Pramāṇa, Vol. 14, No. 1, January 1980, pp. 1–10. © Printed in India

Helicon-phonon interaction for oblique propagation in potassium

ROSCHEN IDICULLA and K S VISWANATHAN
Department of Physics, University of Kerala, Kariavattom, Trivandrum 695 581

MS received 7 May 1979

Abstract. The disperstion equation for oblique propagation of the wave in the xy plane for helicon-phonon interaction have been derived and numerical studies have been carried out on the nature of variation of the four different modes with the magnetic field and the inclination of the magnetic field with the direction of propagation.

Keywords. Helicon-phonon interaction; oblique propagation; dispersion equation; potassium.

1. Introduction

In two earlier papers (Viswanathan 1975; Viswanathan and Sekher 1976), the dispersion equation for helicon-phonon interaction for propagation of a wave parallel to the direction of the magnetic field as well as at an angle to the magnetic field were derived. The results of the numerical studies in the case of cubic crystals were also presented for the nature of variation of the four modes with the magnetic field (Sekher and Viswanathan 1978; Idiculla and Viswanathan 1978). These results have clearly demonstrated that in the resonance region where the velocity of the helicon is nearly equal to the velocity of one of the acoustic waves, mode conversion takes place and the interacting modes are of a hybrid nature, behaving like neither helicons nor phonons. No numerical study has so far been carried out on the nature of the helicon-phonon interaction for oblique propagation, when the helicon mode is propagated at an angle ϕ to the direction of the impressed magnetic field. Such studies could throw light on the difference brought about in the nature of the helicon-phonon interaction by the inclination of the magnetic field to the direction of propagation. In fact, the numerical study for oblique propagation is much more complicated than the case of parallel propagation, in view of the complexity of the dispersion equation in this case. In the present paper, we report the results of the numerical study of the helicon-phonon interaction for propagation, of the wave inclined to the magnetic field at angles $\phi=0°$, 10°, 20°, 30°, and 40° respectively for the metal potassium. Non-local effeots have been neglected. The numerical studies reveal that for oblique propagation, the helicon interacts with the longitudinal mode also even when the propagation is along a symmetry axis. Also, we show that as the inclination is increased, the modes are hybrid for a larger range of the magnetic field. In §4, we reproduce a few figures giving the nature of variation of the velocities of the four modes with ϕ, and these figures show that helicon-phonon interaction makes the acoustic modes also sensitive to the inclination of the magnetic field.

Our numerical studies also show that for parallel propagation along the principal axes, the helicon interacts with only one of the two shear modes, namely the mode that has the same circular polarisation as that of the helicon, and the interaction lifts the degeneracy of these modes.

2. The dispersion equation for oblique propagation in the xy plane of a cubic crystal

The general dispersion equation for oblique propagation is given by (Viswanathan and Sekher 1976)

$$\begin{vmatrix} a_{11}-\frac{i\omega H^2}{C^2}\sin^2\phi\,\sigma'_{33} & a_{12}+\frac{i\omega H^2}{2C^2}\sin 2\phi\,\sigma'_{33} & a_{13}-\frac{i\omega H^2}{C^2}\sin\phi\,\sigma'_{32} & \frac{H}{2C}\sin\phi\left(\sigma'_{33}+\frac{i\sigma'_{32}}{\cos\phi}\right) \\ a_{21}+\frac{i\omega H^2}{2C^2}\sin 2\phi\,\sigma'_{33} & a_{22}-\frac{i\omega H^2}{C^2}\cos^2\phi\,\sigma'_{33} & a_{23}+\frac{i\omega H^2}{C^2}\cos\phi\,\sigma'_{32} & -\frac{H}{2C}\cos\phi\left(\sigma'_{33}+\frac{i\sigma'_{32}}{\cos\phi}\right) \\ a_{31}-\frac{i\omega H^2}{C^2}\sin\phi\,\sigma'_{23} & a_{32}+\frac{i\omega H^2}{C^2}\cos\phi\,\sigma'_{23} & a_{33}-\frac{i\omega H^2}{C^2}\sigma'_{22} & \frac{H}{2C}\sigma'_{-} \\ -\frac{4\pi\omega^2}{C^3}\sigma'_{23}H\sin\phi & \frac{4\pi\omega^2 H}{C^3}\cos\phi\,\sigma'_{23} & -\frac{4\pi\omega^2}{C^3}H\sigma'_{22} & -\frac{1}{2i}\left(k^2\cos\phi-\frac{4\pi\omega\sigma'_{-}}{C^2}\right) \end{vmatrix} = 0$$

where a_{ik} denotes the elements of a matrix A given by

$$a_{ik} = (\tilde{C}_{1i1k}\, k^2 - \rho\omega^2\, \delta_{ik}). \tag{2}$$

C_{1i1k} denotes the elastic constant referred to a set of orthogonal axes e_1, e_2, e_3 such that the direction of propagation coincides with the e_1 axis. σ'_{ik} is the element of the conductivity tensor of the medium referred to a set of orthogonal axes e'_1, e'_2, e'_3 in which e'_1 coincides with the direction of the external magnetic field making an angle ϕ with the e_1 axis, while e'_3 coincides with e_3. The components of the conductivity tensor are given by

$$\begin{aligned}
\sigma'_{22} &= (\mathrm{Ne}^2\, \nu/m\, \Omega^2),\\
\sigma'_{33} &= \frac{\mathrm{Ne}\, C}{H}\,\frac{3\pi}{8}\, kR\cos\phi\tan^2\phi + \frac{\mathrm{Ne}^2\,\nu}{m\,\Omega^2},\\
\sigma'_{23} &= -\,\sigma'_{32} = (\mathrm{Ne}\; C/H),\\
\sigma'_{-} &= \left(\sigma'_{23} + \frac{i\,\sigma'_{22}}{\cos\phi}\right).
\end{aligned} \tag{3}$$

Collision introduces a small damping factor to the wave but the damping rate is so small that we can ignore it. Besides, we neglect the nonlocal effects also so that the component σ'_{23} alone survives in the conductivity tensor.

The elements of the matrix A in the case of wave propagation along a direction e_1 inclined at an angle θ to the x axis in the xy plane, are well known and are quoted in Auld (1973). Using these expressions and writing $k^2 = x_R + ix_I$ we can write the dispersion equation (1) as

$$\begin{vmatrix}
a_1(x_R+ix_I)+b_1 & a_5(x_R+ix_I) & ib_2 & ib_3\\
a_5(x_R+ix_I) & a_2(x_R+ix_I)+b_1 & ib_4 & ib_5\\
-ib_2 & -ib_4 & a_3(x_R+ix_I)+b_1 & b_5\\
b_6 & b_7 & 0 & ia_4(x_R+ix_I)+ib_8
\end{vmatrix} = 0, \tag{4}$$

where
$$\begin{aligned}
a_1 &= C_{11} - \left(\frac{C_{11}-C_{12}}{2} - C_{44}\right)\sin^2 2\theta,\\
a_2 &= C_{44} + \left(\frac{C_{11}-C_{12}}{2} - C_{44}\right)\sin^2 2\theta,\\
a_3 &= C_{44},\\
a_4 &= \tfrac{1}{2}\cos\phi,
\end{aligned} \tag{5}$$

$$a_5 = -\left(\frac{C_{11}-C_{12}}{2} - C_{44}\right) \sin 2\theta \cos 2\theta,$$

$$b_1 = -\rho\omega^2;\ b_2 = \sin\phi\,(\text{Ne}\,\omega H/C);\ b_3 = \tan\phi\,(-\text{Ne}/2);$$

$$b_4 = \cos\phi\,(-\omega H\,\text{Ne})/C;\ b_5 = \text{Ne}/2;\ b_6 = (-4\pi\omega^2\text{Ne}\sin\phi)/C^2;$$

$$b_7 = (4\pi\omega^2\text{Ne}\cos\phi)/C^2;\ b_8 = (-2\pi\omega\text{Ne})/CH.$$

The determinental equation (4) can be expanded in powers of x_R and x_I. It is a complex equation and hence by equating the real and imaginary parts separately to zero, one could obtain two equations from which x_R and x_I could be determined. After heavy algebraic work one obtains

$$A_1(x_R^4 + x_I^4 - 6x_R^2 x_I^2) + A_2 x_R(x_R^2 - 3x_I^2) - A_3(x_R^2 - x_I^2)$$
$$- A_4 x_R + A_5 = 0 \tag{6a}$$

and

$$4A_1 x_R x_I(x_R^2 - x_I^2) - A_2 x_I(3x_R^2 - x_I^2) + 2A_3 x_R x_I + A_4 x_I = 0 \tag{6b}$$

where the coefficients A_1, A_2, A_3, A_4 and A_5 are given by

$$A_1 = a_1 a_2 a_3 a_4 - a_3 a_4 a_5^2, \tag{7a}$$

$$A_2 = a_1 a_2 a_3 b_8 + a_1 a_2 a_4 b_1 + a_1 a_3 b_1 b_4 + a_2 a_3 a_4 b_1$$
$$- a_3 a_5^2 b_8 - a_4 a_5^2 b_1, \tag{7b}$$

$$A_3 = a_1 a_2 b_1 b_8 + a_1 a_3 b_1 b_8 + a_1 a_4 b_1^2 + a_2 a_3 b_1 b_8 + a_2 a_4 b_1^2$$
$$+ a_3 a_4 b_1^2 - a_1 a_4 b_4^2 - a_1 a_3 b_5 b_7 - a_5^2 b_1 b_8 + a_4 a_5 b_2 b_4$$
$$+ a_3 a_5 b_5 b_6 + a_4 a_5 b_2 b_4 - a_2 a_4 b_2^2 + a_3 b_3 a_5 b_7 - a_2 a_3 b_3 b_6, \tag{7c}$$

$$A_4 = (a_1 b_1^2 b_8 + a_2 b_1^2 b_8 + a_4 b_1^3 - a_1 b_4^2 b_8 - a_4 b_1 b_4^2$$
$$+ a_1 b_7 b_4 b_5 - a_1 b_1 b_5 b_7 - a_3 b_1 b_5 b_7 + a_5 b_2 b_4 b_8 - a_5 b_4 b_5 b_6$$
$$+ a_5 b_1 b_5 b_6 + a_5 b_2 b_4 b_8 - a_5 b_2 b_5 b_7 - a_2 b_2^2 b_8 + a_2 b_2 b_5 b_6$$
$$- a_4 b_1 b_2^2 + a_5 b_1 b_3 b_7 - a_2 b_1 b_3 b_6 - a_3 b_1 b_3 b_6), \tag{7d}$$

$$A_5 = (b_1^3 b_8 - b_1 b_4^2 b_8 + b_1 b_4 b_5 b_7 - b_1^2 b_5 b_7 - b_1 b_2^2 b_8 + b_1 b_2 b_5 b_6$$
$$+ b_2^2 b_5 b_7 - b_2 b_4 b_5 b_6 - b_1^2 b_3 b_6 - b_2 b_3 b_4 b_7 + b_3 b_4^2 b_6). \tag{7e}$$

An obvious solution of (6b) is $x_I=0$. This means that undamped propagation is possible even if the wave vector is inclined to the magnetic field provided we neglect nonlocal effects in the expression for the components of the conductivity tensor. The real part x_R of k^2 is then determined by the equation

$$A_1 x_R^4 + A_2 x_R^3 - A_3 x_R^2 - A_4 x_R + A_5 = 0. \tag{8}$$

This equation is a quartic in x_R and gives four real solutions for it.

3. Numerical results

We denote by θ the angle between the direction of propagation of the wave and the x axis, whereas ϕ denotes the angle which the magnetic field makes with the direction of propagation of the wave. A computer programme was written to obtain the four real roots of the quartic equation (6a) for different values of θ, ϕ and the magnetic field for the metal potassium. Our numerical calculations were carried out for a typical value $\omega = 2\pi \times 10^8$ Hz for the frequency of the exciting wave. The values of the other constants used in the calculations are as follows:

$$C_{11} = 0{\cdot}457 \times 10^{11} \text{ dynes/cm}^2,$$

$$C_{12} = 0{\cdot}374 \times 10^{11} \text{ dynes/cm}^2,$$

$$C_{44} = 0{\cdot}263 \times 10^{11} \text{ dynes/cm}^2,$$

$$\rho = 0{\cdot}91 \text{ gm/cm}^3,$$

and

$$N = 1{\cdot}40 \times 10^{22}.$$

Computer calculations were carried out for values of the magnetic field ranging from $0{\cdot}2 \times 10^5$ G to $4{\cdot}4 \times 10^5$ G at intervals of $0{\cdot}2 \times 10^5$ G. The computer could mix up the roots for the different modes, especially in the resonance region, and a proper assignment of the nature of any mode, whether it is a helicon or elastic wave, could be ascertained only by studying the eigen vector of this mode. (Idiculla and Viswanathan 1978).

The velocity of a pure helicon mode has been given by Kaner and Skobov (1971) as

$$U_H^2 = |\cos\phi| \, (\omega \, CH/4\pi \, \mathrm{Ne}), \tag{9}$$

and this indicates a parabolic increase of the velocity with H. The velocity of a non-interacting phonon mode is independent of H. As H is increased, the velocity of the helicon increases parabolically until it becomes resonant with the velocity of one of the acoustic modes. Then interaction takes place and the helicon no longer shows the regular parabolic increase with H, while at the same time, the interacting acoustic mode shows a variation in the velocity with H. Thus in the interaction region, neither of the modes shows the characteristic behaviour of a helicon or a phonon but are of a hybrid nature. When the interaction region is passed, the helicon again shows the parabolic increase with H until it interacts with another acoustic mode.

Figures 1(a) to 1(c) give the variation in velocities of the four modes with the magnetic field, when the magnetic field is inclined at angles 0°, 20° and 40° respectively with the direction of propagation, which coincides with the x axis (i.e. $\theta=0$, $\phi=0°$, 20° and 40°). From figure 1(a) it follows that the velocity of the mode marked as (3) increases parabolically with the magnetic field until H is nearly equal to $0{\cdot}8\times10^5$ G. Thus mode (3) represents the helicon till this value of the magnetic field. Between $0{\cdot}8\times10^5$ G and $1{\cdot}4\times10^5$ G, resonant interaction takes place and mode (1), which has been behaving like a phonon, starts showing a small increase with the magnetic field. Thus in the range of H between $0{\cdot}8\times10^5$ G and $1{\cdot}4\times10^5$ G, the modes (1) and (3) are hybrid waves, made up of both electromagnetic as well as elastic vibrations. After $H=1{\cdot}4\times10^5$ G, mode (1) shows helicon-like behaviour, and modes (2), (3) and (4) exhibit the characteristics of acoustic waves for all higher values of the magnetic field, thus showing that the helicon does not interact with the remaining two acoustic modes.

In figure 1(b), which corresponds to the case $\theta=0$, $\phi=20°$, the same pattern of behaviour is followed as in figure 1(a), until the field reaches the value $H=1{\cdot}4\times10^5$ G. After this, mode (1) behaves like a helicon till $H=2\times10^5$ G. Between $H=2{\cdot}0\times10^5$ G and $H=2{\cdot}4\times10^5$ G, we find that the modes (1) and (2) exhibit hybrid nature. In this region, interaction takes place between the helicon and the longitudinal acoustic mode, which had till now been represented by mode (2). From $H=2{\cdot}4\times10^5$ G onwards mode (2) shows the characteristics of a helicon, while mode (1) represents the longitudinal phonon mode.

As can be seen from figure 1(c), the primary consequence of increasing the angle between the magnetic field and the direction of propagation is that the waves exist as hybrid modes over larger ranges of the magnetic field. The region of interaction between modes (3) and (1) ranges between $H=1{\cdot}0\times10^5$ G and $H=1{\cdot}8\times10^5$ G. After

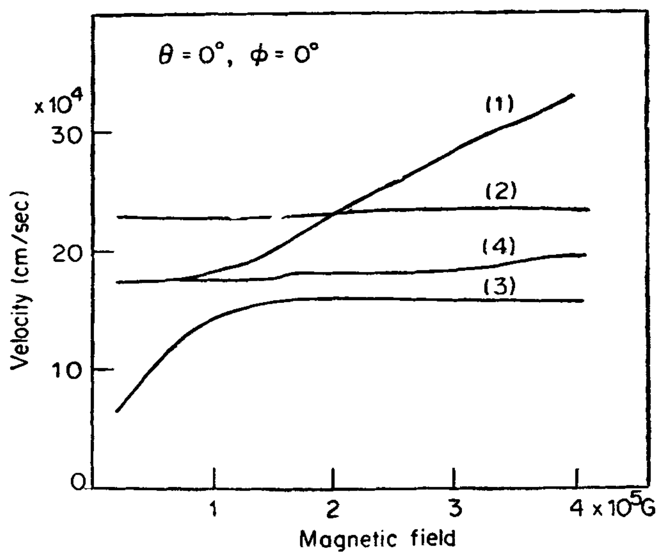

Figure 1a.

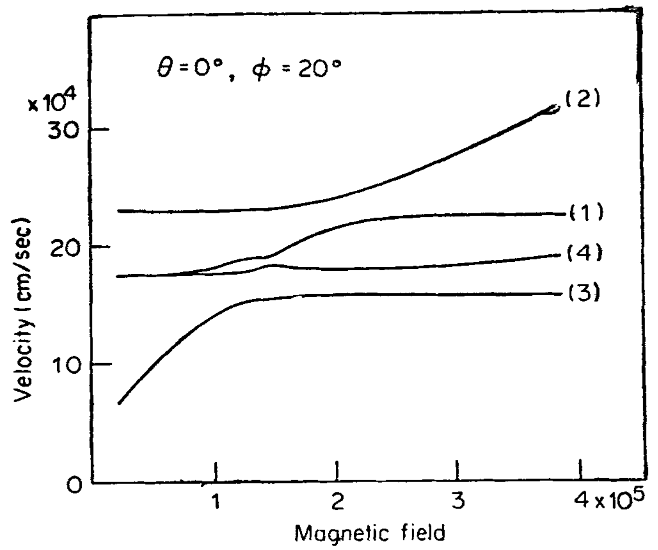

Figure 1b.

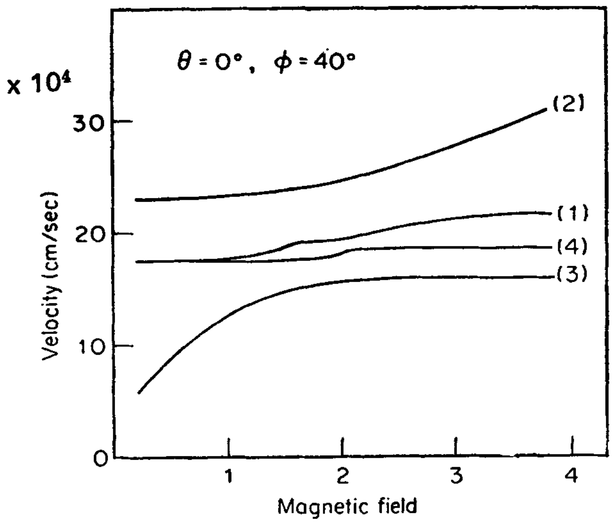

Figure 1c.

Figure 1. Variation in the phase velocity with magnetic field for potassium **a.** $\theta=0$, $\phi=0°$; **b.** $\theta=0$, $\phi=20°$; **c.** $\theta=0$, $\phi=40°$.

this, mode (3) behaves like a phonon. But mode (1), which started showing the characteristics of a helicon, interacts with the longitudinal phonon mode (2), and retains its hybrid nature until $H=2{\cdot}6\times10^5$ G. After this, the velocity of mode (2) increases parabolically with the magnetic field just like a helicon. We note that the mode (4), which represents one of the shear modes, takes no part in the interaction and remains almost unaffected by the magnetic field. Since the velocities of the two shear modes are equal for wave propagation along the x axis, the two transverse modes can be resolved into a left circularly polarised mode and another right circularly polarised mode. Since the helicon is right circularly polarised, it will interact with the right circularly polarised acoustic mode only. This statement can be mathematically proved also, starting from the dispersion equation for parallel propagation (Viswanathan 1975). Thus of the two shear modes propagating with the same velocity along the x axis, only one mode will interact with the helicon. This accounts for the fact that the velocity of the mode (4) exhibits no change with the magnetic field. Again when the acoustic mode regains its phonon characteristics after passing through the interaction region, its velocity is lowered slightly. The helicon-phonon interaction thus lifts the degeneracy in the velocities of the two shear modes.

In figures 2(a) to 2(c), we plot the variation in the velocities of the four modes for wave propagation at an angle 30° with the x axis for different orientations of the magnetic field. The behaviour of the four modes is similar to the previous case.

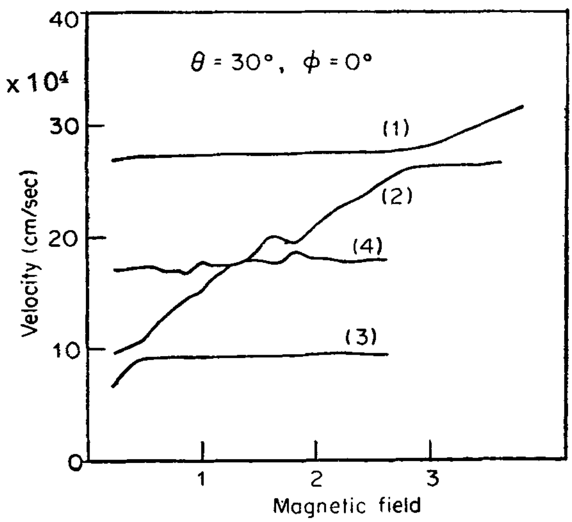

Figure 2a.

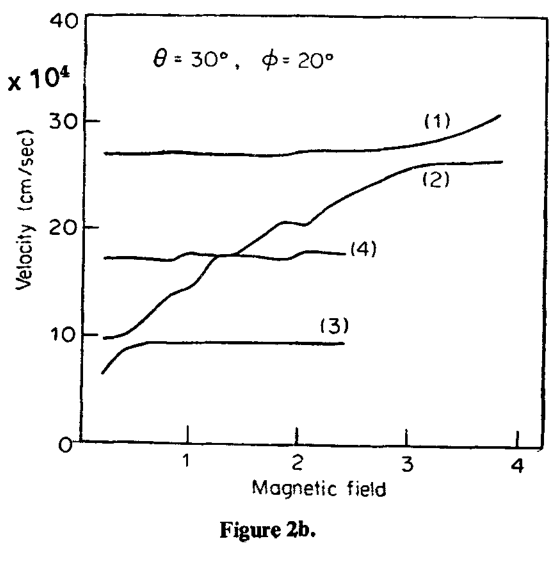

Figure 2b.

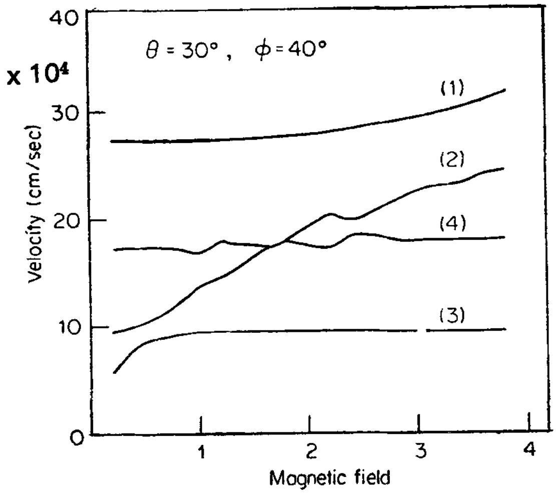

Figure 2c.

Figure 2. Variation in the phase velocity with magnetic field for potassium. **a.** $\theta = 30°$, $\phi = 0°$; **b.** $\theta = 30°$, $\phi = 20°$; **c.** $\theta = 30°$, $\phi = 40°$.

Again, the mode (4), representing a quasi-shear elastic wave, does not interact with the helicon, except showing some kinks in the resonant region.

4. Variation of the velocities of the four modes with the inclination of the magnetic field

From equation (9), it follows that the velocity of a pure helicon is proportional to $\sqrt{\cos\phi}$ ($\phi<\pi/2$), while a phonon should show no variation both with the direction as well as magnitude of the magnetic field. If one plots the graph of the function $y(\phi)=[V(0)-V(\phi)]/V(\phi)$ against $1-\sqrt{\cos\phi}$ for different values of ϕ and for any value of θ, the graphs for the helicon should be a straight line passing through the origin inclined at 45° with the x axis, while the graphs for the phonons should coincide with the x axis.

In figure 3, we plot the variation of $[V(0)-V(\phi)]/V(0)$ against $1-\sqrt{\cos\phi}$ for $\theta=0$ and for the magnetic field $H=1\times10^5$ G, when helicon-phonon interaction is present. The mode marked as (3) in the figure represented the helicon for lower values of the magnetic field, but its graph deviates from the line $y=x$. Similarly the modes marked as (1) and (2), which represent a right circularly polarised shear mode and a longitudinal mode respectively, are inclined both to the x axis, whereas in the absence of interaction, their graphs should coincide with the x axis. We have also drawn the graphs of $y(\phi)$ for different values of θ and H, but for lack of space we do not reproduce them. All these figures clearly demonstrate that in the interaction region, the graphs of $y(\phi)$ against $(1-\sqrt{\cos\phi})$ are curves inclined both to the x axis as well as to the line $y=x$ and depart significantly from the behaviour of a pure helicon or phonon in the absence of interaction.

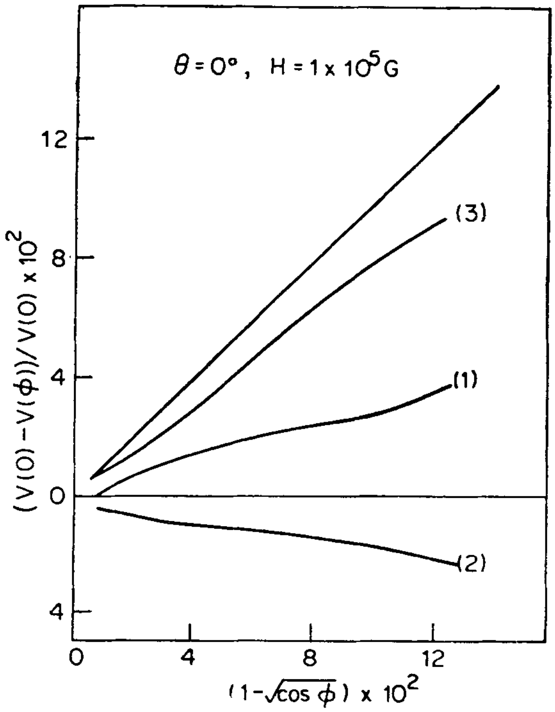

Figure 3. Variation of the parameter for propagation of the wave along the x axis and for the magnetic field $H = 1 \times 10^5$G.

5. Concluding remarks

Our numerical investigations show that in the resonant region the waves are of a hybrid nature and that the helicon interacts with the longitudinal mode also at inclined magnetic fields. For parallel propagation along a symmetry axis, the helicon interacts only with the shear mode that has the same sense of circular polarisation, and the helicon-phonon inteıaction removes the degeneracy in the velocities of the two modes. Further, the helicon-phonon interaction has the effect of lowering the values of the velocities of the interacting shear and longitudinal modes. All the four modes show variation with the inclination of the magnetic field near the resonant region.

References

Auld B A 1973 *Acoustic fields and waves in solids* (New York: Wiley Interscience) Vol. 1
Kaner E A and Skobov V G 1971 *Plasma effects in metals* (London: Taylor and Francis)
Idiculla R and Viswanathan K S 1979 *Can. J. Phys.* **57** 353
Sekher R and Viswanathan K S 1978 *J. Phys.* **F8** 1099
Viswanathan K S 1975 *J. Phys.* **F5** L107
Viswanathan K S and Sekher R 1976 *J. Phys.* **F6** 993