_1 (Hey and Mandula 1972). For transversely polarised protons, the contribution of G_1 and G_2 is of the same order but the asymmetry, in this case, is itself of order Q^{-1}. So we focus our attention on G_1 throughout.

The quark-parton approach is outlined in § 2, where also the two regularisation prescriptions used to calculate the structure functions are discussed. The results of the computations together with a brief sketch of the calculations is presented in § 3. Consequences of these results for the Bjorken sum rule (Bjorken 1970) and for the asymmetry for longitudinally polarised protons using G_1 is discussed in the last section. Emphasis is placed on consequences which are independent of the regularisation procedure.

2. The quark-parton approach

The cross-section for inclusive scattering of a polarised electron on a polarised nucleon (N), $e^-(k, s_e) + N(P, s) \rightarrow e^-(k') +$ anything, is given by

$$d\sigma(e\, s_e, Ns) = \frac{4\alpha^2 M}{Q^4} \cdot \frac{1}{[(k.P)^2 - m_e^2 M^2]^{1/2}} \frac{d^3k'}{k'^0} L_{\mu\nu}(s_e)\, T^{\mu\nu}(s), \tag{1}$$

where k, P and k' are the four-momenta of the incident electron, the nucleon N and the final electron respectively. Moreover s_e^μ and s^μ denote the spin four vectors of the incoming electron and nucleon while the other spins are averaged over. The space like four-momentum $q = k - k'$ is transferred from the leptonic vertex to the hadronic vertex by the virtual photon, so that $Q^2 = -q^2 > 0$. Furthermore, M is the nucleon mass, m_e the electron mass, k'^0 the energy of the final electron and α the fine-structure constant. The leptonic tensor $L^{\mu\nu}$ and the hadronic tensor $T^{\mu\nu}$ come from the leptonic and hadronic vertices respectively. Both these tensors have symmetric and antisymmetric parts, under $\mu \longleftrightarrow \nu$, which are denoted by superscripts S and A. Thus

$$L_{\mu\nu} = L^{(S)}_{\mu\nu} + L^{(A)}_{\mu\nu}(s_e), \tag{2a}$$

$$T_{\mu\nu} = T^{(S)}_{\mu\nu} + T^{(A)}_{\mu\nu}(s). \tag{2b}$$

Only the antisymmetric parts depend on the spin polarisation; the symmetric parts give the spin averaged contribution. Explicitly

$$L^{(S)}_{\mu\nu} = \tfrac{1}{2}\,[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}\,(k.k' - m_e^2)], \tag{3a}$$

$$L^{(A)}_{\mu\nu} = -\frac{i}{2} m_e\, \epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, s^\beta_e. \tag{3b}$$

The full hadronic tensor is

$$T_{\mu\nu} = \frac{4\pi^2}{M} P^0 \int d^4 x \exp(iq\cdot x)\, \langle P, s\,|\,J_\mu(x)\, J_\nu(0)\,|\,P, s\rangle \tag{4}$$

$$= \frac{P^0}{M} \sum_n \langle P, s\,|\,J_\mu\,|\,n\rangle \langle n\,|\,J_\nu\,|\,P, s\rangle\, (2\pi)^6\, \delta^4\,(P + q - P_n), \tag{4a}$$

where J_μ is the hadron electromagnetic current and P^0 is the nucleon energy. Current conservation and Lorentz invariance, etc. enable one to write (Altarelli and Parisi 1977)

$$T^{(S)}_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(P_\mu - \frac{P\cdot q q_\mu}{q^2}\right)\left(P_\nu - \frac{P\cdot q q_\nu}{q^2}\right)\frac{W_2}{M^2}, \tag{4b}$$

and $$T^{(A)}_{\mu\nu} = -\, i\, \epsilon_{\mu\nu\alpha\beta}\, q^\alpha \left[s^\beta \frac{V_1}{M} + (p\cdot q s^\beta - q\cdot s P^\beta)\frac{V_2}{M^3}\right]. \tag{4c}$$

The structure functions W_1, W_2, V_1 and V_2 are in general functions of Q^2 and $\nu = M^{-1} P.q$. However, in the scaling limit (of large ν and Q^2)

$$\mathscr{F}_1 \equiv 2MW_1 \qquad \mathscr{F}_2 \equiv \nu W_2/x_H, \tag{5a}$$

$$G_1 \equiv \nu V_1, \qquad MG_2 \equiv \nu^2 V_2, \tag{5b}$$

are expected to be functions only of the hadronic scaling variable

$$x_H = Q^2/2P\cdot q = Q^2/2M\nu. \tag{6}$$

In the quark-parton approach the hadronic tensor, for a fast nucleon, is given as the incoherent sum of the contributions of the individual partons (quarks and gluons) weighted by their distributions $G_N^q(y)$ (for a quark q) and $G_N^g(y)$ (for a gluon g) where y is the fraction of the nucleon momentum the parton carries. Thus, to order α_s

$$T_{\mu\nu} = \sum_q \int_{x_H}^{1} dy\, G_N^q(y)\, [T_{\mu\nu}(q \to q) + T_{\mu\nu}(q \to qg)] + \int_{x_H}^{1} dy\, G_N^g(y)\, T_{\mu\nu}(g \to q\bar{q}). \tag{7}$$

The sum over q includes sum over quarks of different flavours including antiquarks $\bar{q} \cdot T_{\mu\nu}(q \to q)$ is the contribution of an individual quark calculated from figure 1a which is independent of the quark-gluon coupling constant g_s ($\alpha_s = g_s^2/4\pi$) and is just the contribution of the naive quark-parton model. Explicitly, (for quark q of charge e_q)

$$T_{\mu\nu}(q \to q) = e_q^2 \frac{P^0}{M p_1^0} \int \frac{d^3 p_2}{p_2^0}\, \delta^4 (p_1 + q - p_2)\, |M(q \to q)|^2_{\mu\nu}, \tag{8}$$

where the initial quark spin is s_1 and its four-momentum $p_1^\mu \simeq y P^\mu$ while the final quark (of momentum p_2) spin is summed over. This and colour sum and average are included in $|M(q \to q)|^2_{\mu\nu}$. A further average over s_1 will give W_1 and W_2. As is well-known the contribution of figure 1a gives

$$\mathscr{F}_1^N(x_H) = \mathscr{F}_2^N(x_H) = \sum_q e_q^2 \int_{x_H}^1 \frac{dy}{y}\, G_N^q(y)\, \delta(1 - z), \tag{9}$$

where $x^H = yz$ and $z = Q^2/2p_1 . q$ is the parton scaling variable. This is the leading contribution obtained in the limit of zero mass (m_q) and zero transverse momentum for the quark-partons. In this limit the quark would be longitudinally polarised, that is $m_q s_1^\mu \simeq \eta_1 p_1^\mu$, $\eta_1 = \pm 1$, one obtains for the spin-dependent structure functions (from figure 1a).

$$T_{\mu\nu}(q \to q) = - i e_q^2 \eta_1 \frac{\delta(1 - z)}{2 M^2 \nu y^2} \epsilon_{\mu\nu\alpha\beta}\, q^\alpha p_1^\beta . \tag{10}$$

Thus giving the scaled functions, $G_2^N = 0$ and

$$2 G_1^N(x_H) = \sum_q e_q^2 \int_{x_H}^1 \frac{dy}{y} \delta(1 - z)\, [G_{N\uparrow}^{q\uparrow}(y) - G_{N\uparrow}^{q\downarrow}(y)], \tag{11}$$

where $G_{N\uparrow}^{q\uparrow}(y)$ ($G_{N\uparrow}^{q\downarrow}$) give the number of quarks of type q with +ve (−ve) helicity in a +ve helicity nucleon with momentum fraction between y and $y + dy$. Clearly $G_{N\uparrow}^{q\uparrow} + G_{N\uparrow}^{q\downarrow} = G_N^q$ is what enters in $\mathscr{F}_1$ and $\mathscr{F}_2$. Moreover, in the popular notation for the distributions in the proton, $G_p^u(y) = u(y)$, etc.

The other two contributions in (7) arise from taking into account one gluon effects and are of order α_s. $T_{\mu\nu}(q \to qg)$ has two contributions (i) from figures 2a and 2b due to the emission of a ‘ real ’ gluon and (ii) from the exchange of a virtual gluon which is obtained from the interference of figures 1a and 1b. The latter or the 'virtual' contribution will be proportional to $\delta(1-z)$. The real contribution from figure 2 is

$$T_{\mu\nu}^{(\text{real})}(q\to qg)=e_q^2\frac{P^0}{Mp_1^0}\left(\frac{4}{3}\frac{\alpha_s}{2\pi}\right)\frac{1}{2\pi}\int\frac{d^3p_2}{p_2^0}\frac{d^3h}{h^0}\delta^4(p_1+q-p_2-h)\left|M(q\to qg)\right|^2_{\mu\nu}. \tag{12}$$

The factor $\frac{4}{3}$ comes from sum and average over colour. In $|M(q\to qg)|^2_{\mu\nu}$ sum over the final quark and gluon (which convert into hadrons) spins is understood, while the average over initial quark spin is taken or not taken according to whether one wants $\mathscr{F}_1$ and $\mathscr{F}_2$ or G_1 and G_2.

The contribution of the interaction of a gluon (in the nucleon) with the electromagnetic current by converting into a $q\bar{q}$ — pair arises for the first time in order α_s.

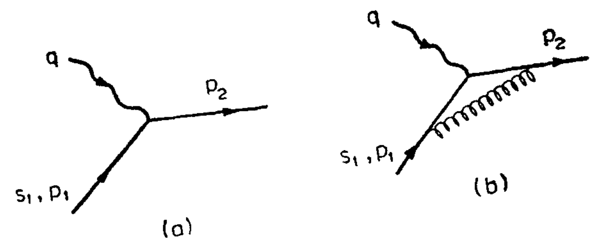

Figure 1.

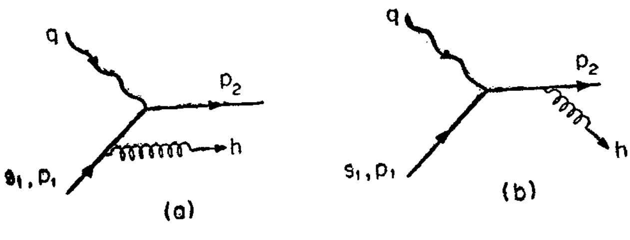

Figure 2.

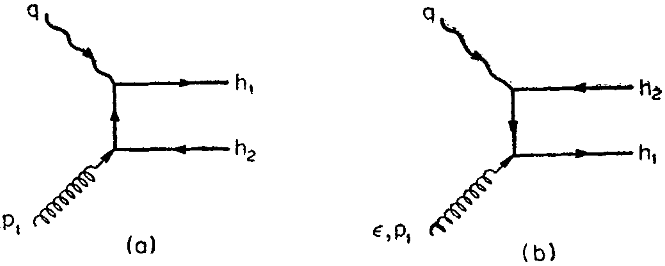

Figure 3.

Figures 1, 2 and 3. The quark-parton diagrams giving the contributions upto order α_s to the hadronic vertex in QCD. The wavy, solid and coiled lines represent the virtual photon, quarks and gluons respectively. The four momenta as indicated are used in the text. The spin (s_1) and polarisation (ϵ) vectors of the initial quark and gluon are also indicated.

This contribution, calculated from figures 3a and 3b, is given by

$$T_{\mu\nu}(g\to q\bar{q}) = \left(\frac{1}{2}\sum_q e_q^2\right)\frac{P^0}{Mp_1^0}\left(\frac{1}{2}\frac{\alpha_s}{2\pi}\right)\frac{1}{2\pi}\int (d^3h_1/h_1^0)\,(d^3h_2/h_2^0)$$

$$\times\, \delta^4(p_1+q-h_1-h_2)\,|\,M(g\to q\bar{q})\,|^2_{\mu\nu}. \tag{13}$$

The factor $\frac{1}{2}$ with α_s is the colour factor, while the sum over the final q and $\bar{q}$ spins is understood. Since the sum over q runs over q and $\bar{q}$ a factor $\frac{1}{2}$ has been included to avoid double counting. Again, the average over the initial gluon polarisation will give $\mathcal{F}_1$ and $\mathcal{F}_2$ while to obtain G_1 and G_2 one has to consider a polarised initial gluon. From equations (7) to (13) the generalisation of the quark-parton approach to higher orders in α_s involving two or more gluons should be clear. We now turn to the method of computation of the order α_s corrections.

The order α_s contributions to $\mathcal{F}_1$ and $\mathcal{F}_2$ in the quark-parton approach have been calculated by Altarelli *et al* (1977) and (1978). As is well-known $\mathcal{F}_1$ and $\mathcal{F}_2$ become functions of x_H and of

$$t \equiv \ln Q^2/\mu^2, \tag{14}$$

where μ is some normalisation mass. These calculations are carried out in the limit of massless partons and infrared singularities appear in the massless theory when a parton emits a collinear parton. One thus needs a regularisation prescription. The leading behaviour, that is, the t-dependent part is independent of the regularisation scheme but the non-leading part depends on the scheme chosen. To illustrate, the general form of the flavour non-singlet contribution is

$$\mathcal{F}_i^N(x_H, t) = \sum_q e_q^2 \int_{x_H}^{1} \frac{dy}{y}\, G_N^q(y)\left[\delta(1-z) + \frac{\alpha_s}{2\pi} t P_{qq}(z) + \alpha_s f_{q,i}(z)\right] \qquad i = 1, 2 \tag{15}$$

The leading contribution given by $P_{qq}(z)$ (obtained earlier by Altarelli and Parisi 1977) is independent of the regularisation prescription and is the same for both $\mathcal{F}_1$ and $\mathcal{F}_2$. So its presence does not affect the Callan-Gross relation $\mathcal{F}_1 = \mathcal{F}_2$. But neither of these statements is true for the non-leading part $f_{q,i}(z)$.* However, quantities which are well defined for the massless theory should also be independent of regularisation. For example, this is true for the difference

$$\alpha_s\,[f_{q,2}(z) - f_{q,1}(z)] = \frac{4}{3}\frac{\alpha_s}{2\pi}\cdot 2z, \tag{16}$$

so that the violation of the Callan-Gross relation is independent of the regularisation scheme adopted. This is borne out by the explicit calculations (Altarelli *et al* 1977, 1978) using the following two regularisation schemes for extracting the leading infrared divergences which we will use in our calculations as well.

* See concluding paragraph.

(i) One writes the amplitudes as for the massless theory but in performing the phase space integrals one takes the initial parton off mass-shell, with $p_1^2 < 0$. In the case of $T_{\mu\nu}$ $(q \to qg)$ it is also necessary to take the final quark off mass-shell with $p_2^2 > 0$ in the intermediate stages of the calculation. One then computes this contribution in limit $\omega = -p_1^2/Q^2$ and $\omega' = p_2^2/Q^2$ tending to zero. The infrared singularities then appear as ln ω and ln ω', respectively. We will refer to this regularisation prescription as $R_{\rm I}$.

(ii) One writes amplitudes as for the massless theory and works with massless partons ($p_1^2 = h^2 = p_2^2 = 0$, etc.) But, now the divergences are regulated by going from 4 to n dimensions. One then works in the limit of $\epsilon = (2 - n/2) \to 0$. The singularities now appear as $1/\epsilon$, $1/\epsilon^2$, etc. This dimensional regularisation prescription will be referred to as $R_{\rm II}$.

Scheme $R_{\rm I}$ has some appeal because the parton in the nucleon is off mass-shell and space-like. Moreover, other calculations using this are available. Scheme $R_{\rm II}$ has the advantage that it is known to preserve gauge invariance and results in considerable technical simplification. Moreover operator product expansion results using it are available for comparison.

We now turn to the presentation of our calculations and results for the spin-dependent structure functions.

3. Calculations and results

A detailed account of the calculation of the spin averaged structure functions using the prescriptions $R_{\rm I}$ and $R_{\rm II}$ is already available (Altarelli *et al* 1977, 1978). The calculations for the spin-dependent structure functions are very similar and so we give only a brief sketch of the calculational details.

3.1. *Flavour non-singlet case*

The flavour non-singlet contribution is given by $T_{\mu\nu}$ $(q \to qg)$. This has two contributions to order α_s: 'real' given by figures 2a and 2b and 'virtual' given by the interference of figures 1a and 1b. In computing these contributions we will take the initial quark to be longitudinally polarised i.e. $m_q s_1^\mu \simeq \eta_1 p_1^\mu$, $\eta_1 = \pm 1$. This as we remarked earlier, is necessitated by the use of the quark-parton approach since one works in the limit of zero parton mass. In addition, we shall see that the cancellation of divergences between the 'real' and 'virtual' contributions takes place to give consistent only for longitudinally polarised initial uqarks.

Prescription R_1: To clarify the procedure of prescription R_1 consider the contribuion of figure 2a to $| M (q \to qg)|^2_{\mu\nu}$. The amplitude is written for massless quarks and gluon. Straightforward evaluation gives the antisymmetric part to be

$$- i\eta_1 \frac{1}{[(p_1 - h)^2]} \epsilon_{\mu\nu\alpha\beta}\, p_2^\alpha \,[2p_1 \cdot (p_1 - h)\,(p_1 - h)^\beta - (p_1 - h)^2\, p_1^\beta]. \quad (17)$$

The denominator comes from the propagator for a massless quark and the square bracket from the evaluation of the trace involved. According to prescription R_I, for a further evaluation of (17) and the phase space-integration in (12) one takes p_1 and p_2 off mass-shell, with

$$\omega \equiv -p_1^2/Q^2 > 0 \text{ and } \omega' = p_2^2/Q^2 > 0. \tag{18}$$

However, the gluon remains massless i.e. $h^2=0$. In doing the kinematics, this means, that one has $(p_1-h)^2 = -\omega Q^2 - 2p_1 \cdot h$ and $(p_2+q)^2 = (\omega'-1)\, Q^2 + 2p_2 \cdot q$, etc. Having evaluated (17) one substitutes it in (12) to obtain the contribution of figure 2a to $T^{(A)}_{\mu\nu}\,(q \to qg)$. The phase space integration in (12) is Lorentz-invariant and is most simply done in the centre mass frame of the incoming quark and the virtual photon. A point to note is that in the evaluation of $|M(q\to qg)|^2$ one should keep the 'double pole' terms like $\omega/(p_1-h)^4$ and $\omega'(p_2+h)^4$ as they contribute in the limit ω, $\omega' \to 0$ to the structure function and its moments. The full 'real' contribution from figures 2a and 2b to the antisymmetric part comes out to be

$$T^{(A)\,(\text{real})}_{\mu\nu}(q \to qg) = -\, ie_q^2\, \eta_1 \frac{1}{2M^2 y^2{}_\nu}\, \epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, p_1^\beta\, V_R(z), \tag{19}$$

$$V_R(z, Q^2) = \mathscr{F}_{2R}(z, Q^2) - \frac{4}{3}\frac{\alpha_s}{2\pi}(1 + z). \tag{20}$$

$\mathscr{F}_{2R}(z)$ is the 'real' contribution (from figure 2) to the spin-averaged structure function $\mathscr{F}_2$ and is given by

$$\mathscr{F}_{2R}(z, Q^2) = \frac{4}{3}\frac{\alpha_s}{2\pi}\Big\{ -\ln \omega \left[\frac{1+z^2}{(1-z)_+} - 2\ln \omega'\, \delta(1-z)\right]$$
$$-\frac{3}{2}\left[\frac{1}{(1-z)_+} - \ln \omega'\, \delta(1-z)\right] - \frac{2(1+z^2)}{(1-z)}\ln z$$
$$+ 1 + 3z + \delta(1-z)\Big\}, \tag{21}$$

where the distribution $(1-z)_+^{-1}$ is defined by

$$\int_0^1 \frac{dz}{(1-z)_+} f(z) \equiv \int_0^1 \frac{dz}{(1-z)} [f(z) - f(1)]. \tag{22}$$

The 'virtual' contribution $V_V(z)$ from the interference of figures 1a and 1b, has the form of (19) except that $V_R(z)$ is replaced by

$$V_V(z, Q^2) = \frac{4}{3}\frac{\alpha_s}{2\pi}\left[-1 - \frac{2\pi^2}{3} - \frac{3}{2}(\ln \omega + \ln \omega') - 2\ln \omega \ln \omega'\right]\delta(1-z). \tag{23}$$

It should be noted that this is exactly the same as the virtual contribution to $\mathcal{F}_2$. This is because the virtual contribution is given effectively by the vertex correction. The only difference that can arise is while taking the spin-average but this also vanishes in the limit ω, ω' tending to zero. The ultraviolet divergence in this computation is regularised by subtracting the contribution at $q^2=0$. The computation is analogous to that in QED (Karplus and Kroll 1950; Jauch and Rohrlich 1955).

The total contribution is given by the sum $V_R(z)+V_V(z)=G_1(z)$. Note that the sum depends on Q^2 through $\ln \omega$, whereas the $\ln \omega'$ terms cancel out completely. In the spirit of the quark-parton model a longitudinally polarised quark would imply a longitudinally polarised nucleon so that only the term with V_1 survives in (4c). Thus

$$2G_1^N(x_H, Q^2) = \sum_q e_q^2 \int_{x_H}^{1} \frac{dy}{y} \left[G_{N\uparrow}^{q\uparrow}(y) - G_{N\uparrow}^{q\downarrow}(y)\right] G_1(z, Q^2), \tag{24}$$

$$G_1(z, Q^2) = \mathcal{F}_2(z, Q^2) - \tfrac{4}{3}(\alpha_s/2\pi)(1+z), \tag{25}$$

$$\mathcal{F}_2(z, Q^2) = -\frac{\alpha_s}{2\pi} P_{qq}(z) \ln \omega + \alpha_s f_{q,2}(z), \tag{26}$$

$$P_{qq}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right], \tag{27}$$

$$f_{q,2}(z) = \frac{4}{3}\cdot\frac{1}{2\pi}\cdot\left[1+3z-\frac{3}{2}\frac{1}{(1-z)_+}-\frac{2(1+z^2)}{1-z}\ln z-\frac{2\pi^2}{3}\delta(1-z)\right]. \tag{28}$$

Thus the leading behaviour of G_1^N given by the coefficient of $\ln Q^2$ is the same as for $\mathcal{F}_2$, the difference being the non-leading terms. Defining moments through $p_{qq}^{(n)} = \int_1^0 dz\, z^{n-1} P_{qq}(z)$, etc., one sees that $\frac{3}{4} P_{qq}^{(n)} = \gamma_{qq}^{(0)(n)}$ is the usual one loop contribution to the anomalous dimension while $f_{q,2}^{(n)}$ is the coefficient function which enters into $\mathcal{F}_2^{(n)}$. From (24) to (27) one sees that the coefficient function $g_{q,1}^{(n)}$ for G_1 is obtained from the nth moment of

$$g_{q,1}(z) = f_{q,2}(z) - \frac{2}{3\pi}(1+z) \tag{28}$$

We now turn to the calculation of the flavour non-singlet contribution using the second regularisation prescription.

Prescription R_{II}: As before, one writes the amplitudes for a massless theory, but now the quarks and gluons are kept massless throughout the computation. However, since the regularisation is done by going to arbitrary dimension n, one has to evaluate the traces as well as the phase space integrations in n dimensions. Note that the presence of γ_5 in the spin-projection operator in the traces has to be treated carefully as γ_5 has no analogue in n dimensions. We use the prescription of t'Hooft and Veltman (1972) to carry out these traces in n dimensions. For example the contribution of figure 2a can be obtained from (17) by multiplying it by $(1/2)(n-2)$ and now in addition $p_1^2=p_2^2=h^2=0$. The phase space integration in n dimensions modifies (12) to

$$T_{\mu\nu}^{(\text{real})}(q\to qg) = e_q^2\,\frac{p^0}{Mp_1^0}\cdot\left(\frac{4}{3}\alpha_s\right)(\mu^2)^{\epsilon}\,4\int\frac{d^n p_2}{(2\pi)^{n-1}}\,\frac{d^n h}{(2\pi)^{n-1}}\times$$

$$\delta(p_2^2)\delta(h^2)\,(2\pi)^n\delta^{(n)}\,(p_1+q-p_2-h)\,|\,M(q\to qg)\,|^2_{\mu\nu}, \tag{29}$$

where $\epsilon=2-\frac{1}{2}n$ and μ is an arbitrary parameter with dimensions of mass which is introduced to keep the quark-gluon coupling constant g_s dimensionless in n dimensions. Straightforward evaluation in the limit $\epsilon\to 0$ again gives (19) and (20) but with

$$\mathcal{F}_{2R}(z,Q^2)=\frac{4}{3}\left(\frac{\alpha_s}{2\pi}\right)(4\pi\mu^2/Q^2)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\cdot\left\{\frac{2}{\epsilon^2}-\frac{1}{\epsilon}\frac{1+z^2}{(1-z)_+}+\frac{3}{2\epsilon}\delta(1-z)\times\right.$$

$$\left.(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-\frac{3}{2}\frac{1}{(1-z)_+}-\frac{(1+z^2)}{1-z}\ln z+3+2z+\frac{7}{2}\delta(1-z)\right\}. \tag{30}$$

The calculation of the virtual contribution is most easily done in the Landau gauge as the quark self-energy (to order α_s) vanishes in this gauge for massless quarks as (Marciano 1975). The effective quark-photon vertex becomes

$$\Gamma^{\mu}(Q^2)=\gamma^{\mu}\left\{1+\frac{1}{2}\cdot\frac{4}{3}\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\right.$$

$$\left.\times\frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8\right]\right\}. \tag{31}$$

Again, $\quad V_V(z)=\mathcal{F}_{2V}(z)$, with

$$V_V(z)=\delta(1-z)\left\{1+\frac{4}{3}\left(\frac{\alpha_s}{2\pi}\right)\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\cdot\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8-\frac{\pi^2}{3}\right]\right\}. \tag{32}$$

Note the double pole in ϵ cancels out in the sum V_R+V_V. The relation between $G_1(z,Q^2)$ and $G_1^N(x_H,Q^2)$ (equations (24) and (25)) still remains true except that $\mathcal{F}_2(z,Q^2)$ is now different. Expanding in ϵ and extracting the finite part according to the minimal-subtraction scheme (t'Hooft 1973), one has

$$\mathcal{F}_2(z,Q^2)=+(\alpha_s/2\pi)\,P_{qq}(z)\,t+\alpha_s f_{q,2}(z), \tag{33}$$

with $P_{qq}(z)$ given by (27) and

$$f_{q,2}(z) = \frac{4}{3}\frac{1}{2\pi}\left\{(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{(1+z^2)}{1-z}\ln z + 3 + 2z - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-z) - \frac{3}{4}P_{qq}(z)(\ln 4\pi - \gamma_E)\right\} \quad (34)$$

The Euler-Mascheroni constant γ_E appears from the expansion $\Gamma(1-\epsilon) = 1 + \gamma_E\,\epsilon + \frac{1}{2}(\gamma_E^2 + \pi^2/6)\,\epsilon^2 + \ldots$ of the gamma function. Equations (33) and (34) agree with the OPE results (Bardeen *et al* (1978)). It is clear that in this case again $g_{q,1}(z)$ is given by (28) with $f_{q,2}(z)$ being given by (34).

We now compare the results obtained from the two prescriptions. From (26) to (33) it is seen that the leading behaviour is exactly the same in both cases, though the non-leading terms differ. However, the difference $G_1 - \mathcal{F}_2$ (given by (25) to order α_s) is independent of the prescription. It is this difference which modifies the Bjorken sum rule (Bjorken 1970) for the flavour non-singlet combination $G_1^p - G_1^n$. Defining effective parton densities through (Altarelli *et al* 1977)

$$\mathcal{F}_2^N(x_H, t) = \sum_q e_q^2\, G_N^q(x_H, t), \quad (35)$$

since the sum over q includes antiquarks, the modified sum rule then reads

$$\int_0^1 [G_1^p(x_H, t) - G_1^n(x_H, t)]\, dx_H = \frac{1}{6}\cdot\frac{G_A}{G_V}\left(1 - \frac{\alpha_s}{\pi}\right). \quad (36)$$

The modifying factor is the same as that for the Gross-Llewellyn Smith sum rule for the structure function $\mathcal{F}_3$ for νN and $\bar{\nu}N$ scattering. This is so because

$$\mathcal{F}_3(z, Q^2) - \mathcal{F}_2(z, Q^2) = G_1(z, Q^2) - \mathcal{F}_2(z, Q^2).$$

OPE results (Kodaira *et al* 1978) using the prescription R_{II} are available for comparison but we are unaware of similar calculations with the prescription R_{I}. We find from (28) and (34), the moments

$$\alpha_s\, g_{q,1}^{(n)} = \frac{4}{3}(\alpha_s/2\pi)\left\{-\frac{9}{2} + \frac{1}{2n} + \frac{1}{n+1} + \frac{1}{n^2} + \frac{3}{2}\sum_{k=1}^{n}\frac{1}{k} - 2\sum_{k=1}^{n}\frac{1}{k^2} - \frac{1}{n(n+1)}\sum_{k=1}^{n}\frac{1}{k} + 2\sum_{k=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{1}{j} + \gamma_{qq}^{(0)(n)}(\gamma_E - \ln 4\pi)\right\}, \quad (37)$$

$$\gamma_{qq}^{(0)(n)} = \frac{3}{2} + \frac{1}{n(n+1)} - 2\sum_{k=1}^{n}\frac{1}{k}. \quad (37a)$$

This agrees with the coefficient function obtained by Kodaira *et al* except for the last term $\gamma_{qq}^{(0)(n)}(\gamma_E - \ln 4\pi)$. Such a difference, depending on $\gamma_{qq}^{(0)(n)}$ can arise due to a different choice of μ^2, the subtraction scale at which the theory is renormalised (Bardeen *et al* 1978). This can be seen directly from (33), (34) and (14). Physical consequences like the modified Bjorken sum rule are identical in the two cases.

We briefly address ourselves to the problem of obtaining G_2 in the quark-parton approach. One possibility is to take $m_q\, s_1^\mu \neq \eta_1\, p_1^\mu$ from the start and keep $m_q\, s_1^\mu$ terms separately but otherwise work as before. Then apart from the overall factors in (19), the expressions now take the form

$$T_{\mu\nu}^{(A)\,(\mathrm{real})}\,(q\to qg) \sim m_q\, \epsilon_{\mu\nu\alpha\beta}\Big[q^\alpha\, p_1^\beta \frac{(s_1\cdot q)}{Q^2} A_R(z) + q^\alpha s_1^\beta B_R(z)$$

$$+\, p_1^\alpha\, s_1^\beta\, C_R(z)\Big], \tag{38a}$$

$$T_{\mu\nu}^{(A)\,(\mathrm{virtual})}(q\to qg) \sim m_q\, \epsilon_{\mu\nu\alpha\beta}\, q^\alpha\, s_1^\beta\, V_V(z). \tag{38b}$$

The precise form of A_R, B_R and C_R is not as relevant as the tensor structure in (38a). G_1 and G_2 can be extracted by considering $s_1^\mu\, T_{\mu\nu}$ and $p_1^\mu\, T_{\mu\nu}$. This gives us G_1 as before but with G_2 in terms of A_R. The first worrying fact is the presence of the $p^\alpha s_1^\beta$ term in (38a) as it does not satisfy current conservation (!). For longitudinal polarisation it does not contribute since $m_q s_1^\mu \simeq \eta_1 p_1^\mu + O\,(p_1^2)$ but for a transversely polarised quark ($\mathbf{s}_1\cdot\mathbf{p}_1 = 0$) it poses a problem. Even if we ignore the C_R term there is a second problem because the ln ω' or $1/\epsilon^2$ terms, depending on the prescription, are present in A_R and $B_R + V_V$. For longitudinal quarks (38a) reduces to (19) with $V_R = (1/2z)$, $A_R + B_R$. It seems then that with the two regularisation procedures adopted, consistent calculations in the quark-parton approach, requires us to work with longitudinally polarised quarks. To obtain G_2 one would presumably have to work with massive quarks with non-zero transverse momentum.

3.2. *Flavour singlet case*

In the infinite momentum frame, given by the electron-nucleon c.m. frame, the gluon momentum $p_1 \simeq y\, P$ has no transverse components, so that we take its polarisation vectors to be $\epsilon_\pm^\mu = (1/\sqrt{2})\,(0,\ 1,\ \pm i,\ 0)$. Calculation of the antisymmetric part $T_{\mu\nu}^{(A)\pm}(g\to q\bar{q})$ from figures 3a and 3b, for the two gluon helicities is straightforward but tedious.

Prescription R_1: The amplitude is written for the massless theory and after evaluation of the trace one takes the initial gluon off mass-shell $p_1^2 < 0$ in doing the kinematics while the final quark-antiquark remain massless ($h_1^2 = h_2^2 = 0$). The phasespace integration in (13) is most easily done in c.m. frame of the gluon and photon, yielding

$$T_{\mu\nu}^{(A)\pm}\,(g\to q\bar{q}) = \pm\, i\Big(\sum_q e_q^2\Big)\Big(\frac{\alpha_s}{2\pi}\Big)\cdot \frac{1}{2M^2 y^2 \nu}\cdot \big\{\epsilon_{\mu\nu\alpha\beta}\, p_1^\alpha\, q^\beta\, C(z)$$

$$-\,\epsilon_{\mu\nu\sigma\beta}\,\epsilon^{12\alpha\beta}\, p_{1\alpha}\, q^\sigma\, D(z)\big\}, \tag{39}$$

$$C(z) = -\frac{z}{2} \ln\left(-\frac{p_1^2}{Q^2}\right) - z \ln z + (1-2z), \tag{40a}$$

$$D(z) = -\tfrac{1}{2}(1-z) \ln\left(-\frac{p_1^2}{Q^2}\right) - (1-z) \ln z. \tag{40b}$$

Gluons with definite helicity will contribute to a nucleon state with a given helicity in the quark-parton approach so that in the scaling limit, one obtains

$$2G_1^N(x_H, Q^2) = \left(\sum_q e_q^2\right)\left(\frac{a_s}{2\pi}\right) \int_{x_H}^1 \frac{dy}{y} \left[G_{N\uparrow}^{g\uparrow}(y) - G_{N\uparrow}^{g\downarrow}(y)\right] [C(z) - D(z)] \tag{41}$$

where $G_{N\uparrow}^{g\uparrow}(y)$ $[G_{N\uparrow}^{g\downarrow}(y)]$

give the number of gluons with +ve (—ve) helicity in a +ve helicity nucleon with momentum fraction between y and $y + dy$.

Prescription R_{II}: The procedure is as in the flavour non-singlet case. The generalisation of the phase space integral should be clear from (29), (12) and (13). Equations (39) and (41) still hold except that instead of (40), now

$$C(z) = \tfrac{1}{2} z \left[t - \ln\left(\frac{z}{1-z}\right) - \ln 4\pi + \gamma_E\right] - \tfrac{1}{2}(1-2z), \tag{42a}$$

$$D(z) = -\tfrac{1}{2}(1-z)\left[t - \ln\left(\frac{z}{1-z}\right) - \ln 4\pi + \gamma_E\right]. \tag{42b}$$

Using the definition of t [equation (14)] one sees that the dependence on Q^2 of G_1 is same in both the prescriptions and is controlled, as expected, by $\Delta P_{qG} = -\tfrac{1}{2}(1-2z)$, and was obtained earlier by Altarelli and Parisi (1977). We have in addition obtained the non-leading terms. For the flavour non-singlet case one has a similar result, since there $\Delta P_{qq} = P_{qq}$.

The effects of these corrections could possibly be seen in the asymmetry discussed below. However, phenomenologically the inclusion of these corrections are a complication since one does not have a good idea of the gluon distribution inside the nucleon. Intuitively, one might expect, since the gluons are in the 'sea', that the sea is unpolarised and that $G_{N\uparrow}^{g\uparrow} \simeq G_{N\uparrow}^{g\downarrow}$, so that the dominant $O(a_s)$ corrections to the asymmetry will come only from the quarks. This expectation has been confirmed by Darrigol and Hayot (1978) who show, using the Altarelli and Parisi equations for the Q^2 dependence, that the variation of G_1 with Q^2 is mainly determined by the valence quark helicity distributions. Thus, in the next section, we mainly discuss the effect of the flavour non-singlet corrections on the asymmetry.

4. Asymmetry

The asymmetry for longitudinally polarised electron and nucleon is defined by

$$A^{eN}(x_H, y_L) = \frac{[d^2\sigma(eN)^-]}{[dx_H\, dy_L]} \bigg/ \frac{[d^2\sigma(eN)^+]}{[dx_H\, dy_L]}, \tag{43}$$

$$\frac{[d^2\sigma(eN)^\pm]}{[dx_H\, dy_L]} = \frac{[d^2\sigma(e\uparrow N\uparrow)]}{[dx_H\, dy_L]} \pm \frac{[d^2\sigma(e\uparrow N\downarrow)]}{[dx_H\, dy_L]}, \tag{44}$$

where $d\sigma(e\uparrow N\uparrow)$ is the differential cross-section when the spin of the electron and the nucleon are parallel and along the direction of the incident electron, while $d\sigma(e\uparrow N\downarrow)$ is the differential cross-section when the spins are antiparallel. The hadronic scaling variable is given by (6) and y_L is the lepton inelasticity. In the laboratory frame $y_L = 1 - E'/E$, where E and E', are the energies of the incoming and outgoing electrons.

The sum of the cross-sections in (44) is just twice the usual spin-averaged and is given by

$$\frac{d^2\sigma(eN)^+}{(dx_H\, dy_L)} = \frac{4\pi\alpha^2}{Q^2 y_L}\left[y_L^2\, \mathscr{F}_1^N + 2(1-y_L)\, \mathscr{F}_2^N - \frac{M}{E} x_H y_L\, \mathscr{F}_2^N\right], \tag{45}$$

where $\mathscr{F}_1$ and $\mathscr{F}_2$ are functions of x_H and $Q^2 = 2\,EM\,x_H y_L$. The precise form of the non-leading terms in the structure functions depends on the regularisation procedure but the difference $\mathscr{F}_1^N - \mathscr{F}_2^N$ is independent of it and a function of x_H above. The difference of the cross-sections

$$\frac{d^2\sigma(eN)^-}{dx_H\, dy_L} = \frac{8\pi\alpha^2}{Q^2}\left[(2-y_L)\, G_1^N - \frac{M}{E}\left(\frac{x_H y_L}{1-y_L}\, G_1^N - 2\, x_H\, G_2^N\right)\right]. \tag{46}$$

For sufficiently high laboratory energy E to make M/E terms negligible

$$A^{eN}(x_H, y_L) = y_L(2-y_L)\frac{2G_1^N(x_H, Q^2)}{[y_L^2\, \mathscr{F}_1^N(x_H, Q^2) + 2(1-y_L)\, \mathscr{F}_2^N(x_H, Q^2)]}, \tag{47}$$

where neglecting the gluonic contribution, $2G_1^N$ is given by (24) to (28) together with the zeroth order contribution $\delta(1-z)$ added to $G_1(z)$. The denominator can be obtained from the measurements of the spin-averaged cross-section. For $\alpha_s = 0$ this gives the naive parton model result. One does not know the quark distributions for a given helicity. So models were considered (Kuti and Weisskopf (1971); Look and Fischbach 1977) to relate $G_{N\uparrow}^{q\uparrow,\downarrow}$ to G_N^q. However to order α_s one has to define Q^2-dependent effective parton densities through $\mathscr{F}_2$ which involves both the quark and gluon distributions in the nucleon and one cannot expect, in general, a simple relation between the polarised and unpolarised distributions. So instead of any detailed phenomenology, unwarranted by the present data, we confine ourselves to some qualitative consequences. For large t, $\mathscr{F}_1 = \mathscr{F}_2$ and

$$A^{eN} = \frac{y_L(2-y_L)}{[y_L^2+2(1-y_L)]} \sum_q e_q^2 \int \frac{dy}{y} [G_{N\uparrow}^{q\uparrow}(y) - G_{N\uparrow}^{q\downarrow}(y)]$$

$$\left(\delta(1-z) + \frac{\alpha_s}{2\pi} t P_{qq}(z)\right) \times \left\{ \int \frac{dy}{y} \left[\sum_q e_q^2 G_N^q(y) \left(\delta(1-z) \right.\right.\right.$$

$$\left.\left.\left. + \frac{\alpha_s}{2\pi} t P_{qq}(z) \right) + \left(\sum_q e_q^2 \right) \frac{\alpha_s}{2\pi} t G_N^q(y) P_{qG}(z) \right] \right\}^{-1} \tag{48}$$

where

$$P_{qG}(z) = \tfrac{1}{2}(z^2 + (1-z)^2), \; z = x_H y^{-1}.$$

The first factor is just the asymmetry for a point-like spin $\frac{1}{2}$ particle. The point to be noticed is that A^{eN} has become independent of Q^2 and the regularisation scheme, since $\alpha_s(Q^2) \sim t^{-1}$ for large t. Further the third term in the denominator is positive, hence we can convert (48) into an inequality by dropping it. If in addition one assumes SU(6) wave-functions for the nucleon (in terms of its valence quarks) to relate $G_{N\uparrow}^{q\uparrow,\downarrow}(y)$ to G_N^q (Kuti and Weisskopf 1971) then

$$A^{ep} \leqslant \frac{y_L(2-y_L)}{y_L^2 + 2(1-y_L)} \frac{5}{9}, \; A^{en} = 0, \tag{49}$$

for large t. The right hand sides in (49) are just the naive quark-parton model results.

Another result which is independent of the regularisation scheme can be obtained by considering the moments of G_1^N obtained from (46) neglecting M/E terms. For fixed Q^2,

$$G_1^{N(n)}(Q^2) = \int_0^1 G_1^{N(n)}(x_H, Q^2)\, x_H^{n-1}\, dx_H = \int \frac{Q^2}{8\pi\alpha^2} \frac{1}{(2-y_L)} \frac{d^2(eN)^-}{dx_H dy_L} dx^H x_H^{n-1}$$

$$= \left[1 + \frac{\alpha_s}{\pi} t P_{qq}^{(n)} + \alpha_s f_{q,2}^{(n)} - \frac{2}{3} \frac{\alpha_s}{\pi} \left(\frac{1}{n} + \frac{1}{n+1} \right) \right] \Delta_q^{N(n)}, \tag{49}$$

$$\Delta_q^{N(n)} \equiv \sum_q e_q^2 \int_0^1 dy\, y^{n-1} [G_{N\uparrow}^{q\uparrow}(y) - G_{N\uparrow}^{q\downarrow}(y)]. \tag{50}$$

For large t, $\Delta_q^{N(n)}$ can be obtained cleanly since $P_{qq}^{(n)}$ is independent of the regularisation prescription. The result for the first moment is rather striking since $P_{qq}^{(1)} = f_{q,2}^{(1)} = 0$, independently of the regularisation procedure. This is because of charge conservation. Thus, for any t,

$$G_1^{N(1)}(Q^2) = \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\Delta_q^{N(1)} - \frac{\alpha_s(Q^2)}{4\pi}\Delta_g^{N(1)}, \tag{51}$$

where we have included the gluonic contribution to G_1. The expression for $\Delta_g^{N(n)}$ is defined analogously to $\Delta_q^{N(n)}$. Note that the first moment of ΔP_{qG} is zero and the non-leading terms contribute the *same* amount in both the prescriptions as can be seen from (40) and (41). In (51) the only Q^2 dependence is through $\alpha_s \sim (\ln Q^2)^{-1}$ so that $G_1^{N(1)}$ would slowly tend to 1 with increasing Q^2. This result is true for both $N = p$ or n. Of course, forming the flavour non-singlet combination in (51) gives just the modified Bjorken sum rule.

In summary, we have obtained the QCD corrections, to order α_s for the spin-dependent structure functions using the quark-parton approach. Two regularisation prescriptions (i) off mass-shell partons and (ii) dimensional regularisation were used to regularise the singularities of the massless theory. The regularisation dependence of the non-leading connections*, though disturbing at first sight, can be removed by defining effective parton densities as in (35). The procedure has been discussed in detail by Altarelli *et al* 1977. However, without defining effective parton densities, the structure functions as calculated (in the form of (15)) can be exploited to give consequences which are independent of the regularisation and such consequences have been emphasised. In particular these are the modified Bjorken sum rule and some interesting consequences for the asymmetry in the scattering of longitudinally polarised electrons and nucleons.

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*In the operator product expansion (OPE) approach the moments of the structure satisfy the renormalisation group equation. On expansion of these moments (to order α_s), the non-leading correction (to the flavour non-single part of $\mathscr{F}_2$) is obtained to be $f_{q,2}^{(n)} + (\gamma_{qq}^{(1)(n)} / 2\beta_0)$, where $\gamma_{qq}^{(1)(n)}$ is the two-loop contribution to the anomalous dimension and $\beta_0 = 11 - \frac{2}{3} N_f$, with N_f = number of flavours. Each term, separately, does depend on the renormalisation prescription as we found for $f_{q,2}^{(n)}$. However, the sum has been shown (Buras 1979, Floratos *et al* 1977) to be independent of the prescription provided of course the same choice is used in computing the two terms.

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