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A Thomas-Fermi type picture and the electromagnetic structure of the nucleon

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Abstract. Using a Thomas-Fermi type picture of the nucleon as a dense system of quarks and antiquarks, we give a rationale for the ' dipole ' nature, scaling and other characteristics of the nucleon electromagnetic form factors. Similar considerations are then given for the electromagnetic structure of the pion.

Keywords. Thomas-Fermi picture; electromagnetic form factors; Fermi momentum; quark system; antiquark system.

1. Introduction

There are several characteristic features which the nucleon electromagnetic structure exhibits to a good approximation (see, for example, Hand 1977). These are (i) the so-called dipole form, (ii) relative smallness of the electric form factor of the neutron and (iii) scaling between the electric and the magnetic form factors. In the present paper, we present a rationale for these aspects in terms of a Thomas-Fermi type of picture (see, for example, Condon and Shortley 1963), imagining the nucleon to be an extended system of quarks and antiquarks.

2. Nucleon as a system of quarks and antiquarks

Let the number-densities of the u and d quarks in a proton be denoted by $n_u(r)$ and $n_d(r)$ respectively and for the antiquarks in a similar way by $n_{\bar{u}}(r)$ and $n_{\bar{d}}(r)$. We take these functions to be spherically symmetric.

The number-densities are constrained by the conditions given by

$$\int_0^{r_0} [n_u(r) - n_{\bar{u}}(r)]\, 4\pi r^2\, dr = 2, \tag{1}$$

$$\int_0^{r_0} [n_d(r) - n_{\bar{d}}(r)]\, 4\pi r^2\, dr = 1, \tag{2}$$

where the upper limit r_0 corresponds to the nucleon radius.

Because of charge-symmetry the quark-distributions in a neutron can be obtained from the functions above through the interchange $u \leftrightarrow d$, $\bar{u} \leftrightarrow \bar{d}$.

Let $p_u^f(r)$ be the fermi momentum of the u-quark in a small volume $d\tau$ (at a distance r from the origin), then

$$n_u \, d\tau = [(p_u^f)^3/(3\pi^2)], \tag{3}$$

with similar expressions for the d-quark and for the antiquarks.

We expect the internal motions of the quarks to be not too large and assume that a non-relativistic picture can be used to obtain the charge and the magnetic moment distributions within a nucleon.

If the u-quark experience a potential $W_u(r)$, one has the relation

$$[(p_u^f)^2/2m] - W_u(r) = -W_u^0, \tag{4}$$

where both $W_u(r)$ and $W_u^0 \equiv W_u(r_0)$ are positive and m is the u (or d) quark mass.

Writing $W_u(r) - W_u^0 = V_u(r)$ and using (3) and (4), we can write the differential equation

$$dn_u(r)/dr = \frac{3}{2}\left(\frac{1}{V_u(r)} \frac{dV_u(r)}{dr}\right) n_u(r) \equiv -w_u(r)\, n_u(r), \tag{5}$$

and, in a similar way, for $\bar{u}$, d and $\bar{d}$ quark-types.

We note that $V_u(r)$ is predominantly given by the strong interaction short-range potential and that $1/w_u(r)$ represents roughly the range of the potential. Similar quantities can be defined for $\bar{u}$, d and $\bar{d}$ in terms of the respective potentials $V_{\bar{u}}(r)$, $V_d(r)$ and $V_{\bar{d}}(r)$. In general, $w_u(r)$, etc. could be complicated functions of the number densities themselves. If we assume a high density expansion of the type

$$w_u(r) = \kappa + \frac{a}{n_u(r)} + \frac{b}{n_u^2(r)} + \ldots + \frac{f}{n_{\bar{u}}(r)} + \frac{g}{n_{\bar{u}}^2(r)} + \ldots,$$

and similarly for $\bar{u}$, d and $\bar{d}$ then in the limit of large densities, these functions tend to some limits like $w_u(r) \to \kappa$ and so on, where κ^{-1} has the interpretation of the 'range' of the corresponding potential. Since this range is expected to be roughly the same for all the quark-types, we can immediately obtain from (5)

$$[n_u(r) - n_{\bar{u}}(r)] \simeq \frac{\kappa^3}{4\pi} \exp(-\kappa r), \tag{6}$$

$$[n_d(r) - n_{\bar{d}}(r)] \simeq \frac{\kappa^3}{8\pi} \exp(-\kappa r), \tag{7}$$

where $r < r_0$ and the normalisations in (6) and (7) have been fixed by the constraints given by (1) and (2).

We would then expect the charge distribution within the proton to be

$$\rho_c^p(r) = \tfrac{2}{3}\,[n_u(r)-n_{\bar{u}}(r)] - \tfrac{1}{3}\,[n_d(r)-n_{\bar{d}}(r)],$$

or

$$\rho_c^p(r) \simeq \frac{\kappa^3}{8\pi}\exp(-\kappa r), \tag{8}$$

and for the charge distribution within the neutron to be

$$\rho_c^n(r) \simeq 0. \tag{9}$$

The spatial Fourier transforms $\int \rho_c^{p,n}(r)\exp[iq\cdot r]\,d^3r$ can be identified with the electric form factors $G_E^{p,n}(q^2)$. The distributions in (8) and (9) give

$$G_E^p(q^2) \simeq \frac{\kappa^4}{(q^2+\kappa^2)^2}, \tag{10}$$

i.e. the 'dipole' form and

$$G_E^n(q^2) \simeq \text{small}, \tag{11}$$

both of these being consistent, to a good approximation, with the experimental observations.

To obtain the magnetic moment distributions, we assume that the ' sea ' of quark-antiquark pairs does not contribute to the spin or the magnetic moment of the nucleon. The proton wave-function is given by (see, for example, Kokkedee 1969)

$$|p, s_z=\tfrac{1}{2}\rangle=\sqrt{\tfrac{1}{18}}\,[2|u\uparrow d\downarrow u\uparrow\rangle+2|u\uparrow u\uparrow d\downarrow\rangle+2|d\downarrow u\uparrow u\uparrow\rangle-$$

$$-|d\uparrow u\downarrow u\uparrow\rangle-|u\uparrow d\uparrow u\downarrow\rangle-|u\downarrow d\uparrow u\uparrow\rangle-|u\uparrow u\downarrow d\uparrow\rangle$$

$$-|d\uparrow u\uparrow u\downarrow\rangle-|u\downarrow u\uparrow d\uparrow\rangle], \tag{12}$$

and one can write the neutron wave-function correspondingly in terms of *d*, *d*, *u* quarks.

The nucleon magnetic moment distributions can be written as

$$\rho_M^p(r)=\tfrac{2}{3}(n_u(r)-n_{\bar{u}}(r)s_u^p-\tfrac{1}{3}(n_d(r)-n_{\bar{d}}(r))s_d^p, \tag{13}$$

$$\rho_M^n(r)=\tfrac{2}{3}(n_d(r)-n_{\bar{d}}(r))s_d^p-\tfrac{1}{3}(n_u(r)-n_{\bar{u}}(r)s_u^p, \tag{14}$$

where $s_{u,d}^p$ can be obtained from (12):

$$s_u^p= (\tfrac{4}{18}+\tfrac{4}{18}+\tfrac{4}{18})e/2m=\tfrac{2}{3}(m_N/m)\,\mu_N, \tag{15}$$

$$s_d^p= (\tfrac{6}{18}-\tfrac{12}{18})\,(e/2m) = -\tfrac{1}{3}(m_N/m)\,\mu_N, \tag{16}$$

where m_N is the nucleon mass and μ_N stands for the nuclear magneton.

Using (6) and (7) in (13) and (14), one then has

$$\rho_M^p(r) \simeq (m_N/m)\, \kappa^3/8\pi \exp(-\kappa r)\mu_N, \tag{17}$$

$$\rho_M^n(r) \simeq -\tfrac{2}{3}\, (m_N/m)(\kappa^3/8\pi) \exp(-\kappa r)\mu_N, \tag{18}$$

wherefrom by integrating over the spatial volume, we obtain the magnetic moments as

$$\mu_p \simeq (m_N/m)\mu_N, \tag{19}$$

$$\mu_n \simeq -\tfrac{2}{3}\, (m_N/m)\, \mu_N. \tag{20}$$

Equations (19) and (20) have the usual feature of the quark model, namely $\mu_p/\mu_n = -\frac{3}{2}$ and also, for $m \simeq 300$ MeV, give $\mu_p = 3\mu_N$ and $\mu_n \simeq -2\mu_N$ to be compared with the experimental values of $\sim 2{\cdot}78\ \mu_N$ and $\sim -1{\cdot}93\ \mu_N$ respectively.

The spatial Fourier transforms $\int \rho_M^{p,n}(r) \exp[iq.r]d^2$ can be identified with the magnetic form factors $G_M^{p,n}(q^2)$ and one has on using (17), (18), (19) and (20)

$$G_M^p(q^2) \simeq \mu_p \frac{\kappa^4}{(q^2+\kappa^2)^2}, \tag{21}$$

$$G_M^n(q^2) \simeq \mu_n \frac{\kappa^4}{(q^2+\kappa^2)^2}, \tag{22}$$

which exhibit the dipole form and satisfy the scaling property

$$G_M^p(q^2)/\mu_p = G_M^n(q^2)/\mu_n = G_E^p(q^2). \tag{23}$$

3. Pion form factor and discussion

What we have shown above is how the basic features of the nucleon electromagnetic form factors can be understood in terms of a simple picture of the nucleon as a dense system of quarks and antiquarks. The parameter r_0 which can be interpreted as the matter radius of the proton, is expected to be $\sim$1 fermi. If we equate the root-mean-squared charge radius (as obtained from (8)) with r_0, we obtain $\kappa^{-1} \sim 0{\cdot}28\, f$, giving roughly the range of the effective potential.

The considerations given in § 2 are expected to apply in other cases. In particular, we may follow the same general lines for the pion electromagnetic form factor as well. Since we are dealing with a spin O-system, it is obvious that in this situation only the charge form factor $F^\pi(q^2)$ is relevant. Thus for π^+ which is imagined as a dense system composed of u and $\bar{d}$ and quark-antiquark pairs, the procedure of § 2 would lead to a dipole form for $F^\pi(q^2)$ as well:

$$F^\pi(q^2) \simeq (\kappa'^4/(q^2+\kappa'^2)^2), \tag{24}$$

where κ'^{-1} gives the range of the corresponding potential, and is expected to be of the same order as κ^{-1}. The form of $F^{\pi}(q^2)$ as given by (24) is consistent with experiments (see, for example, Marshall 1977) with $\kappa'^{-1} \sim 0{\cdot}2f$. It is interesting to note that this feature may be connected with the empirical observation that the pion electromagnetic structure is very close to that given by the Dirac isovector form factor.

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