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Asymmetry of mass and charge division in spontaneous fission

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Abstract. The order-disorder model has been used to explain asymmetry of mass and charge division and related phenomena in fission. According to this model the fission process involves two steps consisting of charge polarisation into two 'impending fragment clusters' with beta stable neutron numbers and subsequent distribution of the balance neutrons between the two. Mode of elemental division of the fissioning nuclei is attributed to the charge polarisation in the first step. Theory of reaction rate has been applied to the system.

The frequency term is obtained by applying the conditional stochastic process under charge polarisation constraint and the energy-dependent term is given by the condition of minimum in free energy of the system. Using this, the relative probability of polarisation into given charge pair is arrived at.

The model uses stable neutron numbers for the charges as the only input. No explicit assumption or quantification on the preference of formation of shell closure species in fission is necessary. The statistics developed on the principle of equal *a priori* probability of all charge polarisation is used. The shell effects come into play only in deciding a stable neutron number for the charges. The total isotopic yield distribution for a number of fission reactions shows asymmetry in the actinide region which reduces with increasing mass/charge of the fissioning nuclide and bunching of the higher z peaks. Although the mass yields obtained therefrom for a number of fission reactions agree with experimental results, the peaks of the distributions are slightly shifted away from the symmetric point and the distributions are somewhat narrower. Charge distribution parameters obtained from these results are also presented.

Keywords. Spontaneous fission; order-disorder model; charge polarisation; stable neutron number; total isotopic yield; asymmetry of mass and charge division; charge distribution.

1. Introduction

One of the most characteristic features of low energy fission in the actinide region is the predominance of asymmetric division of the fissioning nucleus. Many authors have attempted to explain the phenomenon, often assuming preference of formation of closed shell species in fission (Faissner and Wildermuth 1964; Ramanna *et al* 1965; Denschlang and Qaim 1969).

Experimental studies give directly yields and many other parameters as functions of fragment mass. This is due to the fact that the parameters can be easily studied experimentally as functions of fragment mass rather than fragment charge. However, Iyer and Ganguly (1971) and Norenberg (1969) preferred the elemental distribution as primary instead of mass distribution. The fissioning nucleus starts moving towards scission, when in the nucleonic re-arrangement the repulsive coulomb interaction between the 'impending fragments' gains ascendancy over the attractive nuclear forces between them. Impending fragment is a specific concept of a polarised state of the compound nucleus in the order-disorder model (ODM) (Iyer and Ganguly 1971) and consists of two clusters each with protons and neutrons corresponding to that for the ground state stability of the nuclei. Therefore the nuclear charges and neutron numbers of the impending fragment pairs are the two important parameters in the description of the fission process. The success of the elemental description in ODM in computing and interpreting a number of experimental results (Iyer and Ganguly 1971; Sharma *et al* 1976) encouraged further investigation of the model.

The present state of prediction of asymmetric division in fission using theoretical models is illustrated in figure 1, where the fragment mass yield distributions by two different models (Maruhn *et al* 1973, Wilkins *et al* 1976) are compared with the experimental distribution for the fission of ^{236}U. In both the cases asymmetric division has been explained qualitatively but the peak-to-valley ratio is either too high or low and peaks are narrower.

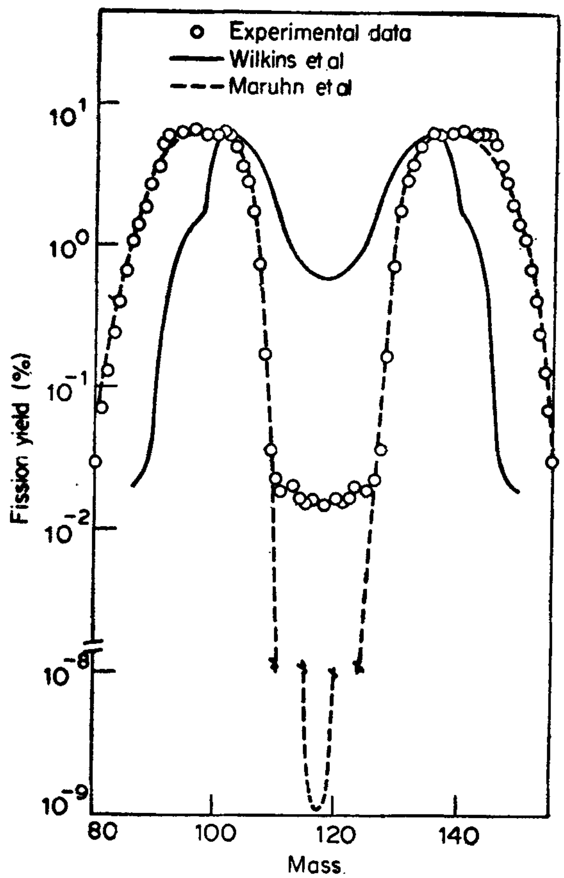

Figure 1. Fragment mass yield distributions for ^{236}U (SF) calculated by Maruhn *et al* (1973) and for ^{235}U (n_{th}, f) calculated by Wilkins *et al* (1976) are compared with experimental ^{235}U (n_{th}, f) data (Schmitt 1970).

2. Order-disorder model

According to the order-disorder model the fission process has two steps:

(i) polarisation into two impending fragment clusters of charges Z_L and Z_H each with the corresponding stable neutron numbers N_L^S and N_H^S, and
(ii) distribution of the balance neutrons, N_{bal} between the two as in order-disorder process; where $N_{\text{bal}} = N_F - N_L^S - N_H^S$ and N_F is the neutron number of the fissioning nucleus.

The second step was earlier used to study charge distribution, neutron evaporation and energy distribution in thermal neutron induced fission (Iyer and Ganguly 1971, 1972) and in higher energy fission (Sharma *et al* 1976). The computations needed experimental mass yield data as input for calculating the independent yields and other fission parameters. This semi-empirical approach was necessary since the statistical methodology remained to be developed for the first step of the model, viz. relative probability of polarisation of the fissioning nucleus into different pairs of charges. The mode of elemental division of the fissioning nuclide is attributed to the charge polarisation in the first step. An elementary mathematical formulation of this step of the model dealt with in the present paper is found to reproduce very well a number of fission data. The systema ics of asymmetry in fission based on ODM was reported earlier (Chakraborty *et al* 1978).

3. Fission as a rate process

3.1. *Basic observations*

The fissioning nucleus proceeds through states of charge polarisation at the very early stages of fission configuration (Ramanna *et al* 1965; Terrell 1959; Iyer and Ganguly 1971). Cluster formations in compound nucleus have been speculated which are assumed to remain essentially unaffected throughout the rest of the fission process (Faissner and Wildermuth 1964; Denschlag and Qaim 1969) and fragment shells develop their characteristics much prior to the actual fragment formation (Schultheis *et al* 1978). The life-time of the compound nucleus is very long compared to the nuclear periods and therefore quasi-stationary equilibrium states are obtained in compound nucleus before scission (Böhr 1956; Fong 1956; Ramamurthy and Ramanna 1969; Jensen and Dossing 1973). Concepts of deformation and fission barrier are all model-dependent and the discrepancies of the various approaches are attributed to the utter complexities of the nuclear many-body problem and the need for rules have been pointed out by Pauli and Ledergerber (1973). Analogies have also been drawn between rates of chemical reaction and fission reaction (Hyde 1962; Thind *et al* 1969).

Based on the basic observations and premises cited above, a quantitative description of fission and charge yield in spontaneous fission is attempted using the ODM.

3.2. *Mathematical formulation*

We take fission as a rate process and apply conditional stochastic process under the charge polarisation constraint.

The relationship for the rate process is given as:

$$R = f \exp\,[-W(Z_L)/kT], \tag{1}$$

where f is the frequency factor determined by the stochastic process involved in the reaction and $\exp\,[-W(Z_L)/kT]$ is the energy-dependent factor. $W(Z_L)$ is the activation energy and T the temperature at which the stochastic distribution takes place.

In a chemical reaction, the reactants mutually interact and also with the ambient. Thus equilibrium conditions are applied to the entire assemblage of interacting chemical species present in the system, whereas in fission reaction a nuclide is an isolated system of nucleons undergoing changes without interaction with other nuclides. Hence basic statistical considerations are applied to each of these systems, then averaged over the entire assembly of the system of isolated nuclides. Equation (1) is thus applied in its present form to the nuclear fission situation with the following stipulations.

In fission reaction having various possible channels without any *a priori* preference for different polarisations, the frequency factor is taken in relative terms, and the exponential factor is understood as the probability of a quasi-equilibrium condition obtained for the given mode of polarisation. The charge polarisation step is further visualised as a thermodynamically quasi-equilibrium adiabatic process at constant volume and its frequency can be obtained from combinatorial considerations. In the energetics of charge polarisation it is reckoned that there is release of energy (E) into the system arising out of charge polarisation and rearrangement of nucleons (increase in entropy-S) and the free energy (F) of the system is given by

$$F = E - TS. \tag{2}$$

Mathematical formulation of the two factors in equation (1) has been carried out by applying the statistics of random selections of nucleons for the charge polarisation. This bears analogy with the order-disorder phenomenon (Glasstone 1952; Frenkel 1946) in binary alloy crystals.

3.2a. *Frequency factor:* The frequency factor f in the random process envisaged corresponds to the relative number of ways in which charge pairs with respective number of neutrons required for beta stable configurations can be chosen from a system comprised of the number of protons and neutrons in the fissioning nucleus. Burnside (1959) has given the expression for the chance of random pick-up of p white and q black balls from a box containing a white and b black (but otherwise identical) balls as:

$$^{a}C_{p} \times {}^{b}C_{q} / {}^{a+b}C_{N}, \tag{3}$$

where $N = p + q$.

The same scheme is applied to the nucleons of the fissioning nucleus. The chance of choosing M_L^S and M_H^S nucleons in a fissioning nucleus of A_F nucleons $=(Z_F+N_F)$ consisting of (Z_L, N_L^S) and (Z_H, N_H^S) respectively is given by:

$$\left[{}^{Z_F}C_{Z_L} \times {}^{N_F}C_{N_L^S} \times {}^{N_F-N_L^S}C_{N_H^S}\right] \Big/ \left[{}^{A_F}C_{M_L^S} \times {}^{A_F-M_L^S}C_{M_H^S}\right] \tag{4}$$

where $\quad M_L^S = Z_L + N_L^S$ and $M_H^S = Z_H + N_H^S$.

The expression is related to the number of ways in which polarisation of the nucleus into $(Z_L+N_L^S)$ and $(Z_H+N_H^S)$ nucleons can be visualised. Each of the possible ways of polarisation relates to one energy state of the polarised nucleus. The number of eigenstates for each of these in an assembly of nucleons composed of two species of fermi particles is given by the product of the number of ways Z_L and N_L^S can be distributed in M_L^S phase space locations and Z_H and N_H^S can be located in M_H^S phase space locations. This gives the multiplicity or statistical weight of the energy levels (cf. appendix.). Thus the frequency factor is given by:

$$f = \left[{}^{Z_F}C_{Z_L} \times {}^{N_F}C_{N_L^S} \times {}^{N_F-N_L^S}C_{N_H^S}\right] \times$$

$$\left[{}^{M_L^S}C_{Z_L} \times {}^{M_H^S}C_{Z_H} \times {}^{N_{\text{bal}}}C_{N_{\text{bal}}}\right] \Big/ \left[{}^{A_F}C_{M_L^S} \times {}^{A_F-M_L^S}C_{M_H^S}\right], \tag{5}$$

or

$$f = D \cdot \left[M_L^S!\, M_H^S! / (Z_L!\, Z_H!\, N_L^S!\, N_H^S!)\right]^2, \tag{6}$$

where D is a constant for a given fissile nuclide.

3.2b. *Exponential energy dependent factor:*

The total number of eigenstates G for (Z_L, Z_H) mode of polarisation of the system is given by (*cf.* appendix):

$$G \propto 1/[Z_L!\, Z_H!\, N_L^S!\, N_H^S!\, N_{\text{bal}}!]. \tag{7}$$

Increase in entropy due to polarisation is then given by:

$$S = k \ln G \times \text{const.} \tag{8}$$

The minimum in free energy corresponding to an equilibrium for a given charge polarisation is obtained by differentiating equation (2) with respect to charge in the polarisation step and equating it to zero

i.e.

$$\delta E/\delta Z_L - kT\,(\delta/\delta Z_L)\,(\ln G) = 0, \tag{9}$$

or

$$(\delta E/\delta Z_L)/kT = (\delta/\delta Z_L)\,(\ln G), \tag{10}$$

Since $\delta E/\delta Z_L = W(Z_L)$, (Iyer and Ganguly 1971)

$$W(Z_L)/kT = (\delta/\delta Z_L)(\ln G). \quad (11)$$

Here $(\delta/\delta Z_L)(\ln G)$ is the slope of the entropy curve at Z_L. The energy-dependent term in (1) is then given by

$$\exp[-(\delta/\delta Z_L)(\ln G)].$$

This is calculated from the beta stable neutron numbers N_L^S and N_H^S together with charges Z_L and Z_H.

4. Estimation of stable neutron number for the charges

Though in general several stable nuclides occur for a given proton number, energetically one particular isotope can be found corresponding to the most beta-stable nuclide. If the mass parabola is taken to be continuous, this minimum may correspond to a non-integral neutron number for a charge. Beta-stable neutron number for the charges can be derived either by using a suitable mass-formula or directly from the experimental beta-decay energies. In the present work, the mass table of Wapstra and Bos (1977) has been used to obtain stable charge for a mass. The stable neutron numbers for integral charges are obtained from these by interpolation.

5. Computational procedure

The expression for the reaction rate derived in § 3.2 can be written as:

$$R = D\left[\left(M_L^S!\, M_H^S!\right) \Big/ \left(Z_L!\, Z_H!\, N_L^S!\, N_H^S!\right)\right]^2 \times \exp[-(\delta/\delta Z_L)(\ln G)], \quad (12)$$

where $$\ln G = C - \left[\ln Z_L! + \ln Z_H! + \ln N_L^S! + \ln N_H^S! + \ln N_{\text{bal}}!\right], \quad (13)$$

and C is a constant.

The factorials for non-integral numbers are calculated using the table of functions.

The frequency distribution calculated as a function of Z using the expression (6) gives qualitative features of yield distribution observed in fission *viz* asymmetry, bunching of higher Z-peaks and reduction in peak-to-valley ratio with increase of mass/charge of the fissioning nuclei.

The exponent of the energy term is obtained from the differential of entropy of polarisation as given in equation (11). The differential at each charge points is calculated and $W(Z_L)/kT$ are thus obtained. Slope at the symmetric charge is obtained by extrapolation.

The calculated rate of reaction R, gives the relative probability of polarisation into a given charge pair or total isotopic yield of elements (i.e. total yield of fragments with the same charge).

The isotopic probability distribution for a given N_{bal}, corresponding to a particular charge polarisation is worked out adopting the procedure described by Iyer and Ganguly (1971). These isotopic probability distributions together with the total isotopic yield gives the independent yield of each fragment. These when added along the isobaric lines give the mass yields.

The independent yields $Y(Z_i, N_j)$ when plotted along the isobaric line gives the charge distribution. The mean (Z_p) of these distributions are given by

$$Z_p = \sum_{i+j=A} Y(Z_i, N_j) \times Z_i \Big/ \sum_{i+j=A} Y(Z_i, N_j). \tag{14}$$

6. Results and discussion

6.1. *Total isotopic yield distributions*

The rate of reaction R, calculated as outlined in § 5 gives the relative probabilities of charge polarisation. This distribution normalised to 200% between the range of

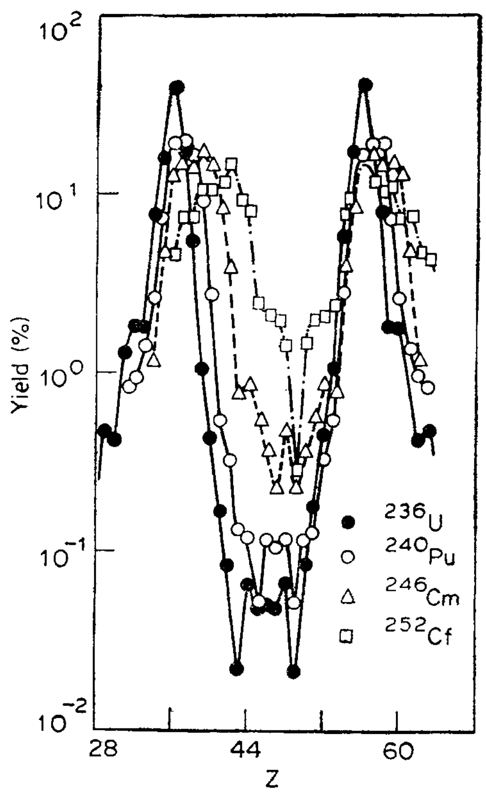

Figure 2. Total isotopic yield distribution for SF of ^{236}U, ^{240}Pu, ^{246}Cm and ^{252}Cf.

charges, gives the total isotopic yield distribution. Since no input of energy from any external source is considered in the formulation of the expression (12), the total isotopic yield distributions obtained are those for spontaneous fission.

The total isotopic yields obtained for ^{236}U, ^{240}Pu, ^{246}Cm and ^{252}Cf are given in figure 2 and those obtained for ^{256}Fm and ^{262}[105] are given in figure 3. In figure 2 the higher-Z-peaks are found to be clustered around $Z=56$. The lower-Z-peaks shift towards the higher-Z-peaks with increasing charge of the fissioning nucleus as needed by complimentarity condition. Peak-to-valley ratio is also found to reduce with increase of charge of the nuclides as observed also in experimental data. For the nuclides whose symmetric charges are beyond $Z = 50$ the ODM shows prominence in symmetric yield and marked reduction in asymmetry as in the case of ^{262}[105]. In the results some structures have also appeared in the symmetric region of ^{236}U and ^{240}Pu. The lower edge of the higher-Z-peaks, in all cases are found to be at the magic number $Z=50$. The shoulder at $Z=50$ observed in the distributions becomes more prominent for the fission of higher mass/charge of the fissioning nuclei. The structures in the symmetric region seem to be an odd-even effect, showing a peak for even charge number and a valley for the odd charge.

6.2. *Mass yield distributions*

The mass yield distributions given here are normalised to 200% within the range of masses shown in the figures that follow. The results obtained for ^{236}U is compared

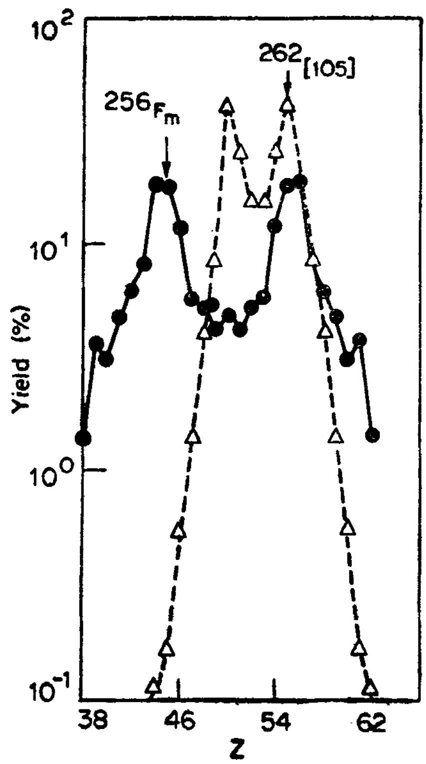

Figure 3. Total isotopic yield distributions for SF of ^{256}Fm and ^{262}[105].

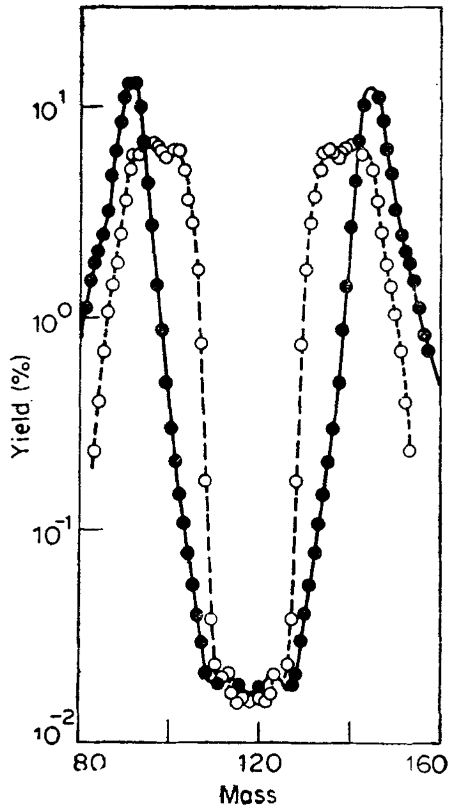

Figure 4. Fragment mass yield distribution for ^{236}U obtained in the present work (solid line with solid circles) is compared with the experimental ^{235}U (n_{th}, f) data (Schmitt 1970), (broken line with open circle).

with experimental data for ^{235}U (n_{th}, f) in figure 4. The heavy fragment peak of the distribution is at $A = 144$. The peak is found to be narrower than the observed experimental mass yield distribution. Full width at one-tenth maximum is 16 in the present results as compared to 19 for the experimental distribution (Von Gunten 1969). It may be noted that below the shoulder at $A_H = 153$ the width of the distribution agrees better. A similar shoulder in the outer wings of the distribution has also been noticed in the results obtained using other models (Maruhn *et al* 1973; Wilkins *et al* 1976) (cf. figure 1). The peak-to-valley ratio for the computed distribution is about 750 against 460 of the experimental data (Schmitt 1970). However, the radio-chemical data give a value of about 630 for peak-to-valley ratio (Meek and Rider 1974). The structure in the symmetric region agrees very well.

The mass yield distribution obtained for ^{240}Pu is compared with the experimental distributions for ^{240}Pu (SF) and ^{239}Pu (n_{th}, f) in figure 5. Here again the peak position in the calculated distribution at $A_H = 146$ is about 7 units away from the peak in the experimental distribution. The peak-to-valley ratio in this case is about 310 as compared to 186 in experimental fragment distribution.

The values obtained in the present work for ^{252}Cf are compared with the experimental data in figure 6. In this case the shift in peak position is less and the width

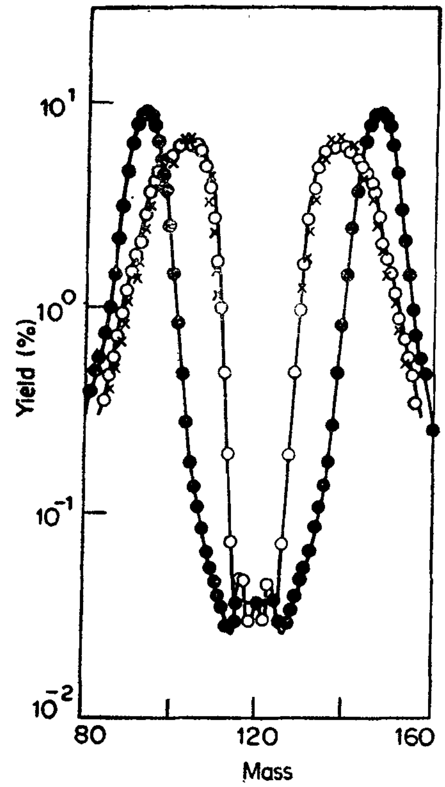

Figure 5. Fragment mass yield distribution for ^{240}Pu obtained in the present work (solid line with solid circles) is compared with the experimental ^{239}Pu (n_{th}, f) data (Schmitt 1970) (solid line with open circles). The experimental ^{240}Pu(SF) data (Deruytter and Penning 1973) is shown with crosses.

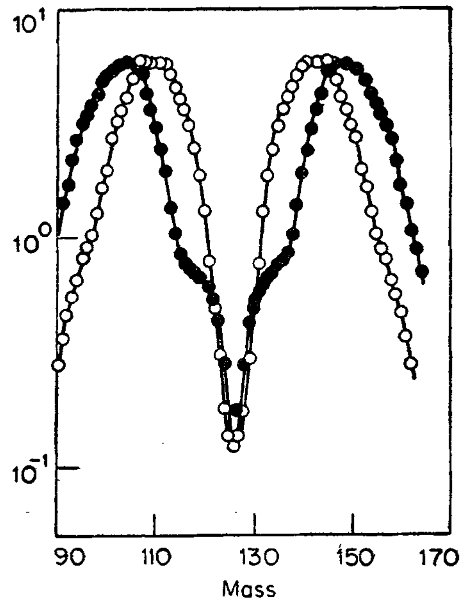

Figure 6. Fragment mass yield distribution for ^{252}Cf obtained in the present work (solid line with solid circle) is compared with the experimental data (Schmitt 1970), (solid line with open circle).

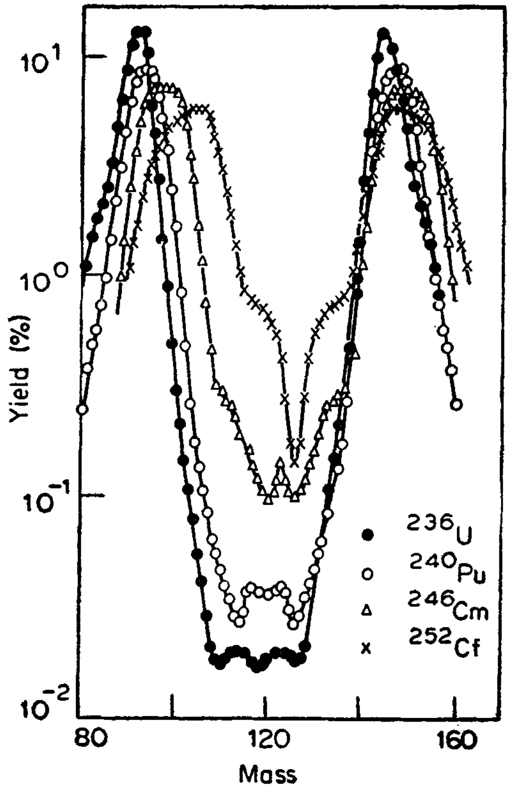

Figure 7. Fragment mass yield distributions for ^{236}U, ^{240}Pu, ^{246}Cm and ^{252}Cf.

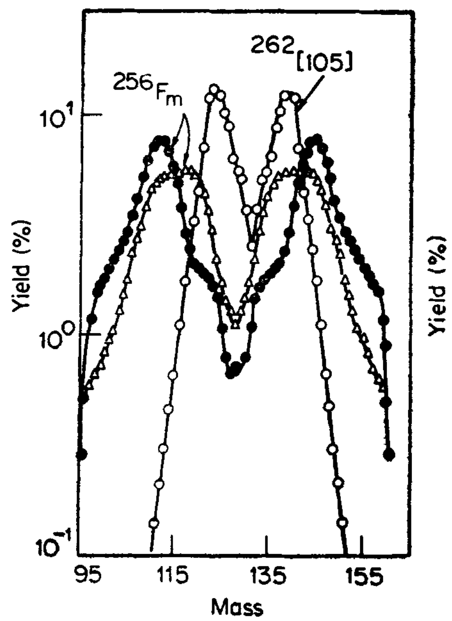

Figure 8. Fragment mass yield distribution for ^{256}Fm (solid line with solid circles) is compared with experimental data (Unik *et al* 1973), (solid line with open triangles). Fragment mass yield distribution of $^{262}[105]$ is shown by solid line with open circles.

of the distribution is in better agreement with the experimental value. The peak-to-valley ratio is about 55 as compared to 60 for the experimental fragment mass yield (Schmitt 1970).

The mass yield distributions for ^{236}U, ^{240}Pu, ^{246}Cm and ^{252}Cf are compared with each other in figure 7. Here, the bunching of higher mass peaks around $A_H = 147$ is clearly seen. The lighter edge of the heavy peak is at the mass number $A = 132$ in all the distributions, which is in agreement with the observation of Pappas *et al* (1969).

Reduction of peak-to-valley ratio with increasing mass and charge of the fissioning nuclide is seen in the distributions. The width of the valley also shows reduction with increasing mass and charge of the nuclides.

Computed mass yields in the fission of transcalifornium nuclides are shown in figure 8. They show marked reduction of peak-to-valley ratio. The reduced asymmetry for ^{262}[105] observed in the present work has also been corroborated in the experimental results obtained by Bemis *et al* (1977). The experimental yield data of ^{256}Fm are also shown in the figure for comparison (Unik *et al* 1973).

The gross features of mass yield distributions as a function of mass of the fissioning

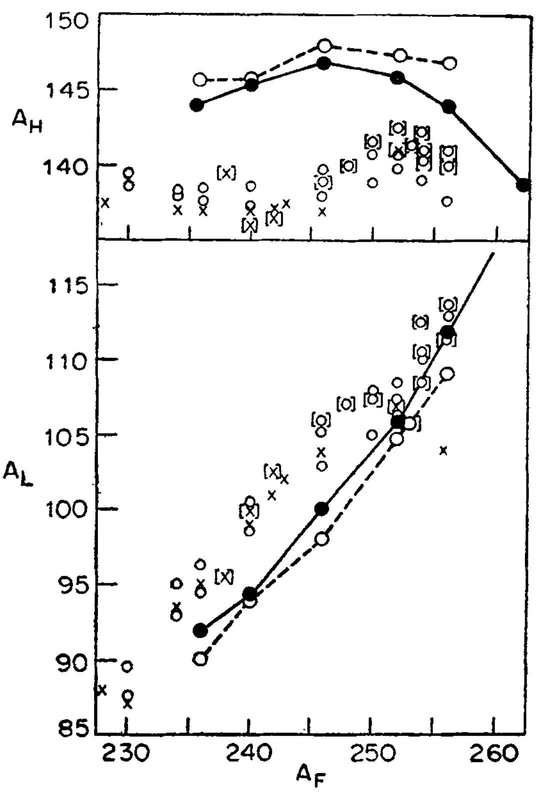

Figure 9. Locations of higher (A_H) and lower (A_L) mass peak positions versus mass of the fissioning nuclei (A_F) obtained in the present work (solid line with solid circles) are compared with earlier values (Chakraborty *et al* 1977) (broken line with open circle) obtained using the mass table of Wapstra and Gove (1971). Experimental values obtained by Von Gunten (1969) (cross) and Unik *et al* (1973) (open circle) are also given. The experimental data given within parenthesis are for SF while those without parenthesis are for thermal fission.

nuclides have also been studied. In figure 9 locations of the lower and higher mass peaks as a function of A_F are shown and compared with those observed for the experimental distribution. Though the actual values themselves differ, the trend of shift of the peaks agrees with that observed experimentally.

As mentioned earlier, the mass data of 1977 (Wapstra and Bos 1977) have been used in arriving at the stable neutron numbers (N^s) used as input in the calculations. The peak characteristics obtained using the 1971 mass tables (Wapstra and Gove 1971) are also shown in figure 9 (Chakraborty 1977). It is seen that the peak characteristics of the predicted results agree better with those of experimental data when the 1977 mass data are used.

Peak-to-valley ratio of the distributions as a function of A_F is compared with the experimental values in figure 10. The experimental values for fragments and products are also presented in this figure. The agreement with the experimental values is satisfactory.

In figure 11 full width of the peaks at one-tenth of the maximum (FWTM) for the distributions is plotted as a function of A_F of the fissioning nuclide and are compared with that obtained from the experimental distributions (Von Gunten 1969; Unik *et al* 1973). The width of the distributions is consistently lower but agrees in trend with that for the experimental data. Since the stable neutron number for the charges

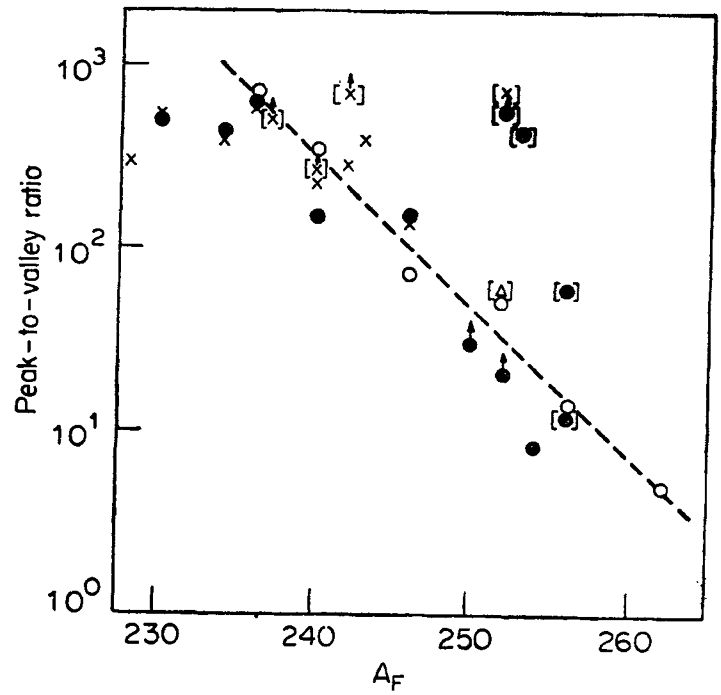

Figure 10. Peak-to-valley ratios versus mass of fissioning nuclei (A_F) obtained in the present work (open circle) are compared with experimental values of Von Gunten (1969) (cross), Unik *et al* (1973), (solid circle) and Schmitt (1970) (triangle). The experimental data within parenthesis are for SF while those without parenthesis are for thermal fission. The broken line represents the least square fit of the values obtained in the present work.

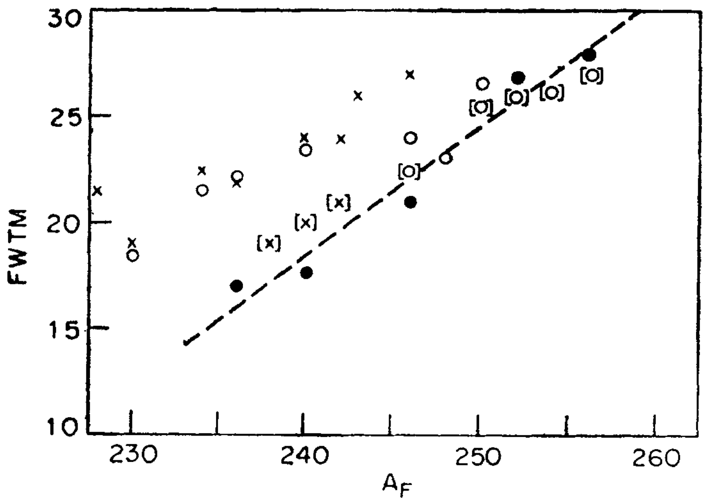

Figure 11. Full width of the peaks at one tenth maximum (FWTM) of the mass yield distribution versus mass of the fissioning nuclei (A_F) obtained in the present work (solid circles) compared with the experimental data of Von Gunten (1969) (cross) and Unik *et al* (1973) (open circle). The experimental values within parenthesis are for SF while those without parenthesis are for thermal fission. The broken line represents the least square fit of the values obtained in the present work.

is the only input data used in the calculations, the results obtained by this model could only be as good as the nuclear stability data. The revised table of 1977 used in the present calculation for obtaining the $N^S_{L,H}$ values for the charges has brought about improvement in the right direction with the experimental data as illustrated in the figures.

6.3. *Isobaric charge distribution in fission*

Isobaric charge distribution in fission is generally studied by plotting the independent yields of isobars as a function of the charge. These distributions are described by a Gaussian characterised by a mean (Z_p) and spread. These parameters are found to vary with mass. The mean (Z_p) is generally expressed in terms of ($Z_p - Z_{UCD}$) where Z_{UCD} is the charge given by the unchanged charge density, (Z_F/A_F).

The (Z_p—Z_{UCD}) function (Wahl plot) for ^{236}U fission is given in figure 12. The parameters calculated from the experimental Z_p value of Notea (1969) compiled after changing it to those of fragments by shifting the mass axis by $\bar{\gamma}$ units (Schmitt 1970) are also shown in the figure. The general trend of the results agree. The large scatter in the experimental data here can be attributed to the effect of neutron evaporation on fragments, leading to drastic difference in charge distributions of fragments and products. The experimental data have been corrected only by shifting the mass axis and this procedure cannot take into account the variation of neutron evaporation from isobaric fragments as discussed by Iyer and Ganguly (1971). The absolute values of (Z_p—Z_{UCD}) for the complementary fragments should be the same and since this is an input condition in the model, it is satisfied, but this requirement is not fulfilled in the values obtained from Notea's (1969) z_p values due to the effect of neutron evaporation from the fragments.

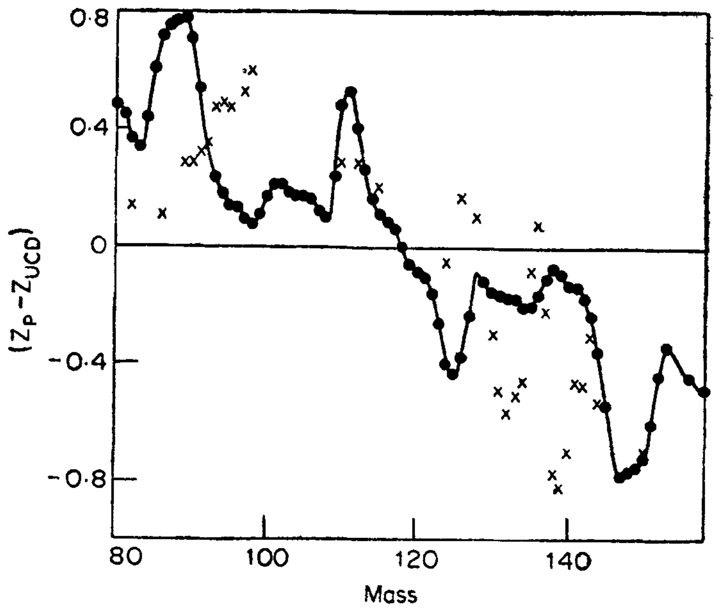

Figure 12. $(Z_p\text{-}Z_{UCD})$ versus fragment mass for ^{236}U obtained in present work (solid line with solid circles) are compared with those calculated from Z_p values for products of Notea (1969), (cross) after changing to fragment mass using $\bar{\gamma}$ (A) values of Schmitt (1970).

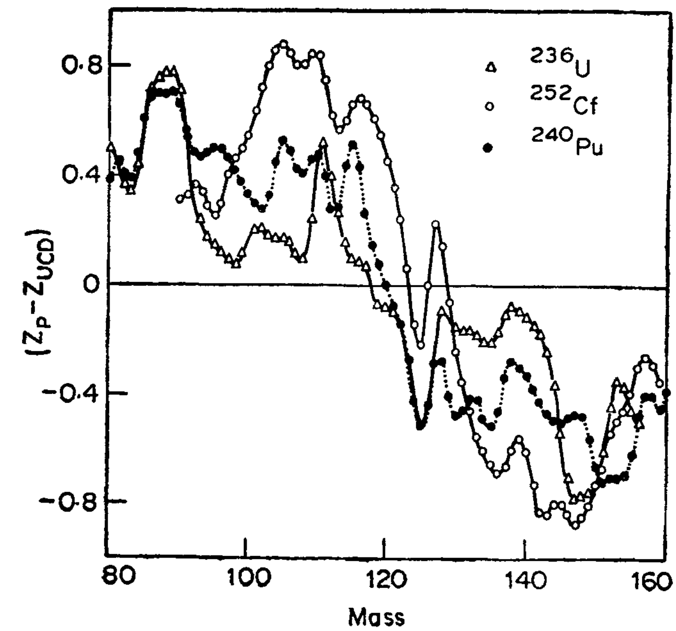

Figure 13. $(Z_p\text{-}Z_{UCD})$ versus fragment mass (A) for ^{236}U, ^{240}Pu and ^{252}Cf.

The Wahl plot for ^{236}U, ^{240}Pu and ^{252}Cf are compared in figure 13. In the heavy fragment region $(Z_p - Z_{UCD})$ values are found to increase with the mass of the fissile nuclide; in other words by and large the distribution for ^{240}Pu is found to lie between that of ^{236}U and ^{252}Cf. This is in line with the conclusion arrived at in the earlier studies using the second step of the ODM (Iyer and Ganguly 1971). However, it may be noted that in the present work no input fission data are used in the calculations.

Appendix

Number of eigenstates for a given charge polarisation

The total number of eigenstates for an assembly of an identical fermi elements where g elementary wave function $(g>n)$ are available to each of the elements is given by:

$$g!/[n!\,(g-n)!].$$

For a number of groups in the assembly of identical elements, the total number of eigenstates is given by

$$G = \Pi_i\, g_i\,!/[n_i\,!\,(g_i - n_i)!],$$

where the total number of elementary wave functions available is $\Sigma\, g_i$ and the total number of elements is $\Sigma\, n_i$.

To apply the formulation to the polarised compound nucleus we proceed as follows:

The assembly of fermi particles in the compound nucleus consists of two species of identical particles of Z_F and N_F. There are two groups for Z_F viz., Z_L and Z_H and three groups for N_F viz., N_L^S, N_H^S and $N_{\rm bal}$. If $g_z \geqslant Z$ and $g_n \geqslant N$ elementary wave functions are available respectively to protons and neutrons in the nucleus then the respective eigenstates for the fermi particles are given by

$$G_Z = \frac{g_z!}{Z_L!\,(g_z - Z_L)!} \times \frac{(g_z - Z_L)!}{Z_H!\,(g_z - Z_F)!}$$

and

$$G_N = \frac{g_n!}{N_L^S!\,(g_n - N_L^S)!} \times \frac{(g_n - N_L^S)!}{N_H^S!\,(g_n - N_L^S - N_H^S)!} \times \frac{(g_n - N_L^S - N_H^S)!}{N_{\rm bal}!\,(g_n - N_F)!}$$

The total number of eigenstates in the assembly is given by

$$G = G_Z\, G_N = \frac{g_z!\; g_n!}{(g_z - Z_F)!\;(g_n - N_F)!} \times \frac{1}{Z_L!\; Z_H!\; N_L^S!\; N_H^S!\; N_{\rm bal}!}.$$

For a given nucleus $g_z!\, g_n!/(g_z - Z_F)!\, [(g_n - N_F)!]$ is a constant and hence equation (7) in the text follows:

i.e. $G \propto 1/[Z_L!\, Z_H!\, N_L^S!\, N_H^S!\, N_{\rm bal}!]$.

Further $A_F (= Z_L + Z_H + N_L^S + N_H^S + N_{bal})$ is also the total number of phase space locations filled up in the nucleus any time and this is a constant for a given fissioning nucleus. We assume that all the phase space locations are available for each of protons and neutrons. In the various possible distributions of Z_F and N_F in A_F phase space locations a polarised state of the fissioning nucleus is characterised by those distributions wherein Z_L and N_L^S are distributed in M_L^S phase space locations, Z_H and N_H^S are distributed in M_H^S phase space locations and N_{bal} in N_{bal} locations. This gives rise to the second factor in the numerator of equation (5).

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