Pramāṇa, Vol. 14, No. 1, January 1980, pp. 25–33. © Printed in India.

Effect of transverse static magnetic field on stimulated Brillouin scattering of electromagnetic wave

S GUHA and N APTE
Department of Physics, Ravishankar University, Raipur 492 010

MS received 25 June 1979; revised 3 October 1979

Abstract. Stimulated Brillouin scattering of a plane polarised electromagnetic wave propagating perpendicular to a static magnetic field has been investigated analytically in an *n*-type piezoelectric semiconductor-plasma. Using coupled mode theory the dispersion relation is obtained and the threshold value of the amplitude of electromagnetic wave for the onset of instability is studied for both the forward and back-scattered modes. The role of the magnetostatic field on the threshold conditions for the unstable mode has been discussed.

Keywords. Stimulated Brillouin scattering; piezoelectric semiconductor; unstable mode; electromagnetic wave; transverse magnetic field.

1. Introduction

The phenomenon of stimulated Brillouin scattering (SBS) in an unmagnetised gaseous plasma in which the enhanced scattering takes place due to the excitation of ion waves at acoustic frequency has been investigated by a number of workers (Lashmore-Davies 1975; Drake *et al* 1974). SBS of circularly polarised electromagnetic waves from ion waves propagating parallel to the external magnetic field in a plasma has been studied by Lee (1974). He has also investigated the stimulated scattering of electromagnetic waves at the upper hybrid frequency from electrostatic waves propagating normal to the external magnetic field. Parametric excitation of Langmuir and acoustic waves in a transversely magnetised piezoelectric semiconductor-plasma has been investigated by Guha and Sen (1979). Recently the phenomenon of SBS in solids has been reviewed by Fabelinskii (1975) and it has been shown that in the non-linear interaction with the medium based on the mechanism of electrostriction, SBS phenomenon creates two waves, one being the frequency-shifted electromagnetic wave and the other an intense sound wave whose frequency can be upto 10^{12} sec^{-1} in a crystal. Such powerful sound generators of various frequencies may be of considerable interest in the investigations of different media by acoustic means (Fabelinskii 1975). In this paper we have investigated SBS in the region $kl \ll 1$ (k and l being the wave number and the electron mean free path respectively) of a plane-polarised electromagnetic wave propagating in a direction perpendicular to a uniform magnetostatic field in a weakly piezoelectric one-component (n-type) semiconductor such that the effect of nonlinear material parameters can be neglected (Thompson and Quate 1971). Using coupled mode theory the parametric decay of the electromagnetic wave into another electromagnetic wave and an acoustic wave (SBS) has been investigated and the threshold value of the pump electric field amplitude and the initial growth rate of parametric instability above threshold have been calculated.

2. Theoretical formulation

We consider the hydrodynamic model of an infinite and homogeneous n-type piezoelectric semiconductor-plasma (electron-plasma) immersed in a uniform magnetostatic field ($\mathbf{B}_0 = B_0\mathbf{z}$). The plasma is subjected to a high frequency electromagnetic wave (pump) which is transversely polarised in a plane perpendicular to $\mathbf{B}_0$ and propagates in the x-direction. Although the acoustic wave can be excited at any angle relative to the field $\mathbf{B}_0$, we have considered the particular geometry where the wave vectors of the excited acoustic wave as well as the scattered electromagnetic wave are along x-direction i.e. perpendicular to $\mathbf{B}_0$. The scattering is completely characterised by the following wave vector and frequency selection rules:

$$\mathbf{k}_{T0} = \mathbf{k}_{T1} + \mathbf{k}_S, \tag{1}$$

and

$$\omega_{T0} = \omega_{T1} + \omega_S. \tag{2}$$

We have assumed the spatial uniformity and the perfect frequency matching and hence, we take equations (1) and (2) to be satisfied exactly. Here $\mathbf{k}_{T0}$ ($\mathbf{k}_{T1}$) and ω_{T0} (ω_{T1}) are, respectively, the incident pump (scattered) wave vector and frequency; $\mathbf{k}_S$ and ω_S are the wave vector and frequency of the acoustic wave that is excited in the semiconductor such that $\omega_S \ll \omega_{T0}, \omega_{T1}$. The subscripts T0, T1 and S are meant for the pump, the scattered wave and the acoustic wave respectively.

The system of equations describing the decay of an electromagnetic wave into another electromagnetic wave and an acoustic wave consists of the following:

$$\frac{\partial n}{\partial t} + n_0 (\nabla . \mathbf{v}) = - \nabla n . \mathbf{v}, \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \nu \mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}_0) = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \frac{e}{m}(\mathbf{v} \times \mathbf{B}), \tag{4}$$

$$\frac{\partial \mathbf{D}}{\partial t} - \frac{1}{\mu_0}(\nabla \times \mathbf{B}) - e n_0 \mathbf{v} = e n \mathbf{v}, \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\nabla \times \mathbf{E}) = 0, \tag{6}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial E}{\partial x}, \tag{7}$$

and

$$D = \epsilon E + \beta \frac{\partial u}{\partial x}. \tag{8}$$

Equation (3) represents the continuity equation for electron in which n_0 and n are the unperturbed and perturbed electron density respectively and $\mathbf{v}$ is the perturbed fluid velocity. The momentum transfer equation for electron is described by (4) where **E, B** and $\mathbf{B}_0$ are the perturbed electric field, magnetic induction and d.c. magnetic field respectively. The electron collision frequency is denoted by ν. Equations (5) and (6) are the Maxwell's equations. The pump field gives rise to strain due to the displacement of the lattice $u(x, t)$, as a result of which an acoustic wave is excited inside the crystal. The equation of motion for $u(x, t)$ in the piezoelectric semiconductor is given by (7) where ρ, c and β are the mass density, the elastic constant and the piezoelectric coefficient of the crystal lattice respectively. The electric displacement D in the piezoelectric semiconductor is represented by eq. (8). $\epsilon(=\epsilon_0\epsilon_L)$ denotes the permittivity of the medium where ϵ_0 and ϵ_L being the permittivity of the free space and the static dielectric constant of the lattice respectively. The rest of the notations have their usual meanings. The effect of pressure term on the electron motion is ignored in the hydrodynamic domain. The terms involving nonlinear contributions are placed on the r.h.s. of eqs (3) to (6). Using the linearised forms of the above mentioned equations appropriate to the electromagnetic and acoustic modes, we have obtained the following normal mode equations:

$$\frac{\partial a_T^\pm}{\partial t} \pm i\omega_T a_T^\pm + \gamma_T a_T^\pm = 0, \tag{9a}$$

and

$$\frac{\partial a_S^\pm}{\partial t} \pm i\omega_S a_S^\pm + \gamma_S a_S^\pm = 0, \tag{9b}$$

where

$$a_T^\pm = v_T^\pm + \alpha_T^\pm B_T^\pm \mp \beta_T^\pm \epsilon E_T^\pm \tag{10a}$$

and

$$a_S^\pm = v_S^\pm + \beta_S^\pm K_p^\pm E_S^\pm, \tag{10b}$$

$a_T^\pm$ and $a_S^\pm$ are the normal mode amplitudes of the transverse electromagnetic and the acoustic wave respectively. γ_T and γ_S are the damping terms. The other notations are explained by Lashmore-Davies (1975). The procedure followed is similar to that of Lashmore-Davies (1975). $K_p^\pm$ is obtained by using eqs (5) and (6) as

$$K_p^\pm = \left(1 \mp \frac{(i K^2 \omega_s)}{2\gamma_s}\right), \quad K^2 = \frac{\beta^2}{\epsilon c}.$$

The electro-mechanical coupling coefficient of the piezoelectric semiconductor. The constants $\alpha_{T,S}^\pm$, $\beta_{T,S}^\pm$ and $\gamma_{T,S}^\pm$ are obtained from the basic equations (3-6) by taking linear combinations adequate to the different normal modes involved in the decay process. For that we have assumed

$$a_{T,S}^\pm \sim \exp\left[i\left(k_{T,S}\, x - \omega_{T,S}\, t\right)\right] \exp\left[-\gamma_{T,S}\, t\right].$$

$a^+_{T,S}$ and $a^-_{T,S}$ correspond to the forward and backward propagating waves with respect to the wave vectors $k_{T,S}$. The constants are

$$a^{\pm}_T = +\frac{ik_T\,\omega_c^2}{\mu_0\,en_0\,\omega_T^2}\left[\frac{1-\dfrac{\omega_T^2}{\omega_c^2}\left(1\pm\dfrac{i\gamma_T}{\omega_T}\right)^2}{\left(1\pm\dfrac{i\gamma_T}{\omega_T}\right)^2}\right],$$

$$\beta^{\pm}_T = \frac{i\,\omega_T}{en_0}\left[\frac{\left(1\mp\dfrac{i\gamma_T}{\omega_T}\right)^2-\dfrac{\omega_c^2}{\omega_T^2}}{1\mp\dfrac{i\gamma_T}{\omega_T}}\right],$$

$$\gamma_T = \frac{\nu\,\omega_p^2\,(\omega_T^2+\omega_c^2)}{2\,(\omega_T^2-\omega_c^2)^2},$$

$$\beta^{\pm}_S = \frac{\nu^2+\omega_c^2-\omega_S^2}{en_0\,\nu}$$

and $$\gamma_s = \frac{K^2\omega_s^2\,(\omega_c^2+\nu^2-\omega_s^2)}{2\omega_p^2\,\nu}$$

where we have assumed

$$\omega_{T0,1} > \nu \text{ and } \omega_p, \nu > \omega_s \cdot \omega_c = \frac{|e|B_o}{m}$$

is the electron cyclotron frequency, ν is the electron collision frequency and $\omega_p=(n_0e^2/m\epsilon_0\epsilon_L)^{1/2}$ represents the electron plasma frequency.

2.1. *Coupled mode equation*

To obtain the nonlinear interactions we have retained the quadratic terms in the wave variables in (3-6). Choosing only those dominant nonlinear terms which have the same space and time dependences as the corresponding normal mode and writing the normal mode amplitudes as follows:

$$a^{\pm}_{T,S}\,(x,t) = A^{\pm}_{T,S}\,(t)\exp\,[i\,(k_{T,S}\,x-\omega_{T,S}\,t)],$$

we have obtained the coupled mode equations. We have not considered the equations for the amplitudes $A^{\pm}_{T0}$ since these equations are not needed to calculate the threshold condition and growth rates for the parametric decay process. Thus we get the following coupled mode equations

$$\frac{\partial A^{\pm}_{T1}}{\partial t} + \gamma_{T1} A^{\pm}_{T1} = -\beta^{\pm}_{T1}\, e n^{*}_{S}\, v_{T0}, \tag{11a}$$

and

$$\frac{\partial (A^{\pm}_{S})^{*}}{\partial t} + \gamma_{S} (A^{\pm}_{S})^{*} = -\frac{e}{m} (v_{T1} B^{*}_{T0} + v^{*}_{T0} B_{T1}). \tag{11b}$$

We can express all the variables on the r.h.s. of eqs (11a) and (11b) in terms of normal mode amplitudes $a^{\pm}_{T0}$, $a^{\pm}_{T1}$, and $a^{\pm}_{S}$ with the aid of the linearised forms of basic equations and the definitions of the normal modes (equations (10)). Thus

$$v^{\pm}_{T0,1} = \frac{\omega_p^2\, a^{\pm}_{T0,1}}{2\omega^2_{T0,1}\left(1 - \dfrac{\omega_c^2}{\omega^2_{T0,1}}\right)^2}, \quad B^{\pm}_{T0,1} = \frac{i\, m\, k_{T0,1}\, \omega_p^2\, a^{\pm}_{T0,1}}{2e\omega^2_{T0,1}\left(1 - \dfrac{\omega_c^2}{\omega^2_{T0,1}}\right)^2},$$

and

$$(n^{\pm}_{S})^{*} = \pm \frac{n_0\, k_S\, \omega_S\, (a^{\pm}_{S})^{*}}{(2\omega_S^2 - \omega_c^2)}, \tag{12}$$

where the superscript asterisk represents the complex conjugate. In obtaining these wave variables we have assumed that the mode amplitudes are small. The effect of collisions on the non-linear terms has been neglected as we have considered weakly damped waves.

In order to calculate the coupling terms on the r.h.s. of (11) we have allowed for the influence of both forward and backward modes. Using (11) and (12) the coupled mode equations are obtained as:

$$\frac{\partial A^{\pm}_{T1}}{\partial t} + \gamma_{T1} A^{\pm}_{T1} = \mp i\, C_{0S}\, [A^{\pm}_{T0}(A^{+}_{S})^{*} + A^{\pm}_{T0}(A^{-}_{S})^{*}], \tag{13a}$$

and

$$\frac{\partial (A^{\pm}_{S})^{*}}{\partial t} + \gamma_{S} (A^{\pm}_{S})^{*} = -i\, C_{01}\, [A^{+}_{T1}(A^{+}_{T0})^{*} + A^{-}_{T1}(A^{-}_{T0})^{*}], \tag{13b}$$

where

$$C_{0S} = \frac{\omega_P^2\, \omega_{T1}\, \omega_S\, k_S \left(1 - \dfrac{\omega_c^2}{\omega^2_{T1}}\right)}{2\omega^2_{T0}(2\omega_S^2 - \omega_c^2)\left(1 - \dfrac{\omega_c^2}{\omega^2_{T0}}\right)^2}, \tag{14a}$$

and

$$C_{01} = \frac{\omega_P^4 \left(-k_{T0} + k_{T1}\right)}{\omega^2_{T1}\, \omega^2_{T0}\left(1 - \dfrac{\omega_c^2}{\omega^2_{T1}}\right)^2 \left(1 - \dfrac{\omega_c^2}{\omega^2_{T0}}\right)^2}. \tag{14b}$$

Introducing the combinations

$$A^{+}_{T} = A^{+}_{T1}(A^{+}_{T0})^{*} \text{ and } A^{-}_{T} = A^{-}_{T1}(A^{-}_{T0})^{*},$$

the four coupled equations describing the parametric interaction then become

$$\frac{\partial A_T^+}{\partial t} + \gamma_{T1} A_T^+ = -i C_{0S} |A_{T0}^+|^2 [(A_S^+)^* + (A_S^-)^*] , \quad (15a)$$

$$\frac{\partial A_T^-}{\partial t} + \gamma_{T1} A_{T0}^- = i C_{0S} |A_T^-|^2 [(A_S^-)^* + (A_S^-)^*], \quad (15b)$$

$$\frac{\partial (A_S^+)^*}{\partial t} + \gamma_S (A_S^+)^* = -i C_{01} A_T^+ , \quad (15c)$$

$$\frac{\partial (A_S^-)^*}{\partial t} + \gamma_S (A_S^-)^* = -i C_{01} A_T^- . \quad (15d)$$

From eqs. (15) the dispersion relation describing the stimulated Brillouin scattering is obtained under the reasonable approximations

$$A_{T0}^{\pm} = \text{constant}, |A_{T0}^{\pm}| \gg |A_{T1}^{\pm}|, |A_{T0}^{\pm}| \gg |A_S^{\pm}| ,$$

and $\quad A_{T1,S}^{\pm} \propto \exp(-i\omega t)$

as $\quad (-i\omega + \gamma_{T1})(-i\omega + \gamma_S) = -C_{0S} C_{10} [|A_{T0}^+|^2 - |A_{T0}^-|^2]. \quad (16)$

We have considered the case of a forward propagating pump wave for which $A_T^-=0$. It can be seen from (15b) and (15d) that the backward modes A_T^- and A_S^+ are not excited while the two modes A_T^+ and A_S^+ are coupled according to the dispersion relation

$$(-i\omega + \gamma_{T1})(-i\omega + \gamma_S) = -C_{0S} C_{01} |A_{T0}^+|^2 . \quad (17)$$

The dispersion relation represented by (17) can now be employed to obtain the threshold electric field E_{th} of the incident electromagnetic wave (pump) necessary for the simultaneous growth of the acoustic and the scattered electromagnetic waves for both the cases of stimulated Brillouin scattering—forward as well as backward.

2.2 *Stimulated Brillouin forward and backscattered modes*

When the scattered wave propagates in the forward direction the frequency and wave vector selection rules which are satisfied by the three nonlinearly interacting waves are the same as given by (1) and (2). Thus (14b) becomes

$$C_{01} = -\frac{\omega_p^4 k_S}{\omega_{T1}^2 \omega_{T0}^2 \left(1 - \frac{\omega_c^2}{\omega_{T1}^2}\right)^2 \left(1 - \frac{\omega_c^2}{\omega_{T0}^2}\right)^2} , \quad (18)$$

where we have used $k_{T0} - k_{T1} = k_S$.

For a back-scattered electromagnetic wave the wave vector selection rule becomes

$$\mathbf{k}_{T0} + \mathbf{k}_{T1} = \mathbf{k}_S.$$

Thus the back-scattered electromagnetic wave amplitude has the space-time dependence $\exp\,[i(-k_{T1}x - \omega_{T1}t)]$. Using this proportionality we get C_{01} same as (18). The dispersion relation for the stimulated Brillouin forward and backward scattered modes is obtained from (17) as

$$(-i\omega + \gamma_{T1})(-i\omega + \gamma_S) = + C_{0S}\, C_{01}\, |A^{+}_{T0}|^2. \tag{19}$$

The threshold electric field required for the onset of Brillouin instability is evaluated by putting $\omega=0$ in (19). Thus we get

$$E_{\text{th}} = \frac{Km\omega_{T0}\left(1 - \frac{\omega_C^2}{\omega_{T0}^2}\right)}{e\omega_p} \left\{ \frac{\omega_S(2\omega_S^2 - \omega_C^2)(\omega_C^2 + \nu^2 - \omega_S^2)\left(1 + \frac{\omega_C^2}{\omega_{T1}^2}\right)}{2\omega_{T1}k_S k_S\left(1 - \frac{\omega_C^2}{\omega_{T1}^2}\right)} \right\}^{1/2} \tag{20a}$$

Equation (19) can be employed to obtain the growth rate of the scattered electromagnetic wave and the acoustic wave well above the threshold by neglecting the damping terms (γ_{T1} and γ_S), i.e.,

$$\omega = \frac{e\omega_P\,|E_{T0}|}{m\omega_{T0}\left(1 - \frac{\omega_C^2}{\omega_{T0}^2}\right)} \left\{ \frac{k_S\, k_{T0}\, \omega_S}{2\omega_{T1}\left(1 - \frac{\omega_C^2}{\omega_{T1}^2}\right)(2\omega_S^2 - \omega_C^2)} \right\}^{1/2}. \tag{20b}$$

It can be seen that the growth rate is unaffected by the presence of the weak collisional damping terms γ_{T1} and γ_S as these terms are present even in the absence of the pump.

2.3 *Case I without magnetic field*

In the absence of the magnetostatic field (i.e. when $B_0 = 0$) if we assume $\omega_{T0} \sim \omega_{T1}$, eqs (20) become

$$E_{\text{th}} = \frac{Km\nu\omega_S}{e\,\omega_P} \left\{\frac{\omega_S\omega_{T0}}{k_S\, k_S}\right\}^{1/2}, \tag{21a}$$

$$\omega = \frac{e\,\omega_P\,|E_{T0}|}{2m\,\omega_{T0}} \left\{\frac{k_S\, k_S}{\omega_{T0}\,\omega_S}\right\}^{1/2} \tag{21b}$$

2.4 *Case II with magnetic field*

The threshold conditions have been studied to see the effect of the transverse magnetostatic field which gives interesting information about stimulated Brillouin scattering in a magnetoactive semiconductor-plasma. Dividing (20) by (21) and assuming $\omega_{T0} \sim \omega_{T1}$, we obtain

$$\frac{(E\text{th})_{B_0\neq 0}}{(E\text{th})_{B_0=0}} = \left(\frac{1}{2}\right)^{1/2}\left[\left(1-\frac{\omega_c^4}{\omega_{T0}^4}\right)(2\omega_S^2-\omega_c^2)\ (\omega_c^2+\nu^2-\omega_S^2)\right]^{1/2}\frac{1}{\nu\omega_S} \tag{22a}$$

and

$$\frac{(\omega)_{B_0\neq 0}}{(\omega)_{B_0=0}} = \frac{2}{\left(1-\frac{\omega_c^2}{\omega_{T0}^2}\right)}\left\{\frac{\omega_S^2}{2\left(1-\frac{\omega_c^2}{\omega_{T0}^2}\right)(2\omega_S^2-\omega_c^2)}\right\}^{1/2}. \tag{22b}$$

The magnetic field is found to add a new dimension to the variety of physically interesting phenomena that may be observed in the scattering. As the cyclotron frequency approaches other characteristic frequencies of the system, the excited fluctuations get significantly modified in the plasma. This has correspondingly changed the threshold conditions.

(i) When $\omega_c \simeq \omega_S$ we have $\omega_c < \nu$; thus we get from equations (22)

$$(E\text{th})_{B_0\neq 0}/(E\text{th})_{B_0=0} = 1/\sqrt{2}, \tag{23a}$$

and

$$(\omega)_{B_0\neq 0}/(\omega)_{B_0=0} = \sqrt{2}, \tag{23b}$$

which shows that if the electron cyclotron frequency and the acoustic wave frequency are of the same order, the growth rate above the threshold is $\sqrt{2}$ times to that in the absence of the magnetic field whereas the electric field of the pump required for the onset of instability is $1/\sqrt{2}$ times to that in the absence of the magnetic field. However, if $\omega_c<\omega_S$, the threshold conditions retain their form which they have in the absence of magnetic field.

(ii) The parametric excitation is found to be impossible in the limit $\omega_c>\sqrt{2}\,\omega_S$ since the threshold electric field and the growth rate become imaginary in this limit as can be seen from (22).

3. Results and discussion

From (20a) we infer that the parametric instability exists only in piezoelectric semiconductor-plasma because in the absence of piezoelectricity i.e., $K=0$, the damping term $\gamma_S=\nu$. Since $\nu>\omega_S$, the acoustic wave becomes highly damped and the assumption of low collisional damping is not valid which is necessary condition for the coupled mode theory. It is evident from equations (21) that the threshold electric field amplitude of the pump is reduced and the growth rate of the unstable mode is enhanced if the crystal has a high carrier concentration. The pump frequency ω_{T0} increases the growth rate (ω) but E_{th} remains unaffected since ω_{T0}/k_{T0} appearing inside the bracket in (21a) is a constant i.e., equal to the velocity of the electromagnetic wave in the crystal if $\omega_{T0} > \omega_p$. Equations (21) and (22) have been studied for the cases of forward and back scattering processes. It is observed that the threshold for the back scattering is smaller than that for forward scattering whereas for the growth rate, the reverse is true. It is also found that as the electron cyclotron frequency approaches the acoustic wave frequency, the threshold electric field is reduced from its value at the limit of vanishing magnetic field but the initial growth rate is enhanced. For the validity of the hydrodynamic model of the plasma taken here i.e. $kl \ll 1$, the maximum value of the acoustic wave frequency (ω_S) can be $\sim 10^{12}$ sec^{-1}. Since for the Brillouin instability to occur, $\omega_c \leqslant \omega_S$, this leads to the upper limit of the magnetostatic field required for SBS.

A typical case of n-CdS is taken for numerical analysis of the threshold conditions for forward scattering. When $B_0 = 0$, $\omega_p = 7 \times 10^{13}$ sec^{-1}, $\omega_S = 10^{11}$ sec^{-1} with a Nd-YAG laser of 1·06 μm as a pump, we get $(E_{th})_{B_0=0} = 6 \cdot 32 \times 10^4$ Vm^{-1}. The growth rate $(\omega)_{B_0=0}$ for the pump electric field above threshold (i.e. of 10^5 Vm^{-1}) comes out to be $1 \cdot 03 \times 10^9$ sec^{-1}. But when $\omega_c \simeq \omega_S$ i.e. $B_0 = 0 \cdot 5$ tesla the threshold electric field is reduced to $4 \cdot 4 \times 10^4$ Vm^{-1}. It is observed that only the acoustic mode is affected by the magnetostatic field since the excitation is possible only when $\omega_c \leqslant \omega_S$. As $\omega_c \leqslant \omega_S \ll \omega_{T0,1}$, the electromagnetic mode is unaffected by the magnetic field. The parametric excitation ceases to occur as soon as the cyclotron frequency becomes greater than the acoustic wave frequency.

Acknowledgements

One of the authors (NA) is thankful to her collegues Dr P K Sen, Dr S K Ghosh and Miss P Namjoshi for many helpful discussions. She is also grateful to the Council of Scientific and Industrial Research, New Delhi for the award of research fellowship. This work is partially supported by the University Grants Commission, New Delhi.

References

Drake J F, Kaw P K, Lee Y C, Schmidt G, Lin C S and Rosenbluth M N 1974 *Phys. Fluids* **17** 778

Fabelinskii T L 1975 *Quantum electronics* eds H Rabin and C L Tang (New York: Academic Press) **1** 363

Guha S and Sen P K 1979 *Phys. Lett.* **A70** 61

Lashmore-Davies C N 1975 *Plasma Phys.* **17** 281

Lee K F 1974 *Phys. Fluids* **17** 1220, 1343

Thompson R B and Quate C F 1971 *J. Appl. Phys.* **42** 907