Pramāṇa, Vol. 13, No. 6, December 1979, pp. 599-606, © printed in India.

Parametric excitation of hybrid mode in magnetised piezoelectric semiconductors

S GHOSH, P K SEN* and S GUHA*
School of Studies in Physics, Vikram University, Ujjain 456 010
*Department of Physics, Ravishankar University, Raipur 492 010

MS received 20 February 1979; revised 8 August 1979

Abstract. Using the hydrodynamic model of homogeneous plasma, the parametric decay of a laser beam into an acoustic wave and another electromagnetic wave has been studied in heavily doped *n*-type piezoelectric semiconductors in the presence of a transverse magnetostatic field. This decay process results in the parametric excitation of the hybrid mode. The threshold electric field necessary for the onset of instability equals to zero. The magnetostatic field couples the acoustic and the electromagnetic waves and in its absence the instability disappears. The growth rate increases with the square of the magnetic field.

Keywords. Parametric interaction; hybrid mode; piezoelectric semiconductor; transverse magnetostatic field.

1. Introduction

During the last few years the topic of parametric excitation of low frequency waves in piezoelectric semiconductors has been extensively studied (Chaban 1968; Epshtein 1969; Kaw 1973; Miranda 1973; Sundaram *et al* 1974; Genkin 1975; Guha and Sen 1979a, b; Guha *et al* 1979a, b). Houck *et al* (1967) have experimentally shown that an external alternating field generates coherent acoustic waves in the metal when a large magnetic field is present. A detailed study of electromagnetic generation of acoustic wave has been made by Maxfield (1971). The investigations made so far deal mainly with the acoustic wave amplification in unmagnetised (Kaw 1973, Sundaram *et al* 1974) as well as magnetised piezoelectric semiconductors (Guha and Sen 1979a, b, Guha *et al* 1979a, b). The investigations made by Guha and Sen (1979a, b) have been carried out in transversely magnetised semiconductors where the primary interest was to study the effect of the transverse magnetostatic field on the threshold value of the electric field amplitude of the spatially uniform high frequency pump wave as well as on the growth rate of the unstable acoustic and the electron plasma waves. Guha *et al* (1979a, b) studied the parametric excitation of the acoustohelicon wave when the magnetostatic field is applied parallel to the direction of the propagation of the acoustic wave. The above investigations have been made when the pump wave is parallel to the propagation vector.

In the present paper we have considered that the pump wave $\mathbf{E}_0 \cos \Omega_0 t$ makes a finite angle θ with the propagation vector $\mathbf{k}$ and is in the x-z plane when $\mathbf{k}$ is along x-axis and the magnetostatic field $\mathbf{B}_0$ is along z-axis. This configuration has been chosen with the primary interest of studying the parametric excitation of the hybrid mode. The analysis gives an interesting result that for $\mathbf{B}_0 \neq 0$, the threshold value of the pump electric field necessary for the onset of instability of the hybrid mode is zero and for a finite value of the electric field amplitude, we get a considerable growth rate of the unstable mode which is quite different from the results of the earlier investigations (where a large electric field amplitude of the order of 10^6 Vm^{-1} was necessary for the onset of instability of the acoustic and the electron-plasma modes).

2. Dispersion relation

We use hydrodynamic model of a homogeneous one-component (electron) plasma of infinite extent in a piezoelectric semiconductor in the region $kl \ll 1$ where k is the acoustic wave number and l is the electron mean free path. The wave vector $\mathbf{k}$ is taken to be along x-axis, the magnetostatic field $\mathbf{B}_0$ along z-axis and the spatially uniform high frequency oscillatory electric field $\mathbf{E}_0 \cos \Omega_0 t$ makes an arbitrary angle θ with the x-axis and is in the x-z plane. The system is not subjected to any static electric field.

We start with the following equations:

$$\frac{\partial \mathbf{v}_0}{\partial t} + \nu \mathbf{v}_0 = \frac{e}{m}(\mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_0), \tag{1}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial E}{\partial x}, \tag{2}$$

$$\frac{\partial E}{\partial x} + \frac{ne}{\epsilon} - \frac{\beta}{\epsilon} \frac{\partial^2 u}{\partial x^2}, \tag{3}$$

$$\frac{\partial n}{\partial t} + v_0 \frac{\partial n}{\partial x} + n_0 \frac{\partial v}{\partial x} = 0, \tag{4}$$

and
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}_0 \cdot \vec{\nabla})\, \mathbf{v} = \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}_0) - \frac{k_B T}{m n_0} \vec{\nabla} n - \nu \mathbf{v}. \tag{5}$$

Equation (1) is the zeroth-order equation of motion for electrons and shows that electrons will oscillate under the influence of the applied electric field. We assume $\Omega_0 \, (\approx \omega_p) \gg \nu$. Equation (2) is the equation of elasticity theory describing the motion of the lattice in a piezoelectric crystal, u is the lattice displacement, ρ is the density of the crystal, c is the elastic constant and β is the piezoelectric coefficient. The space-charge field E is determined by the Poisson equation (3) in which the second term on the right-hand side gives the piezoelectric contribution to polarisation, n is the electron density perturbation and ϵ is the dielectric constant of the semiconductor. T is the electron temperature and has been assumed to be equal to the lattice temperature and k_B represents the Boltzmann's constant. Equations (4) and (5) are the continuity and momentum transfer equations for electrons respectively.

We assume that the low frequency perturbations are proportional to the factor $\exp [i(\Omega t - kx)]$ and $\Omega \ll \nu$ where ν is the phenomenological electron collision frequency. We obtain the components of the perturbed velocity $\mathbf{v}$ using (5) as

$$v_x = \frac{e}{m} \frac{\nu}{\nu^2 + \omega_c^2} \left[b\, E_x + \frac{\omega_c}{\nu} E_y - i \frac{k^2 v_\theta^2}{\omega_p^2} \frac{\beta}{\epsilon} ku \right], \tag{6}$$

and

$$v_y = \frac{e}{m} \frac{\nu}{\nu^2 + \omega_c^2} \left[-\frac{\omega_c}{\nu} b\, E_x + E_y + i \frac{\omega_c}{\nu} \frac{k^2 v_\theta^2}{\omega_p^2} \frac{\beta}{\epsilon} ku \right] \tag{7}$$

where $b = 1 + k^2 v_\theta^2/\omega_p^2$, $\omega_p = (n_0\, e/m\epsilon)^{1/2}$

is the electron plasma frequency, $\epsilon = \epsilon_0\, \epsilon_l$, ϵ_l being the lattice dielectric constant, ϵ_0 is the absolute permittivity, n_0 the equilibrium electron concentration, $v_\theta = (k_B T/m)^{1/2}$ is the electron thermal velocity and $\omega_c = -eB_0/m$ is the electron cyclotron frequency. The perturbed electron concentration n has two components associated with the fast and the slow mode represented by n_f and n_s respectively. These are obtained by using (1) and (3) to (5). Following the method of Guha *et al* (1979a), we get

$$\frac{\partial^2 n_f}{\partial t^2} + \nu \frac{\partial n_f}{\partial t} + \omega_R^2\, n_f + \frac{\omega_p^2\, \omega_c^2\, b}{\nu^2 + \omega_c^2} n_f = i k n_s \left(\frac{e}{m} E_{0x} - \omega_c\, v_{0y} \right), \tag{8}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu \frac{\partial n_s}{\partial t} + \omega_R^2\, n_s + \frac{\omega_p^2\, \omega_c^2\, b}{\nu^2 + \omega_c^2} n_s$$

$$+ \frac{n_0\, e^2\, \beta^2 \left[1 + \dfrac{\omega_c^2}{\nu^2 + \omega_c^2} \dfrac{k^2 v_\theta^2}{\omega_p^2} \right] k^2}{m\, \epsilon^2\, \rho\, (\Omega^2 - k^2 c_s^2 - \beta^2 k^2/\rho\, \epsilon)} n_s = i k n_f \left(\frac{e}{m} E_{0x} - \omega_c\, v_{0y} \right), \tag{9}$$

in which $\omega_R^2 = \omega_p^2 + k^2 v_\theta^2$ is the frequency for dispersive electron-plasma wave. In obtaining (8) and (9), we have used (2) and (3) which yield

$$u \left(\Omega^2 - k^2 c_s^2 - \frac{\beta^2 k^2}{\rho \epsilon} \right) = n \frac{e \beta}{\rho\, \epsilon}, \tag{10}$$

in which $c_s = (c/\rho)^{1/2}$ is the sound velocity and the Poisson equation is written as

$$\partial E_f/\partial x = e n_f/\epsilon, \tag{11}$$

and

$$\partial E_s/\partial x = (e n_s/\epsilon) - (\beta/\epsilon)\, (\partial^2 u/\partial x^2). \tag{12}$$

The fast component of the perturbed electron concentration n_f will have two components at side band frequencies $(\Omega + \Omega_0)$ and $(\Omega - \Omega_0)$. Higher order components of the frequencies *viz.*, $\Omega \pm p\,\Omega_0$ (where p is a positive integer and greater than 1) are negligible because they are non-resonant in contrast to that with $p = 1$.

Thus taking the proportionality of these forced waves as

$$\exp\,[i\,\{(\Omega \pm \Omega_0)\,t - kx\}]$$

one gets from equation (8)

$$n_f = -\,i\frac{kn_s\,\bar{E}}{\Omega_0}\left[\frac{1}{\Omega - i\nu + \delta} - \frac{1}{\Omega - i\nu - \delta}\right], \tag{13}$$

where $\bar{E} = \frac{e}{m}E_{0x} - \omega_c\,v_{0y}, \quad \delta = \Omega_0 - \bar{\omega}_R$

and $\bar{\omega}_R = \omega_R\,[1 + \omega_c^2/(\nu^2 + \omega_c^2)]^{1/2}.$

In obtaining (13) we have assumed that

$$\Omega_0 \approx \omega_p\,(\approx \omega_R) \gg \nu \gg \Omega$$

and thus for such a case (known as collision-dominated plasma when $\nu \gg \Omega$), one can write equation (13) in the simplified form as,

$$n_f = -\frac{2\,i\,\delta\,k\,n_s\,\bar{E}}{\Omega_0\,(\nu^2 + \delta^2)}\,. \tag{14}$$

The r.f. current density can be obtained from the equation

$$\mathbf{J} = e\,(n_0\,\mathbf{v} + n\,\mathbf{v}_0), \tag{15}$$

where $\mathbf{v}_0$ is the zeroth-order velocity of the electrons. Using (1) the components of the zeroth-order velocity are given by

$$v_{0x} = \frac{e}{m}\left(\frac{\bar{\Omega}_0}{\bar{\Omega}_0^2 + \omega_c^2}\right)E_0\cos\theta,$$

$$v_{0y} = -\frac{e}{m}\left(\frac{\omega_c}{\bar{\Omega}_0^2 + \omega_c^2}\right)E_0\cos\theta,$$

and $$v_{0z} = \frac{e}{m\,\bar{\Omega}_0}E_0\sin\theta,$$

where $\bar{\Omega}_0 = \nu + i\,\Omega_0.$

The forced waves will have two components of the velocity and the concentration, i.e., $n = n_s + n_f$ and $\mathbf{v} = \mathbf{v}_s + \mathbf{v}_f$.

Using these relations, one gets from equation (15)

$$\mathbf{J} = e\,[(\mathbf{v}_s + \mathbf{v}_f)\,n_0 + (n_s + n_f)\,\mathbf{v}_0]. \tag{16}$$

Using equation (14) we can have

$$\mathbf{J} = e\left[(\mathbf{v}_s + \mathbf{v}_f)\,n_0 + \left\{1 - \frac{2\,i\,\delta\,k\,\bar{E}}{\Omega_0\,(\nu^2 + \delta^2)}\right\}\,n_s\,\mathbf{v}_0\right]. \tag{17}$$

Now from equation (10) we notice that the low frequency acoustic wave oscillation is related to the density perturbation component n_s and thus we obtain

$$\mathbf{J} = e\left[(\mathbf{v}_s + \mathbf{v}_f)\,n_0 + \left\{1 - \frac{2\,i\,\delta\,k\,\bar{E}}{\Omega_0\,(\nu^2 + \delta^2)}\right\} \times \frac{(\Omega^2 - k^2\,c_s^2 - \beta^2\,k^2/\rho\,\epsilon)}{(\beta e/\rho\,\epsilon)}\right]\,u\,\mathbf{v}_0. \tag{18}$$

The current density perturbation $\mathbf{J}$ can be expressed in terms of the electric field perturbation $\mathbf{E}$ by using equations (6), (7) and Maxwell's equation

$$\vec{\nabla} \times \mathbf{H} = \mathbf{J} + (\partial\,\mathbf{D}/\partial\,t), \tag{19}$$

which yields finally

$$u = \frac{J_x}{k\,\Omega\,\beta} - \frac{i}{k\,\Omega\,\beta}\left(\frac{k^2}{\mu_0\,\Omega} - \epsilon\,\Omega\right)E_x,$$

and consequently, after simplification one obtains the components of the r.f. current density as

$$J_x = (1/P)\left[\left\{\frac{\omega_p^2\,\epsilon\nu}{\nu^2+\omega_c^2} - i v_{ox}\,\frac{\rho\epsilon^2}{k\beta^2\,\Omega^2}\left(\Omega^2 - k^2\,c_s^2 - \frac{\beta^2\,k^2}{\rho\epsilon}\right) \times \left(1 - \frac{2i\,\delta\,k\bar{E}}{\Omega_0\,(\nu^2+\delta^2)}\right)\left(k^2\,c_i^2 - \Omega^2\right)\right\}E_x + \left\{(\omega_p^2\,\epsilon\,\omega_c)\,/\,(\nu^2+\omega_c^2)\right\}E_y\right], \tag{20}$$

and
$$J_y = \frac{\omega_p^2\,\epsilon}{\nu^2 + \omega_c^2}\,(-\,\omega_c\,E_x + \nu E_y), \tag{21}$$

where $P = 1 - \{(v_{0x}\rho\epsilon)/(k\Omega\beta)\}\left(\Omega^2 - k^2c_s^2 - \frac{\beta^2k^2}{\rho\epsilon}\right)\left[1 - \frac{2i\,\delta\,k\bar{E}}{\Omega_0\,(\nu^2+\delta^2)}\right].$

The general wave equation is given by

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} = i\,\Omega\,\mu_0\,\mathbf{J} - \mathbf{E}\frac{\Omega^2}{c_l^2}\mathbf{E}. \tag{22}$$

Using (20) to (22) the dispersion relation is obtained as

$$\left[k^2\,c_l^2 - \Omega^2 + i\,\frac{\nu\,\omega_p^2\,\Omega}{\nu^2 + \omega_c^2}\right]\left[\frac{\omega_p^2\,\nu}{\nu^2 + \omega_c^2} - i\,\frac{v_{0x}\,k^2\,c_l^2}{K^2k\,c_s^2\,\Omega^2}\left(\Omega^2 - k^2\,c_s^2 - \frac{\beta^2\,k^2}{\rho\,\epsilon}\right)\right.$$

$$\left.\times\left(1 - \frac{2\,i\,\delta\,k\,\bar{E}}{\Omega_0\,(\nu^2 + \delta^2)}\right)\right] + i\,\frac{\omega_p^4\,\omega_c^2\,\Omega}{(\nu^2 + \omega_c^2)^2} = 0, \tag{23}$$

where $K^2 = \beta^2/\epsilon\,c$.

Equation (23) represents a general dispersion relation of a hybrid wave in the region $kl \ll 1$ for piezoelectric semiconductors in the presence of a pump field and a transverse magnetostatic field.

3. Results and discussion

One may rewrite the dispersion relation (23) as follows:

$$[k^2\,c_l^2 - \Omega^2 + i\,\nu\,\Omega\,a^2]\;[\nu\,a^2\,K^2\,k\,\Omega^2\,c_s^2 - i\,v_{0x}\,k^2\,c_l^2$$

$$\times\,(\Omega^2 - k^2\,c_s^2 - K^2\,k^2\,c_s^2)\,A] + i\,a^4\,\omega_c^2\,\Omega^3\,K^2\,k\,c_s^2 = 0, \tag{24}$$

where $a^2 = \omega_p^2/(\nu^2 + \omega_c^2)$ and $A = 1 - \frac{2\,i\,\delta\,k\,\bar{E}}{\Omega_0\,(\nu^2 + \delta^2)}$.

In the absence of magnetostatic field i.e., $\omega_c = 0$ we obtain two independent modes given by

$$k^2\,c_l^2 - \Omega^2 + i\,\nu\,\Omega\,a^2 = 0, \tag{25}$$

and

$$\nu a^2K^2k\Omega^2c_s^2 - iv_{0x}\,k^2c_l^2\,(\Omega^2 - k^2c_s^2 - K^2k^2c_s^2)\,A = 0. \tag{26}$$

It can be observed from equation (26) that for $K = 0$ and $E_0 = 0$ we get the usual dispersion relation for sound wave propagation $\Omega^2 = k^2c_s^2$ in isotropic medium.

Thus for $K \neq 0$ and $E_0 \neq 0$ we obtain from equation (24), under the assumption $k^2c_l^2 \gg \Omega^2$

$$(k^2c_l^2 + i\nu\Omega a^2)\,(\Omega^2 - k^2c_s^2) = \frac{k^2c_s^2}{kv_{0x}c_l^2A}\,[k^5c_l^4v_{0x}A + a^4\omega_c^2\Omega^3]. \tag{27}$$

We now address ourselves to the principal point of this paper, the question of parametric excitation of hybrid mode in the presence of the high frequency oscillatory electric field. Equation (27) represents the general dispersion relation. In order to explore the possibility of instability, we solve the dispersion relation (27) for complex values of $\Omega(= \Omega_r + i\Omega_i)$ with real positive values of the wave number k. It is well known that the propagating mode exhibits instability only when $\Omega_i < 0$. We assume that the phase velocity of the propagating mode $v_\phi\ (= \Omega_r/k)$ is nearly equal to that of the acoustic wave c_s (i.e., $v_\phi \approx c_s$) and take the reasonable approximation $k^2c_l^2 \gg \nu\,\Omega\,a^2$ as $kc_l \gtrsim \omega_0 \gg \nu \gg \Omega$ and $a^2 \approx 1$ in a magnetoplasma with $\omega_c \sim \omega_p$. Under this condition, equating the imaginary parts of equation (27), one obtains

$$\Omega_i = \frac{2K^2kc_s}{G}\,a^4\omega_c^2c_s^3v_{0x}k^2c_l^4\left\{\frac{2\,\delta\,k\,\bar{E}}{\Omega_0(\nu^2+\delta^2)}\right\}, \tag{28}$$

where $$G = (2v_{0x}k^2c_l^4)^2 + \left\{\frac{4v_{0x}k^2c_l^4\,\delta\,k\,\bar{E}}{\Omega_0(\nu^2+\delta^2)}\right\}^2.$$

Equation (28) shows that the mode will be unstable ($\Omega_i < 0$) only when $\delta < 0$ as all the other terms are positive. Thus we obtain the necessary condition of instability as

$$\bar{\omega}_R > \Omega_0. \tag{29}$$

The condition (29) can be satisfied easily in n-type semiconductors like n-InSb with electron concentration of the order of $10^{22}\ m^{-1}$ when the crystal is irradiated with a 1·06 μm Nd: YAG laser. Equation (28) shows that the growth rate of the unstable mode varies as the square of the magnetostatic field. The equation further shows that in the absence of static magnetic field the instability disappears. Equation (28) gives zero threshold value of the high frequency oscillatory electric field. A similar threshold electric field value was reported by Sanuki and Schmidt (1977) while studying the parametric decay of lower hybrid wave into electromagnetic waves in gaseous plasmas.

The heating of the electrons due to the electric field E_0 enters only through the thermal correction for dispersive electron-plasma wave. We have assumed $kl \ll 1$ and $\omega_p \gg \nu$, therefore, $k\lambda_D \ll 1$ where λ_D is the Debye wavelength. Thus one can assume $k^2v_\theta^2 \ll \omega_p^2$ whence ω_R becomes equal to ω_p in the long wave length region.

4. Conclusions

The present investigation dealt with the decay of a high frequency electromagnetic wave (pump) into a low frequency acoustic wave and another electromagnetic wave with a frequency of the order of the pump frequency. In the presence of magneto-static field these modes are coupled and consequently gives a hybrid mode. We have studied the parametric excitation of this hybrid mode which propagates with a phase velocity nearly equal to the acoustic wave velocity in the lattice. The threshold value of the pump field for the onset of instability is found to be zero. The present study is the first such report in semiconductor-plasmas.

Acknowledgements

The authors thank the referee for critical comments. The work was carried out under a research project supported by the University Grants Commission. One of the authors (PKS) is thankful to the Council of Scientific and Industrial Research for a fellowship.

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