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# Coulomb energy of $^3$He and charge-symmetry breaking in nuclear forces

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**Abstract.** We report a calculation of Coulomb energy of $^3$He with two-term separable potentials of a form proposed recently, and a perturbative calculation of the charge-symmetry-breaking potential, again in a two-term separable form. By using potential sets adjusted to various values of proton-neutron effective range difference, some limit is sought to be placed on this quantity, about which nothing is known experimentally.

**Keywords.** Nuclear structure; charge-symmetry-breaking; mass difference; coulomb energy; two-term separable potential.

## 1. Introduction

The question of charge symmetry of nuclear forces has remained unresolved despite more than a decade of activity in this field. In the past it was simply assumed that the (nuclear) force between two protons is the same as between two neutrons. The binding energy difference, $\triangle_{BE}$ between $^3$H and $^3$He, or other mirror nuclei for that matter, was sought to be explained on the basis of coulomb forces between protons. Though several calculations that have been performed have produced different values for the coulomb energy of $^3$He (Okamoto and Pask 1971), it is quite clear that a small discrepancy $\triangle = \triangle_{BE} - E_c$ remains between the experimental value of $\triangle_{BE}$ and the theoretical value of $E_c$, which requires the introduction of charge-symmetry-breaking (CSB) forces. Since $\triangle$ turns out to be positive, it follows that the *n-n* force should be somewhat stronger than the *p-p* force. From a theoretical standpoint as well it is quite clear that there is a certain amount of CSB component in the nuclear force arising due to the *n-p* and $\pi^{\pm} - \pi^c$ mass differences, and (mainly) due to the electromagnetic mixing within the various meson multiplets. Now, this CSB component should also make the *n-n* and *p-p* scattering lengths, $a_{nn}$ and $a_{pp}$, different from each other. Since $a_{nn}$ is a poorly determined parameter, earlier it was *presumed* that $|a_{nn}| > |a_{pp}|$, to maintain consistency with the positive sign of $\triangle$. Since the last few years, a better determination of $a_{nn}$ has been made and it appeared that $|a_{nn}| < |a_{pp}|$ (Henley 1972). However this was not confirmed by the Lousanne $\pi^- d \to \gamma nn$ measurements reported on at the Graz conference of 1978. Thus the situation is again fluid. Further separation of electromagnetic effects from the *p-p* data is a highly model-dependent procedure and may entail an error of as much as $\pm$ 5 fm in $a_{pp}$ (Sauer 1976). If indeed $|a_{nn}| < |a_{pp}|$, we have the paradoxical situation that whereas

scattering length measurements require the $p$-$p$ force to be more attractive than the $n$-$n$ force, the binding energy differences require just the opposite. To circumvent this difficulty it has been suggested that whereas $|a_{nn}| < |a_{pp}|$, the $n$-$n$ and $p$-$p$ effective ranges, $r_{nn}$ and $r_{pp}$ respectively, could be different from each other so as to make the $n$-$n$ force in a three-nucleon system effectively stronger than the $p$-$p$ force (Gibson and Stephenson 1975). At the moment a small difference (of either sign) between $r_{nn}$ and $r_{pp}$ is quite consistent with experiment, since $r_{nn}$ is even more poorly known that $a_{nn}$. But there is no reliable estimate of the difference $\triangle r(=r_{pp}-r_{nn})$ required to produce a positive $\triangle$, offsetting the effect of scattering length difference $\triangle a(=a_{nn}-a_{pp})$; which presumably acts in the opposite direction.

A number of calculations have been performed to obtain the CSB potential from the mixing of various meson multiplets (Friar and Gibson 1978). Most of these attempts have been hampered by a lack of precise experimental information, known uncertainties in coupling constants, crude wave functions and other calculational inaccuracies. In their recent analysis Friar and Gibson (1978) find that their CSB potential yields $|a_{nn}| > |a_{pp}$ by about 1 fm and $r_{nn} < r_{pp}$ by about 0·02 fm. They find the $V_{\mathrm{CSB}}$ contribution to the $^3$H – $^3$He binding energy difference to be $\sim$ 50 keV. However, they calculate only the $\rho$-$\omega$ mixing effect, and even there all the required parameter are not known with great reliability.

The objective of the present calculation is two-fold: Firstly, we report a new calculation of $E_c$ with a set of separable potentials introduced by us recently and which provide a more accurate description of the three-nucleon properties than did the earlier sets (Mehdi and Gupta 1976). Secondly, we report a perturbative calculation of $E_{\mathrm{CSB}}$, the contribution of CSB potential to the binding energy. For this purpose we represent the CSB potential phenomenologically in a separable form rather than derive it from any underlying theory of charge symmetry-breaking. We introduce a large set of such potentials $V_{\mathrm{CSB}}$ yielding different values of $\triangle r$ and $\triangle a$ and then find the expectation value of $V_{\mathrm{CSB}}$ in the 3-$N$ ground-state wave function. We do this to find the sensitivity of $E_{\mathrm{CSB}}$ to $\triangle a$ and $\triangle r$ and thereby place some possible limits on the latter.

## 2. Results and discussion

This approach, which avoids the various uncertainties in calculating $V_{\mathrm{CSB}}$ from the mixing of multiplets, has its own model-dependence to contend with. We have introduced a large set of one-term and two-term CSB potentials of various shapes into the various sets of basic $N$-$N$ potential to get an idea of this model-dependence. As we shall see, despite the model-dependence, we are able to place rather narrow limits on $\triangle r$. Also, in the evaluation of $V_{\mathrm{CSB}}$ from 'basic principles' both $\triangle r$ and $\triangle a$ get fixed. On the other hand, we keep $\triangle a$ also as a free parameter and give it both positive and negative values.

Most calculations involving separable potentials have been performed for the following form of the potential function:

$$\text{Shape I: } g(p) = (p^2 + \beta^2)^{-1}.$$

**Table 1.** The Coulomb energy of $^3$He for various sets of shape II singlet potentials with (III-V) and without (I, II) repulsive term. For parameters of these potential sets and the two-body data to which they are fitted, see Mehdi and Gupta (1976).

| Set | I | II | III | IV | V |
|---|---|---|---|---|---|
| $V_c$ (in MeV) | 0·743 | 0·733 | 0·734 | 0·722 | 0·708 |

Kharchenko *et al* (1968) introduced a set of potentials of the general form $(p^2+\beta^2)^{-n}$. We used the above form for $n=2$:

$$\text{Shape II: } g(p) = (p^2+\beta^2)^{-2}$$

for the calculation of 3-body binding energy and electromagnetic form factors (Mehdi and Gupta 1976), wherein we included a repulsive term in the singlet potential as well. We found that shape II potentials give a significant improvement over those of shape I. We have now calculated $E_c$ for all the five sets that we introduced, and our results are shown in table 1.

Results of $E_c$ with shape I potentials were reported in previous papers (Mehdi and Gupta 1974; Gupta and Mitra 1967) and on comparison we find that shape II lowers $E_c$ by about 7–10%. This brings our result somewhat closer to that obtained by variational calculations or by Faddeev calculations with local potentials. However, the effect of including a repulsive term (sets III-V) is only 2–4% as against 10% with potentials of shape I. Accepting the value 0·71 MeV of $E_c$ (which is the lowest of all the sets!), the gap $\triangle$ is 50 keV, to be filled in by the addition of CSB forces. The inclusion of a tensor force is likely to bring $E_c$ still lower by about 5%, thereby increasing this gap to say about 70 to 80 keV. This is to be compared with $\triangle\sim$120 keV obtained by variational and other methods. An almost model-independent calculation by Brandenburg *et al* (1978) yields $E_c\sim$6·5 MeV and after including many other effects, the final contribution of $E_{\text{CSB}}$ as 83 keV.

The gap $\triangle$, whether 70 keV or 120 keV, represents the contribution of the CSB potential, and is very small compared to the contribution of the main potential term to the binding energy of the 3-nucleon system [(small) binding energy=(large) potential energy—(large) kinetic energy]. Thus a perturbative calculation of $E_{\text{CSB}}$ should prove quite adequate. In this spirit, we take the $s$-wave singlet $N$-$N$ potential to be of the form

$$V_{\text{singlet}} = P_\sigma^- P_\tau^+ [V_s + (\tau_3^{(1)} + \tau_3^{(2)}\ V)_{\text{CSB}}],$$

with the charge symmetry conserving part $V_s$ given by

$$\langle p \mid V_s \mid p' \rangle = \lambda_s [g(p)\, g(p') - nf(p) f(p')],$$

and the CSB part by

$$\langle p \mid V_{\text{CSB}} \mid p' \rangle = \lambda_{\text{CSB}} [h(p)\, h(p') - n_1 h_1(p)\, h_1(p')].$$

For $V_s$ we take the various sets used earlier in the calculation of binding energy and form factors (Mehdi and Gupta 1976). For CSB we consider one-term ($n_1=0$) as well as two-term potentials ($n_1\neq 0$). For $n_1=0$, we consider three different forms for $h(p)$ to study the effect of its variation on $E_{\rm CSB}$:

$$A : h(p) = (p^2 + \beta_h^2)^{-1},$$

$$B : h(p) = (p^2 + \beta_h^2)^{-2},$$

$$C : h(p) = p^2 (p^2 + \beta_h^2)^{-2}.$$

The parameter $\beta_h$ is varied to obtain various values of $\triangle r$ for a fixed $\triangle a$. However, even a wide variation in $\beta_h$ exhibits hardly any variation in $\triangle r$. For this reason, we show in table 2 our results of $E_{\rm CSB}$ for only one value of $\beta_h(9{\cdot}0a)$. For different sets and shape for $h(p)$, $\triangle r$ varies from $-0{\cdot}009$ fm to $-0{\cdot}020$ fm and correspondingly $E_{\rm CSB}$ from $-40$ keV to $-100$ keV. However, the important point is that the sign of $\triangle$ is *negative*. Thus this phenomenological potential tends to increase the gap between $E_c$ and $\triangle_{\rm BE}$ rather than plug it. Now the sign of $\triangle$ depends directly on the sign of $\lambda_{\rm CSB}$, and $\lambda_{\rm CSB}$ is proportional to $(\triangle a)^{-1}$ as long as $\lambda_{\rm CSB}\ll\lambda_s$. Thus no amount of variation is $\beta_h$ or the shape of $h(p)$ can induce a change in the sign of $\lambda_{\rm CSB}$ and consequently of $E_{\rm CSB}$ as long as we take $|a_{nn}|<|a_{pp}|$. Therefore, the only way in which such a phenomenological potential can yield the right sign for $\triangle a$ as well as for $\triangle$, if at all, is to have a more complicated CSB potential, to wit a potential with at least two-terms. Then one may be able to adjust the parameters (which will be four in number) so as to get a wide range of $\triangle r$ for a given $\triangle a$, and thereby the correct sign for $\triangle$.

For the two-term $V_{\rm CSB}$ we consider the following shapes:

$$h(p) = (p^2 + \beta_h^2)^{-2}; \quad h_1(p) = p^2 (p^2 + \beta_{\bar{h}}^2)^{-2}.$$

Since there are in all four adjustable parameters. $\lambda_{\rm CSB}$, $n_1$, $\beta_h$, $\beta_{\bar{h}}$, we fix $\beta_h$ and $\beta_{\bar{h}}$ arbitrarily and adjust $\lambda_{\rm CSB}$ and $n_1$ to get the predetermined values of $\triangle a$ and $\triangle r$. The process can then be repeated for a different set of $(\beta_h, \beta_{\bar{h}})$. Our results for two-term $V_{\rm CSB}$ are shown in tables 3 and 4.

The correct sign for $\triangle$ is obtained for those CSB potentials which produce positive $\triangle r(r_{nn}<r_{pp})$, which indicates a stronger $n$-$n$ potential. The dependence of effective

**Table 2.** The contribution $E_{\rm CSB}$ of (one-term) CSB potential to the binding energy difference of $^3$H and $^3$He for various shapes of $V_{\rm CSB}$ and various sets of singlet potentials. $\beta_h=9{\cdot}0\,a$ and $\triangle a=0{\cdot}8$ fm for all the sets. $\triangle r$ is in fm, and $E_{\rm CSB}$ in keV.

| $V_{\rm CSB}$ | Shape A | | Shape B | | Shape C | |
|---|---|---|---|---|---|---|
| $V_s$ | $\triangle r$ | $E_{\rm CSB}$ | $\triangle r$ | $E_{\rm CSB}$ | $\triangle r$ | $E_{\rm CSB}$ |
| I | 0·009 | 44 | 0·013 | 50 | 0·020 | 98 |
| II | 0·010 | 44 | 0·014 | 50 | 0·020 | 98 |
| III | 0·010 | 42 | 0·013 | 46 | 0·019 | 94 |
| IV | 0·010 | 42 | 0·014 | 46 | 0·019 | 90 |
| V | 0·010 | 42 | 0·013 | 46 | 0·017 | 94 |

range on the strength and range of the potential is rather complicated. Just below the threshold for bound state, where the scattering length is large and negative, as is the case in the present situation, the effective range decreases as the range (or strength) of the potential is increased. This can be verified explicitly for square well or separable potentials which are amenable to analytic treatment. Thus a smaller effective range implies a stronger potential.

For a given $\triangle a$, we find that $E_{CSB}$ is very sensitive to $\triangle r$ and varies linearly with it, changing by 800 keV as $\triangle r$ changes from $-0{\cdot}1$ to $+0{\cdot}1$ fm. On the other hand, $E_{CSB}$ is much less sensitive to $\triangle a$, even less than that expected from the fact that the scattering lengths are large compared to effective ranges. Thus $E_{CSB}$ changes by less than 20 keV as $\triangle a$ changes from $-0{\cdot}7$ to $+0{\cdot}7$ fm. It is quite clear, therefore, that even large swings in the value of $\triangle a$ (due to the difficulty of eliminating electromagnetic mixing effects as mentioned earlier) are not likely to affect the calculation of the CSB corrections very much. For the sets I and II calculations have been performed for two different values of $\beta_h$ and $\beta_{\tilde{h}}$: (i) $\beta_h = \beta_{\tilde{h}} = \beta_f$: (ii) $\beta_h = \beta_{\tilde{h}} = 9{\cdot}0\alpha$. From table 3 we find that the results are quite sensitive to the values of these ranges, varying by as much as 15–20%.

**Table 3.** Contribution $E_{CSB}$ of (two-term) CSB potential to the binding energy difference of $^3$H and $^3$He for a one-term singlet potential. $\triangle r$ and $\triangle a$ are in fm and $E_{CSB}$ in keV.

| | Set-I | | | | Set-II | | | |
|---|---|---|---|---|---|---|---|---|
| | $\beta_h=\beta_{\tilde{h}}=7{\cdot}411\alpha$ | | $\beta_h=\beta_{\tilde{h}}=0{\cdot}0\alpha$ | | $\beta_h=\beta_{\tilde{h}}=7{\cdot}160\alpha$ | | $\beta_h=\beta_{\tilde{h}}=9{\cdot}0\alpha$ | |
| $\triangle r$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ |
| $-0{\cdot}10$ | 342 | 310 | 401 | 390 | 329 | 302 | 400 | 402 |
| $-0{\cdot}05$ | 180 | 148 | 206 | 192 | 196 | 144 | 206 | 198 |
| $0{\cdot}00$ | 16 | $-16$ | 9 | $-9$ | 14 | $-14$ | 5 | $-5$ |
| $0{\cdot}025$ | $-66$ | $-98$ | $-92$ | $-109$ | $-65$ | $-94$ | $-96$ | $-106$ |
| $0{\cdot}050$ | $-148$ | $-180$ | $-192$ | $-208$ | $-144$ | $-174$ | $-198$ | $-208$ |
| $0{\cdot}10$ | $-310$ | $-342$ | $-390$ | $-408$ | $-302$ | $-332$ | $-400$ | $-410$ |

**Table 4.** Contribution $E_{CSB}$ of (two-term) CSB potential to the binding energy difference of $^3$H and $^3$He for two-term singlet potential. $\triangle r$ and $\triangle a$ are in fm and $E_{CSB}$ in keV.

| $\triangle r$ | Set-III | | Set-IV | | Set-V | |
|---|---|---|---|---|---|---|
| | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ | $\triangle a=0{\cdot}7$ | $\triangle a=-0{\cdot}7$ |
| $-0{\cdot}10$ | 352 | 332 | 374 | 376 | 380 | 386 |
| $-0{\cdot}05$ | 184 | 160 | 220 | 186 | 200 | 192 |
| $0{\cdot}00$ | 12 | $-12$ | 7 | $-7$ | 7 | $-7$ |
| $0{\cdot}025$ | $-74$ | $-98$ | $-90$ | $-104$ | $-92$ | $-106$ |
| $0{\cdot}050$ | $-160$ | $-184$ | $-186$ | $-200$ | $-192$ | $-204$ |
| $0{\cdot}10$ | $-332$ | $-356$ | $-377$ | $-390$ | $-388$ | $-400$ |

The actual value of $\triangle r$ which will yield the correct value of $\triangle$ depends upon the magnitude of $\triangle$. However, because of the high sensitivity of $E_{CSB}$ to $\triangle r$ this places rather narrow limits on the latter—a perusal of tables 3 and 4 indicates that $\triangle r$ should be between 0·02 to 0·04 fm to produce $\triangle$ of the right order (between 70 and 120 keV). We should expect $r_{nn}$ to be *less* than $r_{pp}$ by about 0·03 fm. This agrees well with the value of $\triangle r$ obtained by Friar and Gibson (1978) for the CSB potential from $\rho$-$\omega$ mixing. Though this is a rather small difference, attempts should be made to find it experimentally.

## References

Brandenburg B A *et al* 1978 *Nucl. Phys.* **A294** 305
Friar J L and Gibson B F 1978 *Phys. Rev.* **C17** 1752
Gibson B F and Stephenson G J Jr 1975 *Phys. Rev.* **C11** 1448
Gupta V K and Mitra A N 1967 *Phys. Lett.* **B24** 27
Henley E M 1972 *Few particle problems in the nuclear interaction* ed I Slaus (North Holland)
Kharchenko V F, Petrov N M and Storozhenko S A 1968 *Nucl. Phys.* **A106** 464
Mehdi S S and Gupta V K 1974 *Phys. Rev. Lett.* **33** 709
Mehdi S S and Gupta V K 1976 *Few body dynamics* eds A N Mitra *et al* (North Holland)
Okamoto K and Pask C 1971 *Ann. Phys.* (*NY*) **68** 18
Sauer P U 1976 *Few body dynamics* ed A N Mitra *et al* (North Holland)