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Coulomb effect in rotating nuclei in the Thomas-Fermi approach

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Abstract. The effect of nuclear Coulomb potential in rotating nuclei has been studied in the Thomas-Fermi approach. The numerical calculations of typical rotating nuclei show that Coulomb potential plays an important role in the rotational mass regions 150–190 and > 224.

Keywords. Coulomb potential; rotating nuclei; nuclear deformations; Thomas-Fermi approach.

1. Introduction

The static properties of nuclei have been explained successfully by the Thomas-Fermi (TF) approximation. Recently Stocker (1977) has applied the TF approach to describe rotating nuclei. In his description, he has not included the effects due to nuclear Coulomb potential. The present work describes not only the rotating nuclei in the TF approach with nuclear Coulomb potential and its exchange effects but also makes numerical calculations of the contributions due to homogeneous and non-homogeneous parts of the nuclear potential energy, Coulomb energy, rotational energy etc., for typical nuclei in the mass regions 19–31, 150–190 and > 224.

In order to describe a rotating nucleus within the TF framework, a linear momentum $\mathbf{D} = \mathbf{D}(\mathbf{r})$ is added to the local momentum at a point $\mathbf{r}$, so that it describes the motion of the nuclear fluid. This is achieved by shifting the centre of the Fermi sphere which represents the momentum distribution around the point $\mathbf{r}$, from the origin in $\mathbf{k}$-space to the point $\mathbf{D} = \mathbf{D}(\mathbf{r})$. The Fermi sphere is shown to be

$$|\mathbf{K}(\mathbf{r}) - \mathbf{D}(\mathbf{r})| \leqslant \mathbf{K}_F(\mathbf{r}), \tag{1}$$

where K_F is the Fermi momentum. The relation between the local velocity of the the fluid and $\mathbf{D}(\mathbf{r})$ in classical picture is $\mathbf{U}(\mathbf{r}) = \hbar\, \mathbf{D}(\mathbf{r})/m$. The angular momentum $\mathbf{J}$ at any point $\mathbf{r}$ of the rotating system is given by $\mathbf{J} = \hbar\, \mathbf{r} \times \mathbf{D}(\mathbf{r})$, and hence calculating the total angular momentum of the rotating system for the state with $J = J_Z$ by summing up all local contributions, one gets

$$J = \hbar \int r \sin\theta\, \mathbf{D}(\mathbf{r})\, \rho(\mathbf{r})\, d^3 \bar{r}, \tag{2}$$

where $\rho(\mathbf{r})$ is the particle density and θ is the azimuthal angle with respect to the rotation axis (Z-axis).

The total kinetic energy of the rotating system is given by

$$E_{\text{kin}} = \frac{\hbar^2}{2m} \int d^3 \bar{r} \left\{ \frac{3\,(3\pi^2)^{2/3}}{5\,(2)^{2/3}} \rho^{5/3}(\mathbf{r}) + D^2(\mathbf{r})\,\rho(\mathbf{r}) \right\}, \tag{3}$$

where the first and second term represent the linear and rotational kinetic energy of the system, respectively. The density ρ and Fermi momentum K_F are related through

$$\rho(\mathbf{r}) = 2/(3\pi^2)\, K_F^3(\mathbf{r}).$$

The potential energy of the system can be written as

$$E_{\text{pot}} = \int d^3 \bar{r} \left\{ \rho(\mathbf{r})\, V(\rho, a) + \frac{\hbar^2}{2m} \eta (\nabla \rho)^2 + C_c\, \rho(\mathbf{r})\, \phi_c(\mathbf{r}) + C_{ce}\, \rho^{4/3}(\mathbf{r}) \right\}. \tag{4}$$

The Thomas-Fermi approximation consists in assuming that the energy dependence in the density is locally the same as that of a homogeneous medium in its ground state. This is equivalent to assuming that the correlations between the nucleons in finite nuclei are the same as in nuclear matter (Brueckner *et al* 1958) and is known as the local density approximation. In accordance with Brueckner *et al* (1968), the first term in equation (4) represents the potential energy which is of the form

$$V(\rho, a) = B_1(a)\, \rho + B_2(a)\, \rho^{4/3} + B_3(a)\, \rho^{5/3}$$

where the coefficients $B_i(a)$ are of the form $B_i(a) = b_i\,(1 + a_i\, a^2)$ and a is the neutron excess parameter: $a = (N-Z)/(N+Z)$. The inhomogeneity correction which takes care of the decrease of the density at the nuclear surface is expressed by the second term of equation (4). The direct Coulomb energy is given by the third term $C_c\, \rho\, \phi_c(\mathbf{r})$ of equation (4) where $C_c = \frac{1}{2}\, [\frac{1}{2}\,(1-a)]\, e$ and

$$\phi_c(\mathbf{r}) = e\, \tfrac{1}{2}\,(1-a) \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}\, d^3 r'.$$

The last term in equation (4) represents exchange Coulomb energy with $C_{ce} = -\,1{\cdot}0636\, [\frac{1}{2}\,(1-a)]^{4/3}\, e^2$.

The total energy is expressed by

$$E(\rho) = \int \epsilon\, [\rho(\mathbf{r})]\, d^3 r, \tag{5}$$

with

$$\epsilon(\rho) = \frac{\hbar^2}{2m} \left\{ \frac{3}{5} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3}(\mathbf{r}) + D^2(\mathbf{r})\, \rho(\mathbf{r}) \right\} + \rho(\mathbf{r})\, V(\rho, a) + \frac{\hbar^2}{8m} \eta (\nabla \rho)^2 + C_c\, \rho(\mathbf{r})\, \phi_c(\mathbf{r}) + C_{ce}\, \rho^{4/3}(\mathbf{r}). \tag{6}$$

In order to minimise the total energy with respect to the local momentum $D(\mathbf{r})$ and the density $\rho(\mathbf{r})$, Lagrange multipliers λ and μ were introduced, respectively, under the constraints of fixed angular momentum and particle number. The energy minimisation with respect to D gives the angular velocity (ω) with which the nucleus rotates as $\omega = \lambda/\hbar$ and its local momentum as

$$D(\mathbf{r}) = \lambda \frac{m}{\hbar^2} r \sin\theta. \tag{7}$$

Minimising the total energy with respect to density variations, one gets the following TF equation

$$\frac{\hbar^2}{2m}(3\pi^2/2)^{2/3}\rho^{2/3} + \frac{m}{2\hbar^2}\lambda^2 r^2 \sin^2\theta + 2B_1\rho + 7/_3 B_2 \rho^{4/3}$$
$$+ 8/_3 B_3 \rho^{5/3} + C_c\phi_c + \frac{4}{3}C_{ce}\rho^{1/3} - \frac{2\hbar^2}{8m}\eta\,\Delta\rho = \mu. \tag{8}$$

Because of Coulomb term, equation (8) is an integro-differential equation. This equation also contains rotational energy term. An exact solution of this equation is rather difficult. An approximate solution of equation (8) can be obtained by assuming

$$\rho(\mathbf{r}) = \rho_0(r)\,[1 + \epsilon\, Y_{20}(\theta)], \tag{9}$$

where $\rho_0(r)$ is the density of the non-rotating nucleus and $\rho_0 \epsilon Y_{20}$ represents a small perturbation of the density ρ_0 due to rotation. This is because of small values of ϵ which is due to small values of angular momentum.

In order to obtain a relation between the unknown parameter ϵ and λ, equation (9) was used in (8) and the resulting expression was linearised with respect to ϵ to get the change $\delta\rho$

$$\delta\rho(r,\theta) = \rho_0\,\epsilon\, Y_{20},$$

$$\delta\rho = \frac{-\rho_0 \frac{m}{2\hbar^2}\lambda^2 r^2 \sin^2\theta}{-\frac{2\hbar^2}{8m}\eta\left[\frac{1}{r}\frac{d^2}{dr^2}(r\rho_0) + \frac{6\rho_0}{r^2}\right] + 2B_1\rho_0 + \frac{28}{9}B_2\rho_0^{4/3} + \frac{40}{9}B_3\rho_0^{5/3} + \frac{4}{9}C_{ce}\rho_0^{1/3} + A + \frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{2/3} \times \frac{2}{3}\rho_0^{2/3}}. \tag{10}$$

where A is the contribution due to the direct Coulomb term $C_c\phi_c$. The calculation is given below.

$$C_c\phi_c = C_c\, e\, \tfrac{1}{2}(1-\alpha)\int \frac{\rho(r')}{|\mathbf{r}-\mathbf{r}'|} d^3 r'$$

Using the expansion of $\dfrac{1}{|\mathbf{r}-\mathbf{r}'|}$

$$C_c\phi_c = C_c\, e\, \tfrac{1}{2}\,(1-\alpha) \left\{ \int_0^r \rho(r')\, d^3 r' \left[\frac{1}{r} + \frac{r'}{r^2}\cos\Theta + \right.\right.$$

$$\left. + \frac{r'^2}{r^3}\,\frac{1}{2}\,(3\cos^2\Theta - 1) + \ldots \right] + \int_r^R \rho(r')\, d^3 r' \left[\frac{1}{r'} + \frac{r}{r'^2}\cos\Theta + \right.$$

$$\left.\left. + \frac{r^2}{r'^3} \times \frac{1}{2}\,(3\cos^2\Theta - 1) + \ldots \right]\right\}. \tag{11}$$

where the first integral (I_1) represents the contribution of all the points inside a sphere of radius r and the second integral (I_2) represents the contribution due to the points between the spheres of radii r and R (nuclear radius) and Θ is the angle between $\mathbf{r}$ and $\mathbf{r}'$ as shown in figure 1.

Analogous to equation (9), the value of $\rho(r')$ is written as

$$\rho(\mathbf{r}') = \rho_0(r')\,[1 + \epsilon\, Y_{20}(\theta')],$$

$$= \rho_0(r')\,[1 + \frac{\epsilon\beta}{2}(3\cos^2\theta' - 1)]. \tag{12}$$

where $\beta = (5/4\pi)^{1/2}$. Inserting equation (12) in I_1 of equation (11), one gets

$$I_1 = C_c\, e\, \tfrac{1}{2}(1-\alpha) \int_0^r \rho_0(r') \left[1 + \frac{\epsilon\beta}{2}(3\cos^2\theta' - 1)\right] \left\{\frac{1}{r} + \frac{r'}{r^2}\cos(\Theta) + \right.$$

$$\left. \frac{r'^2}{r^3} \times \frac{1}{2}\,(3\cos^2\Theta - 1) + \ldots \right\} d^3\bar{r}'$$

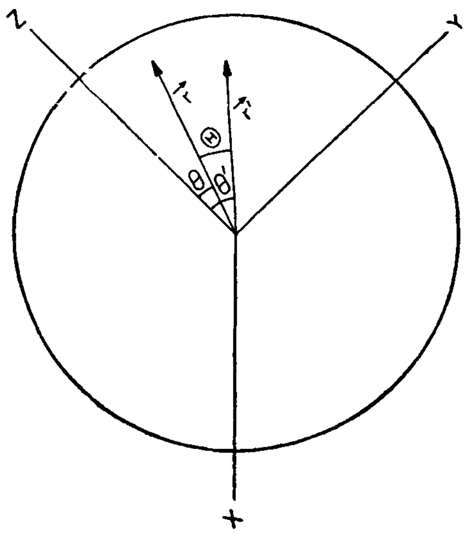

Figure 1. θ is the angle between the rotational axis (z-axis) and $\mathbf{r}$, θ' is the angle between the rotation axis and $\mathbf{r}'$ and Θ is the angle between $\mathbf{r}$ and $\mathbf{r}'$.

which gives the perturbed part of I_1 as

$$I_1 = C_c e\tfrac{1}{2}\,(1 - a) \int_0^r \rho_0\,(r')\,\frac{\epsilon\beta}{2}(3\cos^2\theta' - 1) \times$$

$$\left\{\frac{1}{r} + \frac{r'}{r^2}\cos\Theta + \frac{r'^2}{r^3} \times \frac{1}{2}(3\cos^2\Theta - 1) + \ldots\right\} d^3 r' \tag{13}$$

The evaluation of equation (13) leads to the perturbed part of

$$I_1 = C_c\, c\,\tfrac{1}{2}\,(1 - a)\,\frac{4\pi\,\epsilon\,Y_{20}(\theta)}{5\,r^3} \int_0^r \rho_0\,(r')\,r'^4\,dr' \tag{14}$$

Similarly, the second integral of equation (11) gives the perturbed part of

$$I_2 = C_c\,\epsilon\,\tfrac{1}{2}\,(1 - a)\,\frac{4\pi\,\epsilon\,Y_{20}(\theta)\,r^2}{5} \int_r^R \frac{\rho_0\,(r')}{r'}\,dr'. \tag{15}$$

The total contribution of $C_c\,\theta_c$ is A $=$ perturbed parts of $(I_1 + I_2)$

$$A = C_c\,c\,\tfrac{1}{2}\,(1 - a)\,\frac{4\pi\,\epsilon\,Y_{20}(\theta)}{5}\left\{\frac{1}{r^3}\int_0^r \rho_0\,(r')\,r'^4\,dr'\right.$$

$$\left. + r^2 \int_r^R \frac{\rho_0\,(r')}{r'}\,dr'\right\}. \tag{16}$$

The comparison of expression (10) for change in density with that of Stocker (1977) shows that the present expression (10) contains additional terms in the denominator due to Coulomb contributions. The effect of these (direct and exchange Coulomb) terms is to increase the magnitude of the denominator of Stocker's (1977) expression for $\delta\rho$ which in effect increases the change in density. This means the addition of Coulomb contribution has increased the deformation of the nucleus which is a well known fact. The Coulomb contribution plays an important role in the heavy mass rotational region. The contribution due to Coulomb terms makes no change in the θ dependence and the conclusions drawn by Stocker (1977) for θ dependence hold true in the present case as well.

To facilitate the numerical evaluation of different terms in the numerator and the denominator of $\delta\rho$ (equation (10)), it is assumed that $\rho_0\,(r)$ equals the nuclear saturation density. Calculations were further simplified by assuming that the density is the same for both neutrons and protons. In evaluating the inhomogeneity term, η has been taken to be 5 fm^3 in accordance with Brueckner *et al* (1968) and the actual evaluation was done on the nuclear surface. The values of B_1, B_2 and B_3 for various

Table 1. Values of various terms in the denominator and numerator of expression (10) for typical rotational nuclei. The last column gives the value of λ for the same nuclei with a constraint that $d\rho/\rho_0$ is 1%. All measurements in meV.

Nucleus	HT	ICT	TCT	KET	RKET	λ
$^{25}_{13}$Al	27·290	−0·141	0·019	0·898	$0{\cdot}318 \times 10^{45}\,\lambda^2$	0·276
$^{160}_{66}$Dy	26·240	−0·006	0·253	0·140	$7{\cdot}001 \times 10^{45}\,\lambda^2$	0·057
$^{183}_{74}$W	26·158	−0·005	0·274	0·123	$8.763 \times 10^{45}\,\lambda^2$	0·051
$^{239}_{94}$Pu	25·906	−0·003	0·329	0·094	$13{\cdot}670 \times 10^{45}\,\lambda^2$	0·041

HT = homogeneous term, ICT = inhomogeneity correction term
TCT = total Coulomb term, KET = kinetic energy term
RKET = rotational kinetic energy term

nuclei were obtained from the nuclear-matter saturation curves of Brueckner *et al* (1968). Numerical calculations of various terms in the denominator of equation (10) were performed for typical rotational nuclei $^{25}_{13}$Al, $^{160}_{66}$Dy, $^{183}_{74}$W and $^{239}_{94}$Pu. The first nucleus occurs in the mass region 19–31, the second and the third nuclei fall in the mass region 150–190 and the last nucleus is in the mass region of > 224. The results are summarised in table 1.

To make an estimation of the numerator of equation (10) for the nuclei given above, θ was taken to be $\pi/2$. The numerical evaluation of this can be done, if λ is known. To have an idea of the magnitude of λ, its value was calculated with a constraint that $d\rho/\rho_0$ is of the order of 1% and the values of λ for different nuclei are also listed in table 1.

It is apparent from table 1 that the contribution due to Coulomb energy is rather small in the low mass regions (19-31), whereas it contributes upto 1% in the mass region 150-190 and greater than 1% in the mass region of > 224 with respect to the contribution due to the homogeneous part of the nuclear potential. Hence the effect of Coulomb energy cannot be neglected in the high mass region. On the contrary the effect of surface energy decreases from low mass to high mass region. One can observe from table 1, that λ decreases as mass number increases. An interesting feature of λ is that its estimated value in table 1 for different nuclei is near about the energy of the first excited state of a rotational band. Further investigation on this matter may prove worthwhile.

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