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Effect of correlations on nuclear muon capture

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Abstract. The process $\mu^- + {}^{12}C \to {}^{12}B + \nu_\mu$ is studied using the modified Hartree Fock wavefunction obtained with the unitary-model-operator-approach starting from the realistic hardcore nucleon-nucleon interaction, with the aim of testing the wavefunctions and obtaining a numerical value for the induced pseudoscalar coupling constant (g_P). These observables, namely, the partial capture rate to the ${}^{12}B(1^+$; g.s), its recoil nuclear polarisation and the total capture rate, which exhaust the available experimental data in the above process have been calculated and compared with the other theoretical and experimental results.

As far as the partial capture rate is concerned the use of the unitary-model operator approach wave functions for ${}^{12}C$ with $b = 2{\cdot}09$ fm and Cohen-Kurath wave function for ${}^{12}B(1^+$; g.s) reduces the pure shell model capture rate by about 30%. The effect of strong configuration mixing in the ground state of ${}^{12}C$ is taken into account by introducing a scale factor ξ similar to the 'amplitude reduction factor' of Donnelly and Walecka. With this ξ the agreement with the experiment both for the partial capture rate and the beta decay 'ft' value is found to be satisfactory.

The ${}^{12}B(1^+$; g.s) recoil polarisation is found to be insensitive to the use of the unitary-model-operator-approach wave functions. When compared with the experimental data, we obtain $g_P = (14{\cdot}9 \pm 1{\cdot}9) g_A$.

The total capture rate is found to be sensitive to the use of the unitary-model-operator-approach wave functions which contain the effect of nucleon-nucleon short range correlations and we obtain a satisfactory agreement with the experiment for ${}^{16}O$ and ${}^{12}C$, thereby revealing the importance of the effect of such correlations in the total capture rate studies.

Keywords. Muon capture; Hartree-Fock wave functions; unitary model operator approach; partial capture rates; total capture rates; recoil polarisation; short range correlations; induced pseudoscalar coupling constant; tensor coupling constant; beta decay 'ft' value.

1. Introduction

The capture of muons from the atomic K-orbit by the nuclear protons is by now a well-understood semi-leptonic strangeness-conserving weak interaction process and can be used as a probe to examine the nuclear models. The various weak hadronic form factors are governed by the usual assumptions about the hadronic vector and axial vector currents, namely conserved vector current (CVC) (Feynman and Gell-Mann 1958) and partially conserved axial-vector current (PCAC) (Gell-Mann and Levy 1958; Goldberger and Treiman 1958) respectively. Consequently the 'first class' form factors (Weinberg 1958) at their static limit are given by

Vector form factor $g_V(0) = 0{\cdot}983$ G,
Axial vector form factor $g_A(0) = -1{\cdot}23\, g_V(0)$,
Weak magnetism form factor $g_M(0) = 3{\cdot}7\, g_V(0)$.

These form factors are, in general, functions of the square of the four-momentum transfer q_μ and are expected to differ very little from their static limit, especially when $q^2 \leqslant 0.76$ GeV^2. The value of the induced pseudoscalar form factor for muon capture by a free proton is given by PCAC as $7.5\, g_A(0)$ and in nuclear muon capture this value has been shown to be (Castro and Dominguez 1977) the upper bound for g_P. The 'second class' induced scalar form factor g_S is expected to be zero under CVC and exact SU(2) symmetry for the nucleons. However the second class induced tensor form factor g_T may or may not exist. In muon capture studies these two always occur in the linear form g_P+g_T and so we take the view that the sum represents an effective coupling whose value is to be found from experimental data. Further we make the justifiable assumption that the static limits of the above free nucleon form factors do not change very much when taken over to the nuclear medium using impulse approximation and neglecting the meson exchange effects. Although the recent studies (Ohta and Wakamatsu 1974; Rho 1974) indicate that the axial vector form factors could be quenched in nuclear medium owing to the non-conservation of the axial vector current, the effect is believed to be very small for light nuclei. Thus, the weak interaction of the problem being more or less understood, the muon capture process can be used to examine the nuclear models. The process,

$$\mu^- + {}^{12}\mathrm{C} \rightarrow {}^{12}\mathrm{B} + \nu_\mu, \tag{1}$$

has been chosen in the present study. Its earlier theoretical and experimental studies are summarised by Mukhopadhyay (1977). The final nucleus could be in any one of the low lying states, 1^+ (g.s.), 2^+, 0^- and 1^-. However it has been experimentally found (Miller *et al* 1972) that about 87% of the transition leads to the ground state of ^{12}B, i.e 1^+, 12% to the 1^- state at 2·62 MeV and the remaining 1% to 2^+ and 0^-. In this study of partial capture rate and recoil polarisation, we confine to the ground state of ^{12}B. This can be described to be a proton-hole in $1p_{3/2}$ state and a neutron-particle in $1p_{1/2}$ state and no other configuration is possible within $1\hbar\omega$ excitation. If calculations are made with this simple picture and with harmonic oscillator basis with $b=1.64$ fm, then the partial capture rate and beta decay 'ft' value turn out to be about five times the experimental value. Foldy and Walecka (1965) made use of the inelastic electron scattering data and the 'ft' value of the beta decay process $^{12}\mathrm{B}(1^+)\rightarrow{}^{12}\mathrm{C}(0^+)+e^-+\tilde{\nu}_e$ to obtain the partial capture rate $\lambda(1^+)$ in a nuclear model independent way, since the matrix element involved in inelastic electron scattering can be related by iso-spin rotation to the Gamow-Teller matrix element in muon capture and beta decay. The direct evaluation of $\lambda(1^+)$ with a satisfactory agreement with the experiment has been carried out for the first time by Hirooka *et al* (1968) who used the general $1p$-shell wave functions of Cohen and Kurath (1965) which contain a very strong ground and excited state correlations. These strong correlations essentially bring a downward renormalisation of the pure shell model wave function for ^{12}C(g.s.) and ^{12}B(g.s) by about 36% and 64% respectively. In an attempt to unify all semi-leptonic processes in $A=12$ system, Donnelly and Walecka (1972) observed that the TDA particle-hole amplitude for ^{12}C(1) or its isobaric analogue ^{12}B(1^+; g.s), gets reduced by an adhoc factor $\xi=2.27$ purely by comparing their calculation with the experimental data and this factor is termed as 'amplitude reduction factor'. There are attempts (Devanathan *et al* 1975) to obtain the partial capture rate using the Helm (1956) model for the nucleus by utilising the inelastic electron scattering data to fix the parameters of this model. These are found to give a satisfactory agreement with the experiment but do not throw much light upon the nuclear structure.

Quite contrary to the above situation of the sensitivity of the $\lambda(1^+)$ on nuclear models, the recoil polarisation of $^{12}B(1^+;$ g.s) has been shown by Devanathan *et al* (1972) to be almost nuclear model insensitive. Recently Parthasarathy and Sridhar (1979) have taken into account the correction to the recoil polarisation of $^{12}B(1^+;$ g.s) coming essentially from the gamma decay of $^{12}B(1^-;$ 2·62 MeV) and showed that such corrections are negligible. Thus it seems, that the best observable to examine (g_P+g_T), is the recoil polarisation of $^{12}B(1^+;$ g.s), which is largely free from the nuclear wave function uncertainties.

The total muon capture rate in ^{12}C has been studied by Foldy and Walecka (1964) by using the fact that most of the dominant contribution comes from the narrow region of giant dipole states and that this can be related to the bremsstrahlung weighted photo-nuclear cross-section. This nuclear model-independent treatment gives a good agreement with the experiment. Detailed pure-shell model calculations have been carried out by Bell and Llewellyn-Smith (1972) who conclude that only when the supermultiplet symmetry is assumed, a good agreement with the experiment could be obtained. Recent studies on total muon capture rates have been summarised by Mukhopadhyay (1977) and the general conclusion is that the total capture rate is sensitive to the ground state wave function and much less to the choice of the average neutrino energy.

The purpose of the present investigation is to evaluate the partial and total capture rates and the recoil polarisation using the modified HF wave functions obtained with unitary-model-operator approach (UMOA) starting from the realistic hard core nucleon-nucleon interaction (Shakin and Waghmare 1966; Shakin *et al* 1967). These UMOA wave functions contain the effects of the short range correlations due to the hard core and our motivation is to see how far the effects of such correlations affect these observables. It has been shown by Kaushal and Waghmare (1970) and Bhalerao and Waghmare (1977) that these wave functions describe well the process of bound pion absorption (which is similar to muon capture except for the nature of the interaction) with the emission of two nucleons. Thus the motivation here is to provide a complete description of the process (1) within the context of the realistic nuclear model calculations.

In § 2, the necessary theory of the process (1) is briefly reviewed and closed expressions for the partial capture rate recoil polarisation and the total capture rate are given. The nuclear model used in our calculation is briefly described in §3 and the numerical results along with the discussion are given in §4.

2. Theory of nuclear muon capture

The effective Hamiltonian for $\mu^- + p \rightarrow n + \nu_\mu$ has been obtained by Fujii and Primakoff (1952) starting from the V-A weak interaction Lagrangian, including the strong interaction effects of the nucleons from general invariance arguments and by making use of the two component wave function for the fermions. It is given by (in units $\hbar = c = m_\mu = 1$),

$$H_{\rm eff} = \tfrac{1}{2}\, \tau_l^{+} (1-\sigma_l \cdot \hat{\nu}) \sum_{i=1}^{A} \tau_i^{-} \{ G_V 1_l \cdot 1_i + G_A \sigma_l \cdot \sigma_i - G_P (\sigma_l \hat{\nu})(\sigma_i \cdot \hat{\nu})$$

$$- \frac{g_V}{M} (\sigma_l \cdot \hat{\nu})(\sigma_l \cdot \mathbf{p}_i) - \frac{g_A}{M} (\sigma_l \cdot \hat{\nu})(\sigma_i \cdot \mathbf{p}_i)\} \, \delta(\mathbf{r}-\mathbf{r}_i), \tag{2}$$

where τ_i, σ_i, 1_i and p_i are the nucleon iso-spin, Pauli spin, unit and momentum operators, τ_l, σ_l and 1_l are lepton iso-spin, Pauli spin and unit operators respectively, $\hat{\nu}$ is the unit vector along neutrino momentum, M is the mass of the nucleon and $\delta(\mathbf{r}-\mathbf{r}_i)$ is due to the local nature of the weak interaction assumed in (2). The effective couplings G_V, G_A and G_P are given by,

$$\left.\begin{aligned} G_V &= g_V (1 + \nu/2M) \\ G_A &= g_A - (g_V + g_M)\, \nu/2M \\ G_P &= (g_P - g_A - g_V - g_M + g_T)\, \nu/2M \end{aligned}\right\}. \tag{3}$$

ν is the magnitude of the momentum carried by the neutrino (and it is also the momentum transfer of the process) and is given by $\nu = m_\mu - \Delta E$, neglecting the binding energy of the muon in the atomic K-orbit, where ΔE is the energy required to excite the final nuclear state. The matrix element for the process (1) can be given by (Devanathan 1968)

$$\mathscr{P} = \langle u_\nu \,|\, \mathbf{\Omega} \,|\, u_\mu \rangle, \tag{4}$$

where u_ν and u_μ are the two-component spinors for neutrino and muon respectively and

$$\Omega = \tfrac{1}{2} (1-\sigma_l \cdot \hat{\nu})(\sigma_l \cdot \mathbf{K} + L), \tag{5}$$

with

$$L = G_V \int 1_i - \frac{g_V}{M} \int \hat{\nu} \cdot \mathbf{p}_i, \tag{6}$$

$$\mathbf{K} = G_A \int \sigma_i - G_P \int (\hat{\nu} \cdot \sigma_i)\, \hat{\nu} - \frac{i g_V}{M} \int \hat{\nu} \times \mathbf{p}_i - \frac{g_A}{M} \int (\sigma_i \cdot \mathbf{p}_i)\, \hat{\nu}. \tag{7}$$

The various integrals in (6) and (7) are essentially the nuclear matrix elements and in general they are given by,

$$\int O_j = \int \langle J_f M_f \,|\, \sum_{i=1}^{A} \tau_i^{-} \exp(-i\, \hat{\nu} \cdot \mathbf{r}_i)\, \phi_\mu(r_i)\, O_j \,|\, J_i M_i \rangle \frac{d\Omega_\nu}{4\pi}, \tag{8}$$

where ϕ_μ is the muon wave function which can be factored out by its average value

$$|\phi_\mu|^2_{\rm av} = \frac{1}{\pi} (Z/a_0)^3 R_\mu, \tag{9}$$

with a_0 as the muonic Böhr radius and R_μ a correction factor for the finite size of the nucleus given by $R_\mu = (Z_{\rm eff}/Z)^3$ where $Z_{\rm eff}$ is the effective nuclear charge as seen by the muon. For ^{12}C, $R_\mu = 0{\cdot}86$. The nuclear matrix elements can be evaluated using the standard angular momentum algebra.

2.1. *Partial muon capture rate*

The expressions for the partial transition rate can be obtained by squaring $\mathcal{P}$ and summing and averaging over the lepton spin states. Then

$$\lambda = \frac{\nu^2}{2\pi} | \mathcal{P} |^2 \rho,$$

where ρ is the density of final states. As far as the transition to ^{12}B (1^+; g.s) is concerned, (an allowed Gamow-Teller; $\triangle J = 1$ and $\triangle\pi = No$) the nuclear matrix element, (Fermi type) $\int 1_i = 0$ and so the partial muon capture rate, becomes,

$$\lambda(1^+) = \frac{\nu^2}{2\pi} | \phi_\mu |^2_{\rm av.} G^2\{G_A{}^2 | \int \sigma_i |^2 + (G_P{}^2 - 2G_P\, G_A) | \int \hat{\nu} \cdot \sigma_i |^2$$

$$+ 2(G_P - G_A) \frac{g_A}{M} \text{R.P.} [(\int \hat{\nu} \cdot \sigma_i)(\int \sigma_i \cdot \mathbf{p}_i)^*]$$

$$+ 2G_A \frac{g_v}{M} \text{R.P.} \; [i \int \sigma_i \cdot (\int \hat{\nu} \times \mathbf{P}_i)^*]\}, \tag{10}$$

where R.P. means the real part. To convert the capture rate from $\hbar = c = m_\mu = 1$ to CGS units, equation (10) must be divided by $\hbar/m_\mu c^2 = 6{\cdot}22 \times 10^{-24}$ sec. The various nuclear matrix elements in (10) have been evaluated by Devanathan *et al* (1972) and expressed in terms of angular momentum coefficients and radial integrals. The terms in (10) which do not contain the nucleon momentum operator are known to give the dominant contribution (about 92%) and they involve the radial integral,

$$\int R_{1p}(r)\, j_l(\nu r) R_{1p}(r) r^2 dr, \tag{11}$$

which will get modified by the use of UMOA wave functions. The mometum-dependent terms are known to contribute not appreciably to capture rate and these are evaluated using the pure shell model and added to the dominant contribution. The capture rate can be studied for various values of $(g_P + g_T)$ and the numerical results are given in § 4.

2.2. *Recoil nuclear polarisation*

It has been found experimentally (Garwin *et al* 1957; Possoz *et al* 1977) that the muon, after very many cascades, when reaches the atomic *K*-orbit, possesses a residual polarisation of 15–20% at the time of capture by the nuclear protons. This residual polarisation of the muon although does not affect the capture rate, causes the well-known parity violating effects like asymmetry in the angular distribution of recoil nucleus, polarisation of the recoil nucleus and the asymmetry in the angular distribution and the polarisation of neutrons emitted (Devanathan and Rose 1967). It seems that of these parity violating effects only the recoil polarisation has been accurately measured, while the other observables involve large experimental uncertainties. The recoil polarisation of $^{12}B(1^+;$ g.s) has been measured by Louvain and Louvain-Saclay-ETH group (Possoz *et al* 1974, 1977) by observing the beta decay of $^{12}B(1^+;$ g.s). The theoretical study of $^{12}B(1^+;$ g.s) recoil polarisation has been carried out in a systematic way by Devanathan *et al* 1972, Parthasarathy and Sridhar (1979) and we give only the relevant expressions here.

Considering a nuclear transition from $|J_iM_i\rangle$ to $|J_fM_f\rangle$, the spin orientation of the final nucleus can be studied by constructing the density matrix ρ_f of the final nucleus in its spin space. The spin orientation of $|J_fM_f\rangle$ can be conveniently represented by a set of tensor operators T_k^μ (rank K and projection μ) whose average expectation value can be given by

$$\langle T_k^\mu\rangle = \mathrm{Tr}\ [T_k^\mu \rho_f]/\mathrm{Tr}\ \rho_f. \tag{12}$$

These tensor operators are defined in the spin space of the final nucleus and obey the normalisation condition,

$$\mathrm{Tr}\ [T_k^{\mu\dagger}\ T_{k'}^{\mu'}] = (2J_f+1)\delta_{kk'}\ \delta_{\mu\mu'}.$$

For unoriented initial nucleus and the transition operator

$$t = \sum_{\lambda,\, m\lambda} t_\lambda^{m\lambda}$$

it has been shown by Devanathan *et al* (1972) that,

$$\mathrm{Tr}[T_k^\mu\rho_f] = \frac{1}{2J_i+1} \sum_{\lambda\lambda'} \sum_{m_\lambda m_{\lambda'}} C(\lambda\lambda' K;\ m_\lambda - m_\lambda' - \mu)\ W(\lambda J_i K J_f;\ J_f \lambda') \frac{[J_f]^3}{[K]}$$

$$(-1)^{\lambda - m_\lambda}\langle J_f \| T_k \| J_f\rangle\ \langle J_f \| O_\lambda \| J_i\rangle\ \langle J_f \|\ O_{\lambda'} \| J_i\rangle^*, \tag{13}$$

and $\mathrm{Tr}\rho_f$ can be obtained from (13) by putting $K=0$. In the case of muon capture process, the transition operator t will be a sum of various terms in (5) to (8) and $\mathrm{Tr}[T_k^\mu\rho_f]$ can be evaluated using the standard angular momentum techniques. It is to be noted here that in this case as the muons are polarised, one must use the projection operator for the muons in finding $|\mathscr{P}|^2$ i.e.

$$|\mathscr{P}|^2 = \tfrac{1}{2}\mathrm{Tr}\ [\Omega(1+\boldsymbol{\sigma}_l \cdot \mathbf{P}_\mu)\ \Omega^\dagger]. \tag{14}$$

The resulting expressions for $\mathrm{Tr}\,[T_k^\mu\rho_f]$ are complicated. However, when an integration over neutrino direction is carried out, one obtains δ_{k1} i.e. only vector polarisation for the recoil nucleus can occur. Further choosing $\mathbf{P}_\mu$ along z-axis, it can be shown that for $^{12}B(1^+; g.s)$,

$$\langle T_1{}^0\rangle = \mathbf{A}/B, \tag{15}$$

where
$$\mathbf{A} = [-iG_A{}^2 \int \boldsymbol{\sigma}_i \times (\int \boldsymbol{\sigma}_i)^* - 2G_A G_P\, i \int \hat{\nu} \times \boldsymbol{\sigma}_i (\int \hat{\nu} . \boldsymbol{\sigma}_i)^*$$

$$-2G_A \frac{g_A}{M} \text{R.P.} \{i \int \hat{\nu} \times \boldsymbol{\sigma}_i (\int \boldsymbol{\sigma}_i \cdot \mathbf{p}_i)^*\} + 2G_A \frac{g_V}{M} \text{R.P.} \{\int \boldsymbol{\sigma}_i (\int \hat{\nu} \cdot \mathbf{p}_i)^*\}$$

$$+2(G_P - G_A) \frac{g_V}{M} \text{R.P.} \{\int \hat{\nu} . \boldsymbol{\sigma}_i (\int (\mathbf{p}_i)^*\}] . \mathbf{P}_\mu \tag{16}$$

and B is given by $\{\ldots.\}$ part of (10). The $^{12}B(1^+; g.s)$ recoil polarisation is then given by

$$\mathbf{P}_N = \sqrt{\frac{2}{3}} \langle T_1{}^0\rangle \mathbf{P}_\mu. \tag{17}$$

The recoil polarisation $\mathbf{P}_N$ to a large extent, is seen to be free from the nuclear model uncertainties. The reason for this is that if we neglect nucleon momentum dependent terms and consider only S-wave neutrino, then the nuclear matrix elements in A and B exactly cancel each other, leaving,

$$\mathbf{P}_N = [(2G_A{}^2 - \tfrac{4}{3} G_A G_P)/3G_A{}^2 + G_P{}^2 - 2G_P G_A)]\,\mathbf{P}_\mu \simeq 0{\cdot}61 \mathbf{P}_\mu,$$

for $(g_P + g_T) = 7{\cdot}5 g_A$. As the nucleon momentum dependent terms and the higher partial waves for neutrino ($l = 2$) are expected to contribute very little, $\mathbf{P}_N$ is nearly free from the nuclear model uncertainties. In our calculation we compare the values of $\mathbf{P}_N$ obtained with pure shell model and UMOA wave functions which contain the effect of short range correlations and then obtain a value for $(g_P + g_T)$ by comparing with the experimental data. This is complementary to the early study of Devanathan *et al* (1972) where the results are computed and compared for Independent Particle Model (IPM) and general $1P$-shell wave functions and that of Parthasarathy and Sridhar (1979) wherein a comparison is made among the results of IPM, particle-hole models of Gillet-Vinhmau, Donnelly and Walker in $2\hbar\omega$ shell model space.

2.3. *Total muon capture rate*

This is defined to be the sum of all partial transition rates to the energetically possible levels of the final nucleus and so is given by

$$\Lambda_T = \frac{|\phi_\mu|^2}{2\pi} G^2 \sum_b \nu_{ab}^2 \,[G_V^2 \,|\int 1_i|^2 + G_A^2 \,|\int \boldsymbol{\sigma}_i|^2$$

$$+ (G_P - 2G_A\, G_P)\, |\int \hat{\nu} \cdot \boldsymbol{\sigma}_i|^2] + \Lambda', \tag{18}$$

where Λ' is the contribution due to the nucleon-momentum dependent terms, a is the initial and b the final nuclear levels. The sum over b cannot be evaluated in all its absoluteness. We adopt here the following simplifying assumptions which are justifiable under some conditions which are also discussed.

(i) The quantity $\nu_{ab}=m_\mu-E_a+E_b=m_\mu-\triangle E_{ba}$ can be replaced by $m_\mu-\widetilde{\triangle E}$ independent of the final nuclear state. It has been realised by Foldy and Walecka (1964) that nearly 90% of the total capture is due to the partial transition to the giant dipole state and so $\widetilde{\triangle E}$ could be a representative value for the narrow band of energies where the giant dipole strength is concentrated. For ^{12}C, the giant dipole state lies at 22·6 MeV and so $\hat{\nu}$ could be 80-82 MeV. One can also interpret $\tilde{\nu}$ to be a parameter to fit the data.

(ii) The levels b of the final nucleus are assumed to form a complete set, so that $\sum_b |b\rangle\langle b| = 1$.

(iii) The operators appearing in $\int 1_i$, $\int \boldsymbol{\sigma}_i$ and $\int \hat{\nu}\cdot\boldsymbol{\sigma}_i$ can be identified with the generators of the Wigner supermultiplet and the consequence of this identification (Foldy and Walecka 1964) is,

$$\sum_b \left|\int 1_i\right|^2 = 3^{-1}\sum_b \left|\int \boldsymbol{\sigma}_i\right|^2 = \sum_b \left|\int \hat{\nu}\cdot\boldsymbol{\sigma}_i\right|^2.$$

This property has been examined by Rho (1965), Walker (1966) and Barrett (1967) using particle-hole formalism and they conclude that this is valid within a deviation of 20% for closed shell nuclei. It has been pointed out by Christillen *et al* (1973) that the effects of assuming supermultiplet symmetry and neglecting Λ', nearly cancel each other.

Under these assumptions, (18) becomes,

$$\Lambda_T = \frac{\tilde{\nu}^2}{2\pi}|\phi_\mu|^2 G^2 (G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A)\,\mathcal{J}, \tag{19}$$

where

$$\mathcal{J} = \sum_{i,j} \langle a|\, \tau_i^+\tau_j^- \exp[i\hat{\nu}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\,|a\rangle$$

which can be written as

$$\mathcal{J} = Z - Q, \tag{20}$$

with
$$Q = -\sum_{i\neq j} \langle a|\, \tau_i^+ \tau_j^- \exp[i\hat{\nu}.(\mathbf{r}_i-\mathbf{r}_j)]\,|a\rangle \tag{21}$$

It is convenient to introduce the quantity Λ_r, the reduced capture rate as

$$\Lambda_T = \frac{m_\mu^2}{2\pi}|\phi_\mu|^2 G^2 (G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A)\, Z\, \Lambda_r, \tag{22}$$

and so $\Lambda_r = (\bar{\nu}/m_\mu)^2\,[1-(Q/Z)]$ (23)

The quantity $[1-(Q/Z)]$ is known as the Pauli exclusion factor (PEF). Using the identity,

$$(2l_1+1)^{1/2}\,(2l_2+1)^{1/2}\;C(l_1l_2l;\,000)=C(j_1j_2l;\,\tfrac{1}{2}-\tfrac{1}{2}0)/W(l_1j_1l_2j_2;\,\tfrac{1}{2}l), \qquad (24)$$

and the shell model for the state a, we obtain,

$$Q=2\sum_{l}\sum_{n_1l_1j_1}^{Z}\sum_{n_2l_2j_2}^{N}\left[\frac{C\,(j_1\,j_2\,l;\tfrac{1}{2}-\tfrac{1}{2}\,0)}{W(l_1\,j_1\,l_2\,j_2;\,\tfrac{1}{2}l)}\right]^2\times$$

$$[\int R_{n_2l_2}\,(r)\,j_l\,(\bar{\nu}r)\;R_{n_1l_1}\,(r)\,r^2\,dr]^2, \qquad (25)$$

where the sum over $n_1l_1j_1(n_2l_2j_2)$ is for all occupied proton (neutron) states. Equation (25) gives,

$$2e^{-2y},\; 6e^{-2y}\,(1+16/27\,y^2) \text{ and } 8e^{-2y}\,(1+y^2) \text{ for } {}^4\text{He},\ {}^{12}\text{C and } {}^{16}\text{O}$$

respectively, with $y=(b\bar{\nu}/2)^2$. In our calculation of the total muon capture rates, we use (25) with the radial integrals evaluated using UMOA wave functions.

3. Nuclear models

In this section, the modified HF formalism using UMOA is briefly reviewed for completeness. It is well known that the realistic nucleon-nucleon potential contains the hard core which causes the serious problem of infinities when the potential is evaluated using harmonic oscillator wave functions in the nuclear structure calculations. Shakin and Waghmare (1966) and Shakin *et al* (1967) have developed a method known as the unitary model operator approach (UMOA) which facilitates one to carry out the variational calculation with a realistic nucleon-nucleon potential which may contain the hard core. The philosophy of this approach is to introduce an 'unitary operator ' exp (iS) which when operated on uncorrelated many-body wave function introduces strong short range correlations in that wave function such that the short range part of the nucleon-nucleon potential which does contain the hard core, induces no energy shift in the correlated wave function with respect to the uncorrelated counter part. This results in the appearance of only the long range part in effective Hamiltonian used in structure calculations.

The nuclear Hamiltonian in the second quantisation form can be written as,

$$H=\sum_{\alpha\beta}a_\alpha^\dagger\,\langle\,\alpha\,|\,t\,|\,\beta\,\rangle\,a_\beta+\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta}a_\alpha^\dagger\,a_\beta^\dagger\,a_\delta\,a_\gamma\,\langle\,\alpha\beta\,|\,V_{12}\,|\,\gamma\delta\,\rangle, \qquad (26)$$

where t is the one-body kinetic energy operator and V_{12} the two-nucleon potential which may contain the hard core. From H, an effective Hamiltonian can be obtained as

$$H_{\text{eff}} = \exp(-iS)\, H \exp(iS), \tag{27}$$

where $\exp(iS)$ is the unitary model operator (Villars 1963; Da Providencia and Shakin 1964) spanning the two-particle space and induces strong short range correlations in the uncorrelated wave function used in (26) as,

$$\psi_{\text{c}}(\mathbf{r}_1 \ldots \mathbf{r}_n) = \exp(iS)\, \phi_{\text{u.c}}(\mathbf{r}_1 \ldots \mathbf{r}_n), \tag{28}$$

where the subscript c and u.c. refer to correlated and uncorrelated respectively. The unitary transformation in (27) can be expanded in terms of one-body, two-body, etc. clusters. The clusters of order greater than two can be neglected for the sufficiently short range nature of the correlations induced by S. As a result,

$$\text{H}_{\text{eff}} = \sum_{\alpha\beta} a_\alpha^\dagger \langle \alpha | t | \beta \rangle a_\beta + \tfrac{1}{2} \sum_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger \langle \alpha\beta | \exp(-iS)(t_1 + t_2 + V_{12}) \exp(iS)$$
$$- (t_1 + t_2) | \gamma\, \delta \rangle\, a_\delta\, a_\gamma \tag{29}$$

where t_1 and t_2 are the kinetic energies of particles 1 and 2 and the kets $|\alpha\beta\rangle$ and $|\gamma\delta\rangle$ are the uncorrelated wave functions. Using (28) and adding the harmonic oscillator potentials U_i for the particles 1 and 2, (29) becomes,

$$\text{H}_{\text{eff}} = \sum_{\alpha,\beta} a_\alpha^\dagger \langle \alpha | t | \beta \rangle a_\beta + \tfrac{1}{2} \sum_{\alpha\beta\gamma\delta} [a_\alpha^\dagger a_\beta^\dagger \langle \psi_c | t_1 + t_2 + U_1 + U_2 + V_{12} | \psi_c \rangle a_\delta a_\gamma$$
$$- a_\alpha^\dagger a_\beta^\dagger \langle \alpha\beta | t_1 + t_2 + u_1 + u_2 | \gamma\, \delta \rangle\, a_\delta\, a_\gamma]. \tag{30}$$

The terms in (30) containing U_i's represent the dispersive effect of the nuclear medium and for short range correlations, this has a small effect on the matrix elements of H_{eff}. The potential V_{12} is separated into v_{12} diagonal ($l=l'$) part and V_T off-diagonal ($l_1 \neq l'$) part which receives contributions only from the tensor force. The diagonal part is now separated into long range part v_{12}^l and short range part v_{12}^s such that v_{12}^s satisfies,

$$(t_1 + t_2 + u_1 + u_2 + v_{12}^s)\, \psi_{kl} = (\epsilon_k + \epsilon_l)\, \psi_{kl}, \tag{31}$$

$$(t_1 + t_2) + u_1 + u_2)\, \phi_{kl} = (\epsilon_k + \epsilon_l)\, \phi_{kl}. \tag{32}$$

Thus, v_{12}^s induces no energy shift in ψ_{kl} with respect to ϕ_{kl}. The solution of (31) and (32) give the distance at which the separation of v_{12} into v_{12}^l and v_{12}^s is made. For some particular states wherein v_{12} is completely repulsive, a short range psuedo-potential V_p is introduced as

$$V_{12} = v_{12}^s + V_p + v_{12}^l - V_p,$$

to carry out the separation. Substituting (31) and (32) in (30), one finds

$$H_{\text{eff}} = \sum_{\alpha\beta} \langle \alpha \,|\, t \,|\, \beta \rangle a_\alpha^\dagger \, a_\beta + \tfrac{1}{2} \sum_{\alpha\beta\gamma\delta} a_\alpha^\dagger \, a_\beta^\dagger \, a_\delta \, a_\gamma \, \langle \psi_c(\alpha\beta) \,|\, v_{12}^l - V_P + V_T^{OD}$$

$$+ V_T^{OD}(Q/e)\, V_T^{OD} + V_P(Q/e) V_P \,|\, \psi_c\,(\gamma\delta) \rangle, \tag{33}$$

where the second order terms in v_T^{OD} and V_P are included; Q is the Pauli operator and e is the energy denominator. This prescription can be applied to any singular or non-singular interaction. Shakin *et al* (1967) have evaluated the matrix elements of the Yale potential for various states of relative angular momentum. In order to keep the separation distance constant and to deal with those states of relative motion where the interaction is completely repulsive, the pseudopotential V_P is taken to be an attractive square-well type. Calculations were made by Kakkar (1969) and Kakkar and Waghmare (1970) using the above prescription for some light nuclei including ^{12}C and ^{16}O. She has also made HF calculations for these nuclei using the Sussex matrix elements. In our present study we use the results of both these calculations. In carrying out the self-consistent variational calculation, the HF single-particle wave functions are expanded in terms of the complete set of orthonormal functions which are chosen to be the harmonic oscillator type. The expansion coefficients are varied to obtain the energy minimum. The details can be found in Kakkar (1969).

$$\psi_{nlj} = \sum_{n'=1}^{N} C_{n'}^{n} \phi_{n'lj}. \tag{34}$$

The coefficients $C_{n'}^{n}$ for ^{12}C are given in table 1.

The matrix element of one-body transition operator between the UMOA single-particle wave functions then have the form,

$$\text{M.E.} = \langle \psi_{\text{UMOA}}^{n} \,|\, 0 \,|\, \psi_{\text{UMOA}}^{n} \rangle$$

$$= \sum_{n', n''} C_{n'}^{n} \, C_{n''}^{n} \, \langle \phi_{n'lj} \,|\, 0 \,|\, \phi_{n''lj} \rangle. \tag{35}$$

Table 1. Expansion coefficients $C_{n'}^{n}$ in equation (36) from Kakkar (1969) for ^{12}C. The upper and lower numbers are the results of the use of Yale and Sussex interaction matrix elements, respectively $b = 2{\cdot}09$ fm.

l	$n \mid n'$	Neutron			Proton		
		1	2	3	1	2	3
$s_{1/2}$	1	0·9357	0·3140	0·1605	0·9379	0.3083	0·1594
		0·9397	0·3139	0·1359	0·9422	0·3075	0·1330
$p_{3/2}$	1	0·9672	0·1215	0·2233	0·9685	0·0844	0·2344
		0·9647	0·2140	0·1532	0·9688	0·1954	0·1525

This will be used in total muon capture rate calculations. For partial muon capture rate the final state being $^{12}B(1^+$; g.s), the UMOA wave functions are used only for ^{12}C, the initial state. As a result, in this case the matrix element will be of the type,

$$\text{M.E.} = \sum C^n_{n''} \langle \phi_{nlj} | 0 | \phi_{n''lj} \rangle. \tag{36}$$

The $^{12}B(1^+$ g.s.) can be described either by pure shell model or by the general $1p$-shell wave functions of Cohen-Kurath (1965). The numerical results are discussed in the next section. It is to be noted here that the use of UMOA wave functions affect only the radial integrals, the angular momentum part of the matrix element remain unchanged.

4. Numerical results and discussion

Equations (10), (15) to (17), (23) and (27) for partial capture rate, recoil polarisation and total capture rate respectively, along with (35) and (36) form the basis of our calculations. In partial capture rate calculations, the nucleon momentum-dependent terms are evaluated in IPM and using the wave functions of Cohen-Kurath (1964).

4.1. *Partial capture rate* $\lambda(1^+)$

The numerical results for $\lambda(1^+)$ are summarised in table 2, along with the configuration mixing factor ξ, for various nuclear models for ^{12}C and ^{12}B. From this table, it is seen that the use of UMOA wave function for ^{12}C decreases $\lambda(1^+)$ by about 15%. The results for Yale and Sussex interaction matrix elements differ only by 5% although their detailed structure is quite different. Describing $^{12}B(1^+$; g.s) by the general $1p$-shell wave functions of Cohen-Kurath, as tabulated by Hirooka *et al* (1968), we find with UMOA description of ^{12}C, only the configuration $|(1p_{1/2})^1 (1p_{1/2})^7; J=T=1\rangle$ of $^{12}B(1^+$; g.s) contributes due to the properties of c.f.p. coefficients. The use of general $1p$-shell have function for ^{12}B, along with the UMOA description of ^{12}C, decreases $\lambda(1^+)$ from the IPM value by about 43%. The 'configuration mixing

Table 2. Partial muon capture rate $\lambda(1^+)$ units of 10^3 $\sec^{-1}$. IPM represents the pure independent particle model with $b=1\cdot64$ fm. Yale and Sussex for ^{12}C represent the use of equation (36) for the p-shell radial wave function with the expansion coefficients given in table 1. CK for ^{12}B represents the use of general $1p$-shell wave function of Cohen-Kurath for the ground state of ^{12}B. The uncertainties in ξ are due to the experimental uncertainties in $\lambda(1^+)$.

Nuclear models		$\lambda(1^+)$	ξ
^{12}C	^{12}B		
IPM	IPM	35·11	2·38 ± 0·06
Yale	IPM	29·26	2·17 ± 0·06
Sussex	IPM	30·67	2·22 ± 0·06
Yale	CK	19·38	1·77 ± 0·04
Sussex	CK	20·38	1·82 ± 0·04

factor ' ξ is found to be 2·38±0·06 when IPM is used for ^{12}C and ^{12}B which agrees with that of Donnelly and Walecka (1972). However, with ^{12}B described by the general 1p-shell wave function and ^{12}C by UMOA wave functions, ξ turns out to be 1·77±0·04 and 1·82±0·04 for Yale and Sussex expansion coefficients respectively. With this ξ, the variation of $\lambda(1^+)$ with (g_P+g_T) is given in figure 1 along with the experimental data of Miller *et al* (1972). Incorporating this value of ξ, the beta-decay 'ft' value for the process $^{12}B(1^+) \rightarrow {}^{12}C(0^+)+e^-+\bar{\nu}_e$ is found to be (10925 ±467) sec, and (10985±518) sec, for Yale and Sussex results which is to be compared with the experimental value (11700±120) sec. Thus, with the use of UMOA wave function for ^{12}C and general 1p-shell wave function for ^{12}B along with the ξ-factor due to the correlations in the ground state of ^{12}C, the muon capture rate and beta decay rate are found to be in agreement with the experimental data.

4.2. $^{12}B(1^+;\ g.s)$ *recoil polarisation*

The numerical values for the ^{12}B(1$^+$; g.s) recoil polarisation evaluated in IPM and the UMOA model using the Yale and Sussex interaction matrix elements are given in table 3 along with the earlier results of Devanathan *et al* (1972) using the general 1p-shell wave functions for both ^{12}C and ^{12}B. From this table, it is clear that recoil polarisation, to a large extent, is insensitive to the choice of the nuclear wave functions.

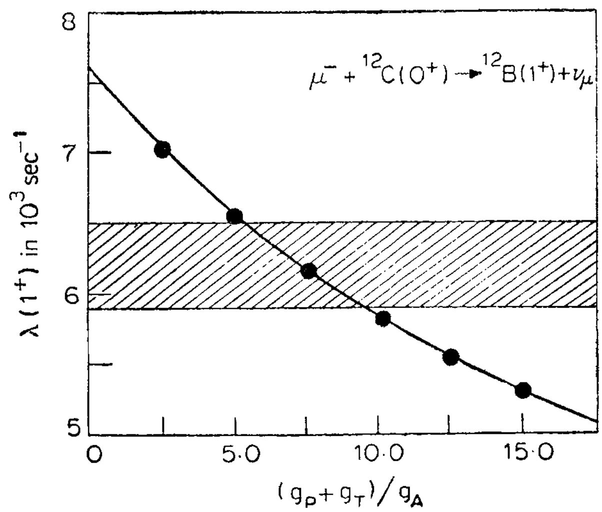

Figure 1. Variation of partial muon capture rate for the process (1) with (g_P/g_T). The dashed portion represents the experimental data of Miller *et al* (1972).

Table 3. ^{12}B(1+) recoil polarisation without momentum-dependent terms. IPM-Yale and Sussex are the same as that in table 2. CK represents the results of Devanathan *et al* (1972) obtained with the general 1p-shell wave functions for ^{12}C and ^{12}B.

$(g_P+g_T)/g_A$	IPM	Yale	Sussex	CK
0	0·6543	0·6534	0·6537	0·6560
7·5	0·5907	0·5864	0·5882	0·5977
15·0	0·4634	0·4580	0·4600	0·4775
22·5	0·2813	0·2459	0·2786	0·3041

However, it is found that as (g_P+g_T) increases, the insensitivity decreases, reaching a deviation of about 17% when $(g_P+g_T)/g_A$ is 22·5. The results for the UMOA wave functions with Yale and Sussex interaction matrix elements do not differ much and generally Sussex results are slightly higher than the Yale results. Having shown that $^{12}B(1^+$; g.s) recoil polarisation is insensitive to the choice of the nuclear wave function, we obtain a value for (g_P+g_T) by comparing with the experiment of Possoz *et al* (1977) as,

$$(g_P+g_T) = (14{\cdot}9\pm1{\cdot}9)\, g_A.$$

Recently Parthasarathy and Sridhar (1979) have shown that the corrections to $\mathbf{P}_N$ coming from the gamma decay of the excited states of ^{12}B are very small. Hence this value for (g_P+g_T) is the best possible value in the process (1). Since the range obtained for (g_P+g_T) is well within the limit (beyond which the deviation from one model to the other is about 15%), this range is claimed to be free from nuclear models. By using the Goldberger-Treiman value for g_P which could be the maximum value of g_P in nuclei (Castro and Dominguez 1977) we obtain a range for the second class induced tensor coupling as $g_T=(7{\cdot}4\pm1{\cdot}9)g_A$ in agreement with the calculations of Kubodera *et al* (1977) who find, in impulse approximation $g_T^{\beta} = (6{\cdot}2\pm1{\cdot}8)g_A$.

4.3. *Total muon capture rate*

The expression for the total capture rate as given in § 2.3 involves the ground state wave function of ^{12}C only. Our motivation here is to examine how far the UMOA wave functions are successful in predicting the total muon capture rate. It is shown in § 2.3 that three approximations have been used in obtaining the closed expression for the total capture rate, each one of which requires a detailed examination which is beyond the scope of the present work. However, we examine them briefly. The first approximation of replacing ν_{ab} by an average value $\bar{\nu}$ can be justified by the observation (Foldy and Walecka 1964) that most of the dominant transition proceeds to the narrow region of giant dipole states and $\bar{\nu}$ can be very nearly given by $(m_\mu - E_{\mathrm{GDR}})$. The second approximation of using the closure property for the excited states of the final nucleus greatly simplifies the problem as otherwise one has to physically carry out the sum which again depends upon the model chosen. The calculations of Luyten *et al* (1963) using shell model gives disagreement with the experiment when the closure property is not used. The third approximation of using Wigner's supermultiplet symmetry is found to be valid upto an uncertainty of 20% for doubly closed shell nucleus (Rho 1965; Walker 1966). In table 4 we give the results of the reduced capture rates as defined in (22) and (23), for ^{16}O using the UMOA wave functions and the expansion coefficients of Kakkar (1969) for Sussex interaction. From this table we find that for $b=2{\cdot}09$ fm (which is the value used in the UMOA calculations of Kakkar) and with $\bar{\nu}=80$ MeV, the agreement with the experiment for Λ_r is satisfactory. This is to be compared with the pure shell model calculation of Bell and Llewellyn-Smith (1972) under the same approximations who find for $b=1{\cdot}76$ fm, $\bar{\nu}=80$ MeV, $\Lambda_r=0{\cdot}122$. Thus the UMOA wave functions represent an improvement over the shell model wave functions. In table 5, the results

Table 4. Reduced capture rates in ^{16}O using the UMOA wave functions for Sussex interaction (Kakkar 1969). The experimental value is from Eckhause *et al* (1966).

ν MeV	Reduced Capture Rates	
	$b = 0.76$ fm	$b=2.09$ fm
80	0·088	0·113
82	0·097	0·124
84	0·106	0·136
	Experiment	0·111 ± 0·04

Table 5. Pauli Exclusion Factor (PEF) for ^{16}O using the UMOA wave functions for Sussex interaction (Kakkar 1969). LFW represents the choice of the parameter a (see text) as 0·416 $A^{1/3}$ fm^2. Empirical represents the use of $a=b^2/2$ where b is the oscillator parameter.

ν MeV	$b = 1.76$ fm			$b = 2.09$ fm		
	LFW	Empirical	UMOA	LFW	Empirical	UMOA
80	0·172	0·266	0·154	0·172	0·359	0·197
82	0·181	0·279	0·161	0·181	0·377	0·206
84	0·190	0·294	0·167	0·190	0·396	0·214

for PEF are given for UMOA wave functions along with the values of Levinger (1960) and Foldy and Walecka (1964) who expand PEF $= 1-Q(\nu)/Z = a\nu^4 + 0(\nu^4)$ $+\ldots$ as a power series in ν^2 and relate a to the bremsstrahlung weighted photonuclear cross-sections. From this table, we find that the UMOA results for PEF generally agree with Levinger-Foldy-Walecka formula. Thus we conclude that the use of UMOA wave functions is indeed successful in predicting Λ_r for doubly closed shell nucleus. ^{16}O for which the approximations made in § 2.3 are generally valid.

Let us now consider ^{12}C, a closed subshell $(1p_{3/2})$ nucleus. The shell model calculations of Bell and Llewellyn-Smith (1972) indicate that although the supermultiplet symmetry is not exact for a closed sub-shell nucleus, it is only when they discard the supermultiplet violation, agreement with the experiment is obtained. Their conclusion is that the very simplest shell model wave functions exaggerates greatly the effect of spin-orbit coupling in destroying the supermultiplet symmetry. So, following Bell and Llewellyn-Smith (1972) we use (27) and (23) to evaluate Λ_r but the UMOA wave functions via (35) and table 1. The numerical results for UMOA wave functions obtained with the Yale and Sussex interaction matrix elements do not differ by more than 2% and so in table 6, we give Λ_r for the results of Sussex interaction matrix elements only. From this table, it is found that for $b=2.76$ fm, (which is the value used in the UMOA calculations) and $\nu=80$ MeV, the agreement with the experiment is satisfactory. (The corresponding value of Bell and Llewellyn-Smith 1972 is 0·113).

Table 6. Reduced capture rates in ^{12}C using the UMOA wave functions for Sussex interaction (see table 1). The experimental values is from Eckhause *et al* (1966).

ν MeV	Reduced capture rates	
	$b = 1·76$ fm	$b = 2·09$ fm
80	0·0975	0·1339
82	0·1097	0·1451
	Experiment	0·125 ± 0·04

Thus the above results can be summarised as follows:

(i) The use of UMOA wave functions which take into account the effect of short range correlations provides an overall improvement over the shell model wave functions.

(ii) The recoil nuclear polarisation of $^{12}B(1^+; g.s)$ is found to be insensitive to the use of UMOA wave functions. The comparison with the experiment yields the result $(g_P+g_T)/g_A = 14·9 \pm 1·9$ free from the nuclear model uncertainties, thus showing that the second class tensor coupling could be as large as $(7·4 \pm 1·9)\ g_A$.

(iii) The UMOA wave functions, with a few reasonable assumptions are successful in predicting the reduced capture rates.

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