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Magnetic moments of baryons in broken SU(4)

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Abstract. Assuming that the anomalous magnetic moment interaction has the form $aT^1_1 + bT^2_2 + cT^4_4 + sT^\alpha_\alpha$ in SU(4), which may arise due to symmetry breaking or some other dynamical effects, we have obtained the magnetic moments and the transition moments of the ordinary and charmed baryons.

Keywords. Magnetic moments; charmed baryons; SU(4) symmetry.

1. Introduction

Magnetic moments of ordinary and charmed baryons have been calculated in both the quark models (Lichtenberg 1977; Singh 1977) and the symmetry schemes (Choudhary and Joshi 1976; Verma and Khanna 1977; Bohm 1978; Dattoli *et al* 1978). With the recent measurement of Ξ^0 magnetic moment (Bunce *et al* 1979) all the magnetic moments of octet baryons (except that of Σ^0) are now available, though the data on $\mu(\Xi^-)$ and $\mu(\Sigma^-)$ are not very accurate. However, the magnetic moments have not been well understood even in the SU(3). The electromagnetic (em) transitions, like the radiative decays of hadrons $V \to P\gamma$ and $\triangle^+ \to P\gamma$, etc are also not explained well theoretically (Bohm and Teese 1977; Edwards and Kamal 1976; Verma *et al* 1978. Since the conventional picture of em hamiltonian is unable to explain the electromagnetic data, it has recently been suggested by Bajaj *et al* (1979) that $aT^1_1 + bT^2_2$ type structure of em current, which can be obtained by including the medium strong breaking effects on the hadronic anomalous em interaction, explains most of the data on radiative decays of uncharmed hadrons. Magnetic moments of the hyperons (Bajaj *et al* 1979) are also understood better here than in the ordinary SU(3) symmetric case. In these considerations transition moment $\langle P | \mu | \triangle^+ \rangle$ is found to be higher than the conventional value by a factor of 1·4, which is very close to the 1·32 as demanded by the experimental situation (Nagels *et al* 1979). Further, in the case of weak radiative decays the modified em hamiltonian indicates a large non-zero asymmetry parameter for $\Sigma^+ \to p\gamma$ decay (Sharma *et al* 1979). In quark model calculations Kamal (1978) has also shown that the anomalous moment of quark transforms like arbitrary combination of λ_3 and λ_8 as a result of em vertex modifications. In an earlier paper, Verma and Khanna (1977) have derived the magnetic moments of charmed baryons in higher symmetry schemes assuming magnetic moment operator to transform like $T^1_1 + T^4_4 - \frac{1}{3}T^\alpha_\alpha$ component of $15 \oplus 1$ representations of SU(4). In this paper, we extend the considerations of Bajaj *et al* (1979) to charm sector by introducing SU(4) symmetry-breaking effects on em vertex. These symmetry-breaking effects are important since SU(4) is a very badly broken symmetry. In addition to SU(4) symmetric consideration, we consider the matrix elements $\langle B | T(H_{\text{em}}, H') | B \rangle$ contributing to magnetic moment, where H' is a symmetry-breaking hamiltonian. In § 2, we obtain electromagnetic hamiltonian modified as a result of SU(4) breaking. In § 3, we derive several relations among the magnetic moments of $J^P=1/2^+$ and $3/2^+$ charmed baryons and the transition moments $\langle 1/2^+ | \mu | 3/2^+ \rangle$.

2. Magnetic moment operator

Magnetic moment has two terms, the Dirac magnetic moment which is directly given by the charge of the particle and the Pauli anomalous magnetic moment. Only the anomalous part of the magnetic interaction is assumed to be modified due to the symmetry-breaking interaction. The Dirac part remains undisturbed in order to keep Gell-Mann-Nishijima relation intact.

The conventional em hamiltonian transforms like $T^1_1 + T^4_4 - \frac{1}{3} T^\alpha_\alpha$ component of $15 \oplus 1$ in SU(4) and the effects of symmetry-breaking interaction arising due to strong interaction dynamics are incorporated by adding the matrix elements $\langle B_f | T(H_{\text{em}} H') | B_i \rangle$, where H' is the symmetry-breaking hamiltonian transforming as T^3_3 and T^4_4 components of 15. In simple symmetry formalism of Muraskin and Glashow (1963), using the λ matrix notation, effective em hamiltonian will then transform as

$$H^{\text{em}}_{\text{eff}} \sim \left[\frac{1}{3}\lambda_0 + \frac{1}{\sqrt{2}}\lambda_3 + \frac{1}{\sqrt{6}}\lambda_8 - \frac{1}{\sqrt{3}}\lambda_{15}\right]$$

$$+ \left[\frac{1}{3}\lambda_0 + \frac{1}{\sqrt{2}}\lambda_3 + \frac{1}{\sqrt{6}}\lambda_8 - \frac{1}{\sqrt{3}}\lambda_{15};\ x\lambda_8 + y\lambda_{15}\right],$$

or

$$H^{\text{em}}_{\text{eff}} \sim \frac{\lambda_0}{3}\left[1 + \sqrt{\frac{3}{2}}x - \sqrt{3}\,y\right]$$

$$+ \frac{\lambda_3}{\sqrt{2}}\left[1 + \sqrt{\frac{2}{3}}x + \sqrt{\frac{1}{3}}y\right]$$

$$+ \frac{\lambda_8}{\sqrt{6}}\left[1 - \sqrt{\frac{2}{3}}x + \sqrt{\frac{1}{3}}y\right]$$

$$- \frac{\lambda_{15}}{\sqrt{3}}\left[1 - \frac{1}{\sqrt{6}}x - \sqrt{3}\,y\right]. \quad (1)$$

We ignore the + terms belonging to higher representations of SU(4). In tensor notation, the magnetic moment operator would thus transform like

$$H_{\text{eff}}^{\text{em}} \sim aT_1^1 + bT_2^2 + cT_4^4 + sT_a^a, \tag{2}$$

where T_1^1, T_2^2, T_4^4 are the components of the same 15 plet and T_a^a is SU(4) singlet piece. A similar transformation property has also been obtained by Buccalla *et al* (1978) using Melosh transformation.

3. Magnetic moments of baryons

In the following we derive various magnetic moments sum rules for $1/2^+$ and $3/2^+$ baryons and their transition moments. The notation used to describe the particle is given in Verma and Khanna (1977).

3.1. $J^P = 1/2^+$ *baryons*

For 20′ multiplet there are two types of $\bar{B}BT$ coupling (D and F type) i.e.

$$(\tfrac{1}{2}\, \bar{B}^a_{[m,\, n]}\, B_b^{[m,\, n]} \pm \bar{B}^m_{[b,\, n]}\, B_m^{[a,\, n]})\, T_a^b. \tag{3}$$

In all there are seven parameters a_D, a_F, b_D, b_F, c_D, c_F and s. We obtain the following relations relating different charm multiplets with uncharmed ones.

3.1a $B(8)$ *multiplet*

$$\mu(\Xi^0)-\mu(\Xi^-) = \mu(\Sigma^+)-\mu(\Sigma^-)+\mu(n)-\mu(p) \tag{4}$$

(−0·65±0·81) (−0·40±0·62)

$$2\mu(\Sigma^0) = \mu(\Sigma^+)+\mu(\Sigma^-)$$

(1·35±0·62)

$$= 4[\mu(\Sigma^+)-\mu(p)]+2[3\mu(\Lambda)-2\mu(\Xi^0)] \tag{5}$$

(1·33±1·4),

$$\langle \Lambda \,|\, \mu \,|\, \Sigma^0 \rangle = (1/2\sqrt{3})\ [2\mu(p)-2\mu(n)+\mu(\Sigma^-)-\mu(\Sigma^+)] \tag{6}$$

(1·5±0·19).

3.1b. $B(3^*)$ *multiplet*

$$6[\mu(\Xi'^+_1) - \mu(\Lambda'^+_1)] = 4[\mu(\Sigma^+)-\mu(p)]+\mu(\Xi^0)-\mu(n). \tag{7}$$

3.1c. $B(6)$ *multiplet*

$$2\mu(\Sigma_1^+) = \mu(\Sigma_1^{++})+\mu(\Sigma_1^0), \tag{8}$$

$$\begin{aligned}\mu(\Omega_1^0)-\mu(\Xi_1^0) &= \mu(\Xi_1^0)-\mu(\Sigma_1^0),\\ &= \mu(\Xi_1^+)-\mu(\Sigma_1^+),\\ &= \tfrac{1}{2}[\mu(\Xi^0)-\mu(n)].\end{aligned} \tag{9}$$

3.1d. $B(3)$ *multiplet*

$$\mu(\Omega_2^+)-\mu(\Xi_2^+) = \mu(\Sigma^+)-\mu(p), \tag{10}$$

$$\mu(\Xi_2^{++})-\mu(\Xi_2^+) = \mu(\Sigma_1^{++})-\mu(\Sigma_1^0)+\mu(n)-\mu(p). \tag{11}$$

3.1e. *Transition moments*

$$\begin{aligned}[\langle\Xi_1'^0|\mu|\Xi_1^0\rangle-\langle\Xi_1'^+|\mu|\Xi_1^+\rangle] &= \langle\Lambda_1'^+|\mu|\Sigma_1^+\rangle,\\ &= \langle\Lambda|\mu|\Sigma^0\rangle,\end{aligned} \tag{12}$$

$$\langle\Xi_1'^0|\mu|\Xi_1^0\rangle = \frac{\sqrt{3}}{8}[4\mu(P)-4\mu(n)+6\mu(\Lambda)-5\mu(\Sigma^+)-\mu(\Sigma^-)]. \tag{13}$$

These relations are valid for both the total and anomalous magnetic moments. Notice that the Coleman-Glashow relation (4) and its charmed analogue (11) are obtained. The relation $2\mu(\Sigma^0)=\mu(\Sigma^+)+\mu(\Sigma^-)$ and its charmed analogue (8) can be obtained at the SU(2) level. For ordinary baryons we get four relations which are experimentally satisfied.

If D/F ratio is assumed to be the same for all the components i.e.

$$a_D/a_F = b_D/b_F = c_D/c_F, \tag{14}$$

we further obtain

$$\begin{aligned}&\frac{4[\mu(p)-\mu(n)]-6\mu(\Lambda)+\mu(\Sigma^+)+5\mu(\Sigma^-)}{4[\mu(p)+\mu(n)]-6\mu(\Lambda)+\mu(\Sigma^+)-3\mu(\Sigma^-)}\\ &\qquad\left(\frac{17\cdot87\pm2\cdot3}{14\cdot03\pm1\cdot57}\right)\\ &\qquad= \frac{4[\mu(n)-\mu(p)]-6\mu(\Lambda)+5\mu(\Sigma^+)+\mu(\Sigma^-)}{4[\mu(p)+\mu(n)]-6\mu(\Lambda)-3\mu(\Sigma^+)+\mu(\Sigma^-)}\\ &\qquad\qquad\left(\frac{5\cdot49\pm1\cdot82}{2\cdot21\pm1\cdot32}\right)\\ &\qquad= \frac{6[\mu(\Lambda_1'^+)-\mu(\Sigma_1^+)]-6\mu(\Lambda)+3\mu(\Sigma^+)+3\mu(\Sigma^-)}{2[3\mu(\Lambda_1'^+)+\mu(\Sigma_1^+)]-6\mu(\Lambda)+\mu(\Sigma^+)-\mu(\Sigma^-)}\end{aligned} \tag{15}$$

Within the large experimental error this relation is satisfied.

It has already been argued (Bajaj and Khanna 1977) that the contribution of the SU(3) singlet term to em hamiltonian for uncharmed particles is small. Therefore the SU(4) em hamiltonian can be put into correspondence with the SU(3) em hamiltonian ($aT_1^1 + bT_2^2$) by ensuring that the effective singlet contribution arising from T_4^4 and SU(4) singlet piece in the SU(4) em hamiltonian does not contribute to the uncharmed particles. This condition relates the two reduced amplitudes $\langle B \| 15 \| B\rangle$ and $\langle B \| 1 \| B\rangle$.

Then, using (14), charmed baryonic magnetic moments can be expressed in terms of one parameter. The calculated magnetic moments of charmed baryons are displayed in table 1. By knowing the magnetic moment of one charmed baryon, the others can be estimated.

3.2. $J^P = 3/2^+$ *baryons*

Magnetic moments for $J^P = 3/2^+$ baryons obtained from the trace $(\bar{D}^{amn} D_{bmn}) T_a^b$ are expressed in terms of four parameters and obey the following relations.

Table 1. Anomalous magnetic moment of charmed baryons

Particles		Anomalous magnetic moment (calculated) (in d_F)
B (6) C = 1	Σ_1^{++}	1·45 − 1·37
	Σ_1^{+}	0·36 − 1·37
	Σ_1^{0}	− 0·73 − 1·37
	Ξ_1^{+}	0·59 − 1·37
	Ξ_1^{0}	− 0·49 − 1·37
	Ω_1^{0}	− 0·25 − 1·37
B (3*) C = 1	$\Xi_1'^{0}$	− 0·11 + 1·79
	$\Xi_1'^{+}$	− 0·74 + 1·79
	$\Lambda_1'^{+}$	− 0·60 + 1·79
B (3) C = 2	Ξ_2^{++}	− 1·41 + 2
	Ξ_2^{+}	0·08 + 2
	Ω_2^{+}	− 0·25 + 2
Transition	$\Lambda_1'^{+}\ \Sigma_1^{+}$	1·50
	$\Xi_1'^{+}\ \Xi_1^{+}$	− 1·11
	$\Xi_1'^{0}\ \Xi_1^{0}$	0·33

3.2a. *D*(10) *multiplet*

$$\mu(\Delta^+)-\mu(\Sigma^{*+}) = \mu(\Delta^0)-\mu(\Sigma^{*0})=\mu(\Delta^-)-\mu(\Sigma^{*-}),$$

$$= \mu(\Sigma^{*0})-\mu(\Xi^{*0})=\mu(\Sigma^{*-})-\mu(\Xi^{*-}),$$

$$= \mu(\Xi^{*-})-\mu(\Omega^-), \tag{16}$$

$$\mu(\Delta^{++})-\mu(\Delta^-) = 3[\mu(\Delta^+)-\mu(\Delta^0)]. \tag{17}$$

3.2b. *D*(6) *multiplet*

$$\mu(\Sigma_1^{*+}) - \mu(\Xi_1^{*+}) = \mu(\Sigma_1^{*0})-\mu(\Xi_1^{*0}),$$

$$= \mu(\Xi_1^{*0})-\mu(\Omega_1^{*0}),$$

$$= \mu(\Delta^+)-\mu(\Sigma^{*+}). \tag{18}$$

3.2c. *D*(3) *multiplet*

$$\mu(\Xi_2^{*++}) - \mu(\Xi_2^{*+}) = \mu(\Delta^+) - \mu(\Delta^0), \tag{19}$$

$$\mu(\Xi_2^{*+}) - \mu(\Omega_2^{*+}) = \mu(\Delta^+) - \mu(\Sigma^{*+}), \tag{20}$$

and

$$\mu(\Delta^{++}) - \mu(\Sigma_1^{*++}) = \mu(\Sigma_1^{*++}) - \mu(\Xi_2^{*++}),$$

$$=\mu(\Xi_2^{*++}) - \mu(\Omega_3^{*++}). \tag{21}$$

3.3. *Transition moments* $\langle 1/2^+ | \mu | 3/2^+ \rangle$

In this case the SU(4) singlet component of em current does not contribute since singlet representation is not present in the direct product $\overline{20'} \otimes 20$. Transition moments $\langle B | \mu | D \rangle$ are obtained from the trace

$$[\epsilon_{bcmn} D^{acd} B_d^{[m,n]}] T_a^b, \tag{22}$$

we get the following relations

$$\langle p | \mu | \Delta^+ \rangle = \langle n | \mu | \Delta^0 \rangle,$$

$$= \frac{2}{\sqrt{3}} [\langle \Lambda | \mu | \Sigma^{*0} \rangle] = -\frac{2}{\sqrt{3}} [\langle \Lambda_1'^{+} | \mu | \Sigma_1^{*+} \rangle],$$

$$= -[\langle \Sigma^+ | \mu | \Sigma^{*+} \rangle + \langle \Sigma^- | \mu | \Sigma^{*-} \rangle], \tag{23}$$

$$2\,[\langle \Sigma^0 | \mu | \Sigma^{*0} \rangle] = [\langle \Sigma^- | \mu | \Sigma^{*-} \rangle - \langle \Sigma^+ | \mu | \Sigma^{*+} \rangle], \tag{24}$$

$$\langle \Sigma^+ | \mu | \Sigma^{*+} \rangle = \langle \Xi^0 | \mu | \Xi^{*0} \rangle = \frac{2}{\sqrt{3}} [\langle \Xi'^{+}_1 | \mu | \Xi^{*+}_1 \rangle], \tag{25}$$

$$\langle \Sigma^- | \mu | \Sigma^{*-} \rangle = -\langle \Xi^- | \mu | \Xi^{*-} \rangle = -\frac{2}{\sqrt{3}} [\langle \Xi'^0_1 | \mu | \Xi^{*0}_1 \rangle], \tag{26}$$

$$\begin{aligned} \langle \Sigma^+_1 | \mu | \Sigma^{*+}_1 \rangle &= [\langle \Omega^+_2 | \mu | \Omega^{*+}_2 \rangle - \langle \Sigma^0 | \mu | \Sigma^{*0} \rangle], \\ &= [\langle \Xi^+_1 | \mu | \Xi^{*+}_1 \rangle + \langle \Xi^0_1 | \mu | \Xi^{*0}_1 \rangle], \\ &= \tfrac{1}{2} [\langle \Sigma^0_1 | \mu | \Sigma^{*0}_1 \rangle - \langle \Sigma^{++}_1 | \mu | \Sigma^{*++}_1 \rangle], \end{aligned} \tag{27}$$

$$\begin{aligned} \langle \Sigma^0_1 | \mu | \Sigma^{*0}_1 \rangle &= \langle \Xi^+_2 | \mu | \Xi^{*+}_2 \rangle, \\ &= [\langle \Omega^+_2 | \mu | \Omega^{*+}_2 \rangle - \langle \Sigma^- | \mu | \Sigma^{*-} \rangle], \end{aligned} \tag{28}$$

$$\langle \Omega^0_1 | \mu | \Omega^{*0}_1 \rangle = \langle \Omega^+_2 | \mu | \Omega^{*+}_2 \rangle \tag{29}$$

$$\begin{aligned} \langle \Sigma^{++}_1 | \mu | \Sigma^{*++}_1 \rangle &= -\langle \Xi^{++}_2 | \mu | \Xi^{*++}_2 \rangle, \\ &= [\langle \Omega^+_2 | \mu | \Omega^{*+}_2 \rangle + \langle \Sigma^+ | \mu | \Sigma^{*+} \rangle], \end{aligned} \tag{30}$$

$$\langle \Lambda'^+_1 | \mu | \Sigma^{*+}_1 \rangle = [\langle \Xi'^0_1 | \mu | \Xi^{*0}_1 \rangle - \langle \Xi'^+_1 | \mu | \Xi^{*+}_1 \rangle]. \tag{31}$$

An interesting feature of this model is that the transition moment $\langle p | \mu | \Delta^+ \rangle$ obtained here agrees well with experiment. Using SU(6) symmetry and the SU(3) symmetric em hamiltonian i.e. T^1_1 one obtains $\langle p | \mu | \Delta^+ \rangle \approx 2{\cdot}6$ which is about 1·6 times less than the experimental value (Pais 1966). More recent experiments (Nagels *et al* 1979) yield

$$\frac{\langle p | \mu | \Delta^+ \rangle_{\text{expt}}}{\langle p | \mu | \Delta^+ \rangle_{\text{SU(6)}}} = 1{\cdot}32. \tag{32}$$

In our considerations

$$\langle p | \mu | \Delta^+ \rangle = (1 + 2/3\ b/a) \langle p | \mu | \Delta^+ \rangle_{\text{conventional}} \tag{33}$$

$(1+2/3\ b/a) \approx 1{\cdot}4$ seems to be a correct multiplication factor to agree with experiment.

4. Conclusion

The electromagnetic phenomena involving strongly interacting particles, like the radiative decays of hadrons, magnetic moments and weak electromagnetic decays, etc are not well explained in the conventional model of electromagnetic current. However, most of the data on this phenomena can be understood if one assumes a $T^1_1 + bT^2_2$ type transformation property of anomalous em interaction. This type of transformation property can be obtained in many ways, for example, by including the symmetry-breaking effects (Bajaj *et al* 1979), using the Melosh transformation (Buccella *et al* 1978), by including anomalous moment arising due to quark interaction with psuedoscalar mesons (Kamal 1978) and assigning different anomalous moment to the flavoured quarks (Franklin 1969).

In this paper the magnetic moments of charmed as well as uncharmed baryons are calculated with the modified form of em current in SU(4). Several relations among the magnetic moments of $J^p = 1/2^+$ and $3/2^+$ baryons and transition moments $\langle 1/2^+ | \mu | 3/2^+ \rangle$ have been obtained which are valid for total as well as anomalous magnetic moment. These relations are naturally different from those obtained earlier in the symmetry scheme by Choudhary and Joshi (1976) and Verma and Khanna (1977). Here we have related the discrepancies present in those relations. Assuming universality for D/F ratio for different components, and null contribution of em singlet component to ordinary baryons, we are able to express the magnetic moment of charmed baryons in terms of one parameter (table 1). Though these moments are not yet observed, we present our relation in order to distinguish the SU(4) symmetric and SU(4) broken results.

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