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General relations among observables in neutral-current phenomena mediated by one or two Z bosons

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Abstract. If μ-e universality is assumed, there are 17 neutral-current parameters of current experimental interest, including the parity-violating nuclear force sector. We deduce the general relations among these parameters implied by gauge models. In single-Z boson models, there are 10 relations, while two-boson models lead to 4 relations. If μ-e universality is abandoned, the number of parameters increases to 31, while the number of relations becomes 21 in single-boson models and 12 in two-boson models. We derive all these relations.

Keywords. Weak neutral currents; gauge models; neutrino interactions; parity-violating nuclear force; μ-e universality.

1. Introduction

Although neutral current is a major prediction of gauge theory, the rich variety of neutral-current phenomena deserve a general analysis not tied down to any particular gauge model. In this paper we consider all the ' observables ' in various neutral-current experiments performed at energies much lower than the Z-boson masses and derive all the relations among these observable parameters implied by the class of single-Z-boson models as well as the class of two-Z-boson models.

Such an analysis was earlier performed by Hung and Sakurai (1977) (see also Wolfenstein 1974, 1975) and more recently by Dass and Ram Babu (1978). However, these authors have not considered all the neutral-current sectors and also they had restricted themselves to the class of single boson models. We provide a more complete list of relations.

The empirically determined neutral-current coupling constants in the neutrino-hadron sector have been found to be in remarkable accord with the prediction of the minimal gauge model of Weinberg (1967) and Salam (1968) which contains only one Z boson. This, coupled with the agreement of the measured polarisation asymmetry in deep inelastic e-d scattering (Prescott *et al* 1978) with Weinberg-Salam model has given rise to a general approval of this minimal model. It may be pointed out however, that neutral current phenomena cover five independent sectors involving as many as 17 independent physically observable coupling parameters and so far only a few of these parameters have been experimentally determined. It is too early to close down our choice to a single model.

In fact there are theoretical reasons for believing that the real unification of weak and electromagnetic interactions might require a group larger than the $SU(2) \otimes U(1)$ of Weinberg and Salam. Further, it has been conjectured that what is being observed at present may be only a substructure of a larger group which may lead to a grand unification of strong, weak and electromagnetic interactions (Pati and Salam 1974; Georgi and Glashow 1974; Fritzsch and Minkowski 1975). These unified models in general involve more than one neutral boson.

It is also worth pointing out that the left-right symmetric models based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ which have been intensively studied in the recent literature contain two Z bosons. Although one special class of these models, namely those with parity-conserving neutral currents have been ruled out by the observation of polarisation asymmetry in deep inelastic electrondeuteron scattering (Prescott *et al* 1978), this by no means rules out all left-right symmetric models. In view of the attractive possibility of attributing the observed parity violation in weak interactions to spontaneous symmetry breaking, the left-right symmetric models have a claim to our attention until they are ruled out by experiment. In fact, a recent analysis of left-right symmetric models (Bajaj and Rajasekaran 1979) shows that even a model with the ratio of the masses of the two Z bosons as small as 1·9 is not ruled out by experimental data on neutral currents.

It follows therefore that, especially at the present juncture, unbiased theoretical investigations of *all* the neutral-current phenomena under the hypotheses of one or more Z bosons are important at least at a phenomenological level. Our purpose is to provide a general framework for such an analysis.

A special feature of the present work is that we include the parity-violating nuclear force as one of the important neutral-current sectors. Although this sector is plagued with the usual problems of nonleptonic physics coupled with the uncertainties of nuclear physics, it is nevertheless necessary to make a systematic attack on it. It may be pointed out that the present state of our knowledge of the parity-violating nuclear force is characterised by glaring discrepancies between theory and experiment (see for instance the review of Henley 1976) and these should be resolved before we can claim to have achieved an understanding of neutral-current interactions. This will also require a sufficiently general framework and this provides an additional motivation for the present analysis.

Throughout the paper, we shall assume only vector and axial vector neutral currents. Although this is not yet established experimentally, it is consistent with all available data.

We divide our work into two parts. In the first part we assume μ-e universality in which case there are 17 neutral-current parameters of current experimental interest. We deduce the general relations among these parameters implied by gauge models. In single Z boson models, there are 10 relations, while two-boson models lead to 4 relations.

In the second part of the paper, we abandon μ-e universality which has not been tested in neutral-current phenomena so far. We then find that there are 31 neutral-current parameters which may be experimentally measured. Single-Z-boson or two-Z-bosons hypotheses lead to 21 or 12 relations among these parameters respectively, all of which are derived here.

It should be remarked that the single-boson relations follow quite simply whereas the derivation of the two-boson relations is quite non-trivial and requires rather lengthy though straightforward algebraic manipulations.

In §§ 2 and 3 respectively, we derive the relations with and without μ-e universality. § 2 also contains an account of the parity-violating nuclear force which is augmented by the discussion in Appendix A. § 4 is devoted to a brief summary and discussion. Appendix B sketches the derivation of the relations in the two-bosons case.

2. Relations among neutral-current parameters with the assumption of μ-e universality

In this section, we assume μ-e universality. There are then five classes of neutral-current experiments, and their effective interactions are given below. Wherever possible, we follow the notation of Hung and Sakurai (1977).

2(a) *Neutrino-induced hadron reactions*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\bar{\nu}\,\gamma_\lambda\,(1+\gamma_5)\,\nu\left[\frac{\alpha}{2}(\bar{u}\,\gamma_\lambda\,u-\bar{d}\,\gamma_\lambda\,d)+\frac{\beta}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\,u\right.$$

$$\left.-\bar{d}\,\gamma_\lambda\,\gamma_5\,d)+\frac{\gamma}{2}(\bar{u}\,\gamma_\lambda\,u+\bar{d}\,\gamma_\lambda\,d)+\frac{\delta}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\,u+\bar{d}\,\gamma_\lambda\,\gamma_5\,d)\right]. \quad (1)$$

(b) *$\nu_\mu e$ scattering and the neutral-current contribution in $\nu_e e$ scattering*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\bar{\nu}\,\gamma_\lambda\,(1+\gamma_5)\,\nu\,\bar{e}\,(g_V\,\gamma_\lambda+g_A\,\gamma_\lambda\,\gamma_5)\,e. \quad (2)$$

(c) *Neutral-current effects in $e^+ e^- \to \mu^+ \mu^-$*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\{h_{VV}\,(\bar{e}\,\gamma_\lambda\,e+\bar{\mu}\,\gamma_\lambda\,\mu)\,(\bar{e}\,\gamma_\lambda\,e+\bar{\mu}\,\gamma_\lambda\,\mu)$$

$$+\,2\,h_{VA}\,(\bar{e}\,\gamma_\lambda\,e+\bar{\mu}\,\gamma_\lambda\,\mu)\,(\bar{e}\,\gamma_\lambda\,\gamma_5\,e+\bar{\mu}\,\gamma_\lambda\,\gamma_5\,\mu)$$

$$+\,h_{AA}\,(\bar{e}\,\gamma_\lambda\,\gamma_5\,e+\bar{\mu}\,\gamma_\lambda\,\gamma_5\,\mu)\,(\bar{e}\,\gamma_\lambda\,\gamma_5\,e+\bar{\mu}\,\gamma_\lambda\,\gamma_5\,\mu)\}. \quad (3)$$

(d) *Parity violation in atoms and in e-N scattering*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\left[\bar{e}\,\gamma_\lambda\,\gamma_5\,e\left\{\frac{\tilde{\alpha}}{2}(\bar{u}\,\gamma_\lambda\,u-\bar{d}\,\gamma_\lambda\,d)+\frac{\tilde{\gamma}}{2}(\bar{u}\,\gamma_\lambda\,u+\bar{d}\,\gamma_\lambda\,d)\right\}\right.$$

$$\left.+\,\bar{e}\,\gamma_\lambda\,e\left\{\frac{\tilde{\beta}}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\,u-\bar{d}\,\gamma_\lambda\,\gamma_5\,d)+\frac{\tilde{\delta}}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\,u+\bar{d}\,\gamma_\lambda\,\gamma_5\,d)\right\}\right]. \quad (4)$$

(e) *Neutral-current contribution to parity-violating nuclear forces*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\Big[\frac{\xi}{4}(\bar{u}\,\gamma_\lambda\, u-\bar{d}\,\gamma_\lambda\, d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\, u-\bar{d}\,\gamma_\lambda\,\gamma_5\, d)+\frac{\eta}{4}(\bar{u}\,\gamma_\lambda\, u+\bar{d}\,\gamma_\lambda\, d)$$

$$(\bar{u}\,\gamma_\lambda\,\gamma_5\, u+\bar{d}\,\gamma_\lambda\,\gamma_5\, d)+\frac{\zeta}{4}(\bar{u}\,\gamma_\lambda\, u-\bar{d}\,\gamma_\lambda\, d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\, u+\bar{d}\,\gamma_\lambda\,\gamma_5\, d)$$

$$+\frac{\rho}{4}(\bar{u}\,\gamma_\lambda\, u+\bar{d}\,\gamma_\lambda\, d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\, u-\bar{d}\,\gamma_\lambda\,\gamma_5\, d)\Big]. \tag{5}$$

It is the primary aim of the neutral-current experiments to determine the 17 parameters α, $\beta \ldots$, ρ occurring in (1) – (5). Additional quark flavours and lepton flavours will bring in more parameters, but neutral-current phenomenology at the present stage cannot determine these additional parameters with any accuracy.

The neutral-current phenomena in the first four sectors (a)-(d) have been discussed in sufficient detail by Hung and Sakurai and in earlier work. Considering the fifth sector (e), namely parity-violating nuclear force, extensive literature exists on this subject also (Fischbach and Tadic 1973; Gari 1973 and 1975). But in most of the earlier work, the emphasis was mainly on testing the Cabibbo form of the charged-current interaction. Neutral-current weak interaction introduces new physics into the parity-violating nuclear force.

The four terms on the right side of (5) have the following isospin properties:

ξ $\quad \triangle I = 0$ and 2,

η $\quad \triangle I = 0$,

ζ and ρ $\quad \triangle I = 1$,

where we have used the coupling parameter to denote the corresponding neutral-current term. Of course, the charged-current also contributes to the parity-violating nuclear force and that should be subtracted from the observed effect to get the neutral-current contribution (see Appendix A). For the $\triangle I = 1$ part, the charged-current contribution is known to be suppressed by the factor $\sin^2 \theta\, c$ (Dashen *et al* 1964), θ_c being the Cabibbo angle. Further, since this term requires the excitation of the strangeness degree of freedom, it will be suppressed in nuclear states. This is in fact the reason why we have dropped the terms involving strange quarks in (5). So, the neutral-current contribution is in fact the dominant one for the $\triangle I = 1$ parity-violating nuclear force (Datta *et al* 1977; Desplanques and Hadjimichael 1976).

Let us suppose that, in general, observed neutral-current effects arise from the exchange of n number of Z bosons of very high mass (alternatively, we may suppose the neutral-current interaction to be due to a number of current $\times$ current terms added together). Then,

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\sum_{i=1}^{n} J_\lambda^{(i)}\, J_\lambda^{(i)}, \tag{6}$$

where
$$J^{(i)}_\lambda = C^{(i)}_0 \bar{\nu}_e \gamma_\lambda (1+\gamma_5) \nu_e + C^{(i)}_0 \bar{\nu}_\mu \gamma_\lambda (1+\gamma_5) \nu_\mu$$
$$+\bar{e}\gamma_\lambda (C^{(i)}_V + C^{(i)}_A \gamma_5) e + \bar{\mu} \gamma_\lambda (C^{(i)}_V + C^{(i)}_A \gamma_5)\mu$$
$$+ \frac{C^{(i)}_\alpha}{2} (\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d) + \frac{C^{(i)}_\beta}{2} (\bar{u}\gamma_\lambda \gamma_5 u - \bar{d}\gamma_\lambda \gamma_5 d)$$
$$+ \frac{C^{(i)}_\gamma}{2} (\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d) + \frac{C^{(i)}_\delta}{2} (\bar{u}\gamma_\lambda \gamma_5 u + \bar{d}\gamma_\lambda \gamma_5 d). \tag{7}$$

Thus, there are 7 current parameters $C^{(i)}_0$, $C^{(i)}_V$, $C^{(i)}_A$, $C^{(i)}_\alpha$, $C^{(i)}_\beta$, $C^{(i)}_\gamma$, $C^{(i)}_\delta$ for each current.

The 17 'observable' parameters defined in (1)—(5) are related to the 'current' parameters $C^{(i)}_0$, $C^{(i)}_V$....by the following equations (where we combine the n parameters $C^{(1)}_0$, $C^{(2)}_0$, ...$C^{(n)}_0$ into the convenient vector notation $\mathbf{C}_0$ and similarly for the others):

$$g_V = 2\mathbf{C}_0 \cdot \mathbf{C}_V, \tag{8a}$$
$$g_A = 2\mathbf{C}_0 \cdot \mathbf{C}_A, \tag{8b}$$

$$h_{VV} = \mathbf{C}_V \cdot \mathbf{C}_V, \tag{9a}$$
$$h_{AA} = \mathbf{C}_A \cdot \mathbf{C}_A, \tag{9b}$$
$$h_{VA} = \mathbf{C}_V \cdot \mathbf{C}_A, \tag{9c}$$

$$\alpha = 2\mathbf{C}_0 \cdot \mathbf{C}_\alpha, \tag{10a}$$
$$\beta = 2\mathbf{C}_0 \cdot \mathbf{C}_\beta, \tag{10b}$$
$$\gamma = 2\mathbf{C}_0 \cdot \mathbf{C}_\gamma, \tag{10c}$$
$$\delta = 2\mathbf{C}_0 \cdot \mathbf{C}_\delta, \tag{10d}$$

$$\tilde{\alpha} = 2\mathbf{C}_A \cdot \mathbf{C}_\alpha, \tag{11a}$$
$$\tilde{\beta} = 2\mathbf{C}_V \cdot \mathbf{C}_\beta, \tag{11b}$$
$$\tilde{\gamma} = 2\mathbf{C}_A \cdot \mathbf{C}_\gamma, \tag{11c}$$
$$\tilde{\delta} = 2\mathbf{C}_V \cdot \mathbf{C}_\delta, \tag{11d}$$

$$\xi = 2\mathbf{C}_\alpha \cdot \mathbf{C}_\beta, \tag{12a}$$
$$\eta = 2\mathbf{C}_\gamma \cdot \mathbf{C}_\delta, \tag{12b}$$
$$\zeta = 2\mathbf{C}_\alpha \cdot \mathbf{C}_\delta, \tag{12c}$$
$$\rho = 2\mathbf{C}_\gamma \cdot \mathbf{C}_\beta, \tag{12d}$$

From (9), we have the following inequalities:

$$\left.\begin{array}{l} h_{VV} \geqslant 0,\ h_{AA} \geqslant 0, \\ h_{VV}\, h_{AA} \geqslant h^2_{VA} \end{array}\right\}. \tag{13}$$

It is to be noted that these inequalities are general and are valid in models with any number of intermediate bosons.

2.1. *Single-boson models*

For single-boson-models, there are 7 current parameters C_0, C_V, C_A, C_α, C_β, C_γ, C_δ and hence their elimination gives us 10 relations among the observable neutral-current parameters. These are

$$\frac{h_{VV}}{h_{VA}} = \frac{h_{VA}}{h_{AA}}, \quad (14a); \qquad \frac{g_V}{g_A} = \frac{h_{VV}}{h_{VA}}, \quad (14b)$$

$$\tilde{\alpha} = 2\alpha \frac{h_{VA}}{g_V}, \quad (14c); \qquad \tilde{\beta} = 2\beta \frac{h_{VA}}{g_A}, \quad (14d)$$

$$\tilde{\gamma} = 2\gamma \frac{h_{VA}}{g_V}, \quad (14e); \qquad \tilde{\delta} = 2\delta \frac{h_{VA}}{g_A}, \quad (14f)$$

$$\xi = 2\alpha\beta \frac{h_{VA}}{g_V g_A}, \quad (14g); \qquad \eta = 2\gamma\delta \frac{h_{VA}}{g_V g_A}, \quad (14h)$$

$$\zeta = 2\alpha\delta \frac{h_{VA}}{g_V g_A}, \quad (14i); \qquad \rho = 2\gamma\beta \frac{h_{VA}}{g_V g_A}. \quad (14j)$$

The first six relations are essentially those obtained by Hung and Sakurai; however, we have replaced a quadratic relation of these authors by a linear one (our (14b)), since the other possibility $g_V/g_A = h_{AA}/h_{VA}$ implied by their quadratic relation is inconsistent with 'factorisability'. The four equations (14g)-(14j) are our new relations for the parity-violating nuclear force. We now turn to the class of two boson models, which is in fact the main concern of our work.

2.2. *Two-boson models*

These in general involve 14 current parameters, 2 for each of the 7 current vectors. However, the 'observables' in (8) to (12) depend only on 'scalar products of the vectors'. The total number of independent scalar variables for a system of 7 two-dimensional vectors is only 13 (7 magnitudes of vectors and 6 relative angles). Hence the 17 observables satisfy four relations and these are found to be*

$$\xi = \pm [- C \tilde{\alpha} \tilde{\beta} + D \{\tilde{\alpha} \beta g_V + \alpha \tilde{\beta} g_A - 2 \alpha \beta h_{VA}\}], \qquad (15a)$$

$$\eta = \pm [- C \tilde{\gamma} \tilde{\delta} + D \{\tilde{\gamma} \delta g_V + \gamma \tilde{\delta} g_A - 2 \gamma \delta h_{VA}\}], \qquad (15b)$$

$$\zeta = \pm [- C \tilde{\alpha} \tilde{\delta} + D \{\tilde{\alpha} \delta g_V + \alpha \tilde{\delta} g_A - 2 \alpha \delta h_{VA}\}], \qquad (15c)$$

$$\rho = \pm [- C \tilde{\gamma} \tilde{\beta} + D \{\tilde{\gamma} \beta g_V + \gamma \tilde{\beta} g_A - 2 \gamma \beta h_{VA}\}], \qquad (15d)$$

*These relations were earlier reported but not published (Parida and Rajasekaran 1978).

where $$C = \frac{g_A^2 h_{VV} + g_V^2 h_{AA} - 2h_{VA} g_A g_V}{2(g_V h_{AA} - g_A h_{VA})(g_A h_{VV} - g_V h_{VA})},$$

$$D = \frac{h_{VV} h_{AA} - h_{VA}^2}{(g_V h_{AA} - g_A h_{VA})(g_A h_{VV} - g_V h_{VA})}.$$

Although the derivation of these relations involves lengthy algebra, it is straightforward and is given in Appendix B.

This set of four relations (15a) to (15d) are then the most general consequences of the hypothesis of two mediating Z bosons, with μ-e universality. Note that the relations now connect the parameters of all the five types of neutral-current experiments. These relations which are generalisations of (14g) to (14j) in fact determine (apart from the sign ambiguity) all the parameters of the parity-violating nuclear force in terms of the parameters measured in the other four neutral-current sectors.

2.3. *Models with three or more Z bosons*

Since the number of 'current parameters' exceeds 17 for $n \geqslant 3$, there are in general no equations relating the 'observable' parameters for gauge models with more than two intermediate bosons. Thus, all the neutral-current 'observables' will become free parameters (except for the inequalities in (13), if eventually we have to turn to a gauge model with more than two Z bosons. So, for $n \geqslant 3$, the discrepancies in the parity-violating nuclear force for example can be resolved in a trivial way.

3. Relations among neutral-current parameters without μ-e universality

In this section, we augment the generality of our analysis even further by abandoning the assumption of μ-e universality. As already mentioned, μ-e universality is so far untested in neutral-current phenomenology. (A partial test will soon be possible, with the accumulation of more scattering events in the $\nu_e\, e$ and $\nu_\mu\, e$ sectors.) We should also remark that in the unified gauge models, there does not seem to be any compelling reason for μ-e universality. With the discovery of newer leptons such as τ, μ-e universality becomes a questionable hypothesis.

If μ-e universality is not assumed, there are nine classes of neutral-current experiments on the whole and the corresponding effective Lagrangians are listed below.

(a) *ν_e-e scattering via neutral current*

$$\mathscr{L} = -\frac{G}{\sqrt{2}} \bar{\nu}_e \gamma_\lambda (1+\gamma_5)\nu_e\, \bar{e}\gamma_\lambda (g_V + g_A \gamma_5)e \tag{16}$$

(b) *ν_μ-e scattering*

$$\mathscr{L} = -\frac{G}{\sqrt{2}} \bar{\nu}_\mu \gamma_\lambda (1+\gamma_5)\nu_\mu\, \bar{e}\, \gamma_\lambda (g'_V + g'_A \gamma_5)e. \tag{17}$$

(c) *Neutral-current effects in Møller and Bhabha scattering*

$e^{\pm}\, e^{\pm} \to e^{\pm}\, e^{\pm}$ and $e^{+}\, e^{-} \to e^{+}\, e^{-}$

$$\mathscr{L} = -\frac{G}{\sqrt{2}}(k_{VV}\,\bar{e}\,\gamma_\lambda\, e\, \bar{e}\,\gamma_\lambda\, e + k_{AA}\,\bar{e}\,\gamma_\lambda\,\gamma_5\, e\,\bar{e}\,\gamma_\lambda\,\gamma_5\, e + 2\,k_{VA}\,\bar{e}\,\gamma_\lambda\, e\,\bar{e}\,\gamma_\lambda\,\gamma_5\, e). \tag{18}$$

(d) *Neutral-current effects in* $e^{+}\, e^{-} \to \mu^{+}\, \mu^{-}$

$$\mathscr{L} = -\frac{G}{\sqrt{2}}(k_{VV}\,\bar{e}\,\gamma_\lambda\, e\,\bar{\mu}\,\gamma_\lambda\,\mu + k'_{AA}\,\bar{e}\,\gamma_\lambda\,\gamma_5\, e\,\bar{\mu}\,\gamma_\lambda\,\gamma_5\,\mu + k'_{VA}\,\bar{e}\,\gamma_\lambda\, e\,\mu\,\gamma_\lambda\,\gamma_5\,\mu + k'_{AV}\,\bar{e}\,\gamma_\lambda\,\gamma_5\, e\,\bar{\mu}\,\gamma_\lambda\,\mu). \tag{19}$$

(e) ν_e*-induced hadron reactions*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\bar{\nu}_e\,\gamma_\lambda\,(1+\gamma_5)\,\nu_e\Big[\frac{\alpha}{2}(\bar{u}\,\gamma_\lambda\, u - d\,\gamma_\lambda\, d) + \frac{\beta}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\, u - \bar{d}\,\gamma_\lambda\,\gamma_5\, d) + \frac{\gamma}{2}(\bar{u}\,\gamma_\lambda\, u + \bar{d}\,\gamma_\lambda\, d) + \frac{\delta}{2}(\bar{u}\,\gamma_\lambda\,\gamma_5\, u + \bar{d}\,\gamma_\lambda\,\gamma_5\, d)\Big]. \tag{20}$$

(f) ν_μ*-induced hadron reactions*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\nu_\mu\,\gamma_\lambda\,(1+\gamma_5)\,\nu_\mu\Big[\frac{\alpha'}{2}\Big(\bar{u}\gamma_\lambda\, u - \bar{d}\gamma_\lambda\, d\Big) + \frac{\beta'}{2}\Big(\bar{u}\gamma_\lambda\,\gamma_5\, u - \bar{d}\,\gamma_\lambda\,\gamma_5\, d\Big) + \frac{\gamma'}{2}\Big(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d\Big) + \frac{\delta'}{2}\Big(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d\Big)\Big]. \tag{21}$$

(g) *Parity violation in atoms and in e-N scattering*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\Big[\bar{e}\gamma_\lambda\gamma_5 e\left\{\frac{\tilde{\alpha}}{2}(\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d) + \frac{\tilde{\gamma}}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d)\right\} + \bar{e}\gamma_\lambda e\left\{\frac{\tilde{\beta}}{2}(\bar{u}\gamma_\lambda\gamma_5 u - \bar{d}\gamma_\lambda\gamma_5 d) + \frac{\tilde{\delta}}{2}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d)\right\}\Big]. \tag{22}$$

(h) *Parity violation in muonic atoms and in* μ*-N scattering*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\Big[\mu\gamma_\lambda\gamma_5\mu\left\{\frac{\tilde{\alpha}'}{2}(\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d) + \frac{\tilde{\gamma}'}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d)\right\} + \bar{\mu}\gamma_\lambda\mu\left\{\frac{\tilde{\beta}'}{2}(\bar{u}\gamma_\lambda\gamma_5 u - \bar{d}\gamma_\lambda\gamma_5 d) + \frac{\tilde{\delta}'}{2}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d)\right\}\Big]. \tag{23}$$

(i) *Neutral-current contribution to parity-violating nuclear forces*

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\Big[\frac{\xi}{4}(\bar{u}\gamma_\lambda u-\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda\gamma_5 u-\bar{d}\gamma_\lambda\gamma_5 d)$$

$$+\frac{\eta}{4}(\bar{u}\gamma_\lambda u+\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda\gamma_5 u+\bar{d}\gamma_\lambda\gamma_5 d)$$

$$+\frac{\zeta}{4}(\bar{u}\gamma_\lambda u-\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda\gamma_5 u+\bar{d}\gamma_\lambda\gamma_5 d)$$

$$+\frac{\rho}{4}(\bar{u}\gamma_\lambda u+\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda\gamma_5 u-\bar{d}\gamma_\lambda\gamma_5 d)\Big]. \tag{24}$$

The total number of neutral-current parameters occurring in (16)-(24) are 31 and all these parameters are to be determined by experiment. The experimental determination of some of these parameters will be hard, but not impossible. For instance, observation of neutral current effects in Møller and Bhabha scattering processes is more difficult than in $e^+e^-\to\mu^+\mu^-$ because of the dominating crossed-channel photon exchange contributions in the former processes. However, because of the fundamental importance of these parameters, these difficulties will be surmounted, hopefully in the near future. On the other hand, there has already been an experimental measurement of the circular polarisation of x-rays from muonic atoms (Abela *et al* 1977). Such measurements will provide information on the parity violation in muonic atoms.

With μ-e universality, the current parameters were defined by 7 different vectors. Without μ-e universality, the current parameters are now given by 10 different vectors $\mathbf{C}_0$, $\mathbf{C}_0'$, $\mathbf{C}_V$, $\mathbf{C}_V'$, $\mathbf{C}_A$, $\mathbf{C}_A'$, $\mathbf{C}_\alpha$, $\mathbf{C}_\beta$, $\mathbf{C}_\gamma$, $\mathbf{C}_\delta$ defined through the following current vector (compare (7)):

$$\mathbf{J}_\lambda = \mathbf{C}_0\,\bar{\nu}_e\,\gamma_\lambda(1+\gamma_5)\,\nu_e + \mathbf{C}_0'\,\bar{\nu}_\mu\,\gamma_\lambda(1+\gamma_5)\,\nu_\mu$$

$$+\,\bar{e}\gamma_\lambda(\mathbf{C}_V+\mathbf{C}_A\gamma_5)e + \bar{\mu}\gamma_\lambda(\mathbf{C}_V'+\mathbf{C}_A'\,\gamma_5)\mu$$

$$+\,\frac{\mathbf{C}_\alpha}{2}(\bar{u}\gamma_\lambda u-\bar{d}\gamma_\lambda d)+\frac{\mathbf{C}_\beta}{2}(\bar{u}\gamma_\lambda\gamma_5 u-\bar{d}\gamma_\lambda\gamma_5 d)$$

$$+\,\frac{\mathbf{C}_\gamma}{2}(\bar{u}\gamma_\lambda u+\bar{d}\gamma_\lambda d)+\frac{\mathbf{C}_\delta}{2}(\bar{u}\gamma_\lambda\gamma_5 u+\bar{d}\gamma_\lambda\gamma_5 d), \tag{25a}$$

and the neutral-current interaction Lagrangian is

$$\mathscr{L} = -\frac{G}{\sqrt{2}}\mathbf{J}_\lambda\cdot\mathbf{J}_\lambda. \tag{25b}$$

The 31 observable parameters for the 9 neutral-current sectors can now be given as scalar products of the current parameter vectors, as follows:—

$\nu_e e \to \nu_e e$

$$g_V = 2\,\mathbf{C}_0 \cdot \mathbf{C}_V, \tag{26a}$$

$$g_A = 2\,\mathbf{C}_0 \cdot \mathbf{C}_A, \tag{26b}$$

$\nu_\mu e \to \nu_\mu e$

$$g'_V = 2\,\mathbf{C}_0' \cdot \mathbf{C}_V, \tag{27a}$$

$$g'_A = 2\,\mathbf{C}_0' \cdot \mathbf{C}_A. \tag{27b}$$

$e^+ e^- \to e^+ e^-$

$$k_{VV} = \mathbf{C}_V \cdot \mathbf{C}_V, \tag{28a}$$

$$k_{AA} = \mathbf{C}_A \cdot \mathbf{C}_A, \tag{28b}$$

$$k_{VA} = \mathbf{C}_V \cdot \mathbf{C}_A, \tag{28c}$$

$e^+ e^- \to \mu^+ \mu^-$

$$k'_{VV} = 2\,\mathbf{C}_V \cdot \mathbf{C}'_V, \tag{29a}$$

$$k'_{AA} = 2\,\mathbf{C}_A \cdot \mathbf{C}'_A, \tag{29b}$$

$$k'_{VA} = 2\,\mathbf{C}_V \cdot \mathbf{C}'_A, \tag{29c}$$

$$k'_{AV} = 2\,\mathbf{C}_A \cdot \mathbf{C}'_V. \tag{29d}$$

$\nu_e h \to \nu_e h$

$$\alpha = 2\,\mathbf{C}_0 \cdot \mathbf{C}_\alpha, \tag{30a}$$

$$\beta = 2\,\mathbf{C}_0 \cdot \mathbf{C}_\beta, \tag{30b}$$

$$\gamma = 2\,\mathbf{C}_0 \cdot \mathbf{C}_\gamma, \tag{30c}$$

$$\delta = 2\,\mathbf{C}_0 \cdot \mathbf{C}_\delta. \tag{30d}$$

$\nu_\mu h \to \nu_\mu h$

$$\alpha' = 2\,\mathbf{C}'_0 \cdot \mathbf{C}_\alpha, \tag{31a}$$

$$\beta' = 2\,\mathbf{C}'_0 \cdot \mathbf{C}_\beta, \tag{31b}$$

$$\gamma' = 2\,\mathbf{C}'_0 \cdot \mathbf{C}_\gamma, \tag{31c}$$

$$\delta' = 2\,\mathbf{C}'_0 \cdot \mathbf{C}_\delta. \tag{31d}$$

$eh \to eh$

$$\tilde{\alpha} = 2\,\mathbf{C}_A \cdot \mathbf{C}_\alpha, \tag{32a}$$

$$\tilde{\beta} = 2\,\mathbf{C}_V \cdot \mathbf{C}_\beta, \tag{32b}$$

$$\tilde{\gamma} = 2\,\mathbf{C}_A \cdot \mathbf{C}_\gamma, \tag{32c}$$

$$\tilde{\delta} = 2\,\mathbf{C}_V \cdot \mathbf{C}_\delta. \tag{32d}$$

$\mu h \to \mu h$

$$\tilde{\alpha}' = 2\,\mathbf{C}'_A \cdot \mathbf{C}_\alpha, \tag{33a}$$

$$\tilde{\beta}' = 2\,\mathbf{C}'_V \cdot \mathbf{C}_\beta, \tag{33b}$$

$$\tilde{\gamma}' = 2\,\mathbf{C}'_A \cdot \mathbf{C}_\gamma, \tag{33c}$$

$$\tilde{\delta}' = 2\,\mathbf{C}'_V \cdot \mathbf{C}_\delta. \tag{33d}$$

$NN \to NN$

$$\xi = 2\,\mathbf{C}_\alpha \cdot \mathbf{C}_\beta, \tag{34a}$$

$$\eta = 2\,\mathbf{C}_\gamma \cdot \mathbf{C}_\delta, \tag{34b}$$

$$\zeta = 2\,\mathbf{C}_\alpha \cdot \mathbf{C}_\delta, \tag{34c}$$

$$\rho = 2\,\mathbf{C}_\gamma \cdot \mathbf{C}_\beta. \tag{34d}$$

We again have the Schwartz inequalities, but now valid for the parameters of the electron-electron sector only:

$$\begin{aligned} &k_{VV} \geqslant 0, \quad k_{AA} \geqslant 0, \\ &k_{VV}\,k_{AA} \geqslant k_{VA}^2. \end{aligned} \tag{35}$$

3.1. *Single boson models*

With single-Z-boson hypothesis there are 10 current parameters C_0, C'_0, C_V, C'_V) C_A, C'_A, C_α, C_β, C_γ and C_δ. Eliminating these from the 31 equations given in (26a) to (34d) yields 21 factorisation relations given below:

$$k_{VV}/k_{VA} = k_{VA}/k_{AA}, \quad \text{(36a)} \qquad k_{VV}/k_{VA} = g_{VV}/g_{AA}, \quad \text{(36b)}$$

$$\tilde{\alpha} = 2\alpha \frac{k_{VA}}{g_V}, \quad \text{(36c)} \qquad \tilde{\beta} = 2\beta \frac{k_{VA}}{g_A}, \quad \text{(36d)}$$

$$\tilde{\gamma} = 2\gamma \frac{k_{VA}}{g_V}, \quad \text{(36e)} \qquad \tilde{\delta} = 2\delta \frac{k_{VA}}{g_A}, \quad \text{(36f)}$$

$$\xi = 2\alpha\beta \frac{k_{VA}}{g_V g_A}, \quad \text{(36g)} \qquad \eta = 2\gamma\delta \frac{k_{VA}}{g_V g_A}, \quad \text{(36h)}$$

$$\zeta = 2\alpha\delta \frac{k_{VA}}{g_V g_A}, \quad \text{(36i)} \qquad \rho = 2\gamma\beta \frac{k_{VA}}{g_V g_A}, \quad \text{(36j)}$$

$$k'_{VV}/k'_{VA} = k'_{AV}/k'_{AA}, \quad \text{(37a)} \qquad k'_{VV}/k'_{AV} = g'_V/g'_A, \quad \text{(37b)}$$

$$k'_{VV}/k'_{AV} = k_{VV}/k_{VA}, \quad \text{(37c)}$$

$$\tilde{\alpha}' = \alpha' \frac{k'_{VA}}{g'_V}, \quad \text{(37d)} \qquad \alpha/g_V = \alpha'/g'_V, \quad \text{(37h)}$$

$$\tilde{\beta}' = \beta' \frac{k'_{AV}}{g'_A}, \quad \text{(37e)} \qquad \beta/g_V = \beta'/g_V, \quad \text{(37i)}$$

$$\tilde{\gamma}' = \gamma' \frac{k'_{VA}}{g'_V}, \quad \text{(37f)} \qquad \gamma/g_V = \gamma'/g'_V, \quad \text{(37j)}$$

$$\tilde{\delta}' = \delta' \frac{k'_{AV}}{g'_A}, \quad \text{(37g)} \qquad \delta/g_V = \delta'/g'_V. \quad \text{(37k)}$$

Recently, Dass and Ram Babu (1978) have proposed tests of single-Z-boson hypothesis without invoking μ-e universality, but they have restricted themselves to purely leptonic processes. The 21 relations we have given above, provide a more complete list of all the factorisation relations.

When μ-e universality is imposed, we have

$$g'_A = g_A, \quad g'_V = g_V,$$

$$k'_{AA} = 2\,k_{AA} = 2\,h_{AA}, \quad k'_{VV} = 2\,k_{VV} = 2\,h_{VV},$$

$$k'_{VA} = k'_{AV} = 2\,k_{VA} = 2\,h_{VA},$$

$$\alpha' = \alpha, \quad \beta' = \beta, \quad \gamma' = \gamma, \quad \delta' = \delta,$$

$$\tilde{\alpha}' = \tilde{\alpha}, \quad \tilde{\beta}' = \tilde{\beta}, \quad \tilde{\gamma}' = \tilde{\gamma}, \quad \tilde{\delta}' = \tilde{\delta}, \tag{38}$$

and the 21 relations given in (36a) to (37k) reduce to the 10 relations (14a) to (14j) given in the last section.

3.2. *Two-boson models*

If the neutral-currents are mediated by two Z bosons, there are 20 current parameters for the 10 two-dimensional vectors. But, in the 31 equations given by (26a) to (34d) the vectors occur as scalar products which remain invariant under rotation in two-dimensional space. Thus, to describe the 31 observable parameters by 10 vectors we require only 19 independent scalar parameters (10 magnitudes of vectors and 9 relative angles). Elimination of these 19 parameters from 31 equations then yields 12 relations among the 31 observable parameters. In Appendix B, a brief sketch of the derivation of these relations is supplied. Here we quote the results:

$$\xi = \pm [- A\tilde{\alpha}\tilde{\beta} + B(\tilde{\alpha}\beta g_V + \alpha\tilde{\beta}g_A - 2\alpha\beta k_{VA})], \tag{40a}$$

$$\eta = \pm [- A\tilde{\gamma}\tilde{\delta} + B(\tilde{\gamma}\delta g_V + \gamma\tilde{\delta}g_A - 2\gamma\delta k_{VA})], \tag{40b}$$

$$\zeta = \pm [- A\tilde{\alpha}\tilde{\delta} + B(\tilde{\alpha}\delta g_V + \alpha\tilde{\delta}g_A - 2\alpha\delta k_{VA})], \tag{40c}$$

$$\rho = \pm [- A\tilde{\gamma}\tilde{\beta} + B(\tilde{\gamma}\beta g_V + \gamma\tilde{\beta}g_A - 2\gamma\beta k_{VA})], \tag{40d}$$

where
$$A = \frac{g_A^2 k_{VV} + g_V^2 k_{AA} - 2k_{VA} g_A g_V}{2(g_V k_{VA} - g_A k_{VV})(g_A k_{VA} - g_V k_{AA})},$$

$$B = \frac{k_{VV}k_{AA} - k_{VA}^2}{(g_V k_{VA} - g_A k_{VV})(g_A k_{VA} - g_V k_{AA})},$$

$$\xi = \pm [- A'\tilde{\alpha}\tilde{\beta} + B'(\tilde{\alpha}\beta' g'_V + \alpha'\tilde{\beta} g'_A - 2\alpha'\beta' k_{VA})], \tag{41a}$$

$$\eta = \pm [- A'\tilde{\gamma}\tilde{\delta} + B'(\tilde{\gamma}\delta' g'_V + \gamma'\tilde{\delta} g'_A - 2\gamma'\delta' k_{VA})], \tag{41b}$$

$$\zeta = \pm [- A'\tilde{\alpha}\tilde{\delta} + B'(\tilde{\alpha}\delta' g'_V + \alpha'\tilde{\delta} g'_A - 2\alpha'\delta' k_{VA})], \tag{41c}$$

$$\rho = \pm [- A'\tilde{\gamma}\tilde{\beta} + B'(\tilde{\gamma}\beta' g'_V + \gamma'\tilde{\beta} g'_A - 2\gamma'\beta' k_{VA})] \tag{41d}$$

where
$$A' = \frac{g'^2_A k_{VV} + g'^2_V k_{AA} - 2k_{VA} g'_A g'_V}{2(g'_V k_{VA} - g'_A k_{VV})(g'_A k_{VA} - g'_V k_{AA})},$$

$$B' = \frac{k_{VV}k_{AA} - k^2_{VA}}{(g'_V k_{VA} - g'_A k_{VV})(g'_A k_{VA} - g'_V k_{AA})},$$

$$\xi = \pm (a\tilde{\alpha}'\tilde{\beta}' + b\tilde{\alpha}'\beta' + c\alpha'\tilde{\beta}' + d\alpha'\beta'), \quad (42a)$$

$$\eta = \pm (a\tilde{\gamma}'\tilde{\delta}' + b\tilde{\gamma}'\delta' + c\gamma'\tilde{\delta}' + d\gamma'\delta'), \quad (42b)$$

$$\zeta = \pm (a\tilde{\alpha}'\tilde{\delta}' + b\tilde{\alpha}'\delta' + c\alpha'\tilde{\delta}' + d\alpha'\delta'), \quad (42c)$$

$$\rho = \pm (a\tilde{\gamma}'\tilde{\beta}' + b\tilde{\gamma}'\beta' + c\gamma'\tilde{\beta}' + d\gamma'\beta'), \quad (42d)$$

where
$$a = \frac{-2(g'^2_A k_{VV} + g'^2_V k_{AA} - 2k_{VA} g'_V g'_A)}{(k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)},$$

$$b = \frac{2\{g'_V (k'_{VV} k_{AA} - k'_{AV} k_{VA}) - g'_A (k'_{VV} k_{VA} - k'_{AV} k_{VV})\}}{(k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)},$$

$$c = \frac{2\{g'_V (k'_{VA} k_{AA} - k'_{AA} k_{VA}) - g'_A (k'_{VA} k_{VA} - k'_{AA} k_{VV})\}}{(k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)},$$

$$d = \frac{2\{(k'_{AA} k'_{VV} + k'_{AV} k'_{AV}) k_{VA} - k'_{AA} k'_{AV} k_{VV} - k'_{VV} k'_{VA} k_{AA}\}}{(k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)}.$$

The 12 relations contained in (40a) to (42d) represent the main results of the present work and they are the general consequences of the two-boson hypothesis without μ-e universality.

We have been able to express these 12 relations conveniently as three groups of four relations each and each group of four relations is analogous to the group of four relations (15a) to (15d) derived on the basis of μ-e universality. If μ-e universality is valid, then, using (38), it is easy to check that the three groups are identical to each other and to the group (15a) to (15d).

Further, it is useful to note that equating the right-hand-sides of the corresponding relations from each group, we get relationships *not* involving the parameters of the parity-violating nuclear force. Thus, we have

r.h.s. of (40a) = r.h.s. of (41a) = r.h.s. of (42a)

r.h.s. of (40b) = r.h.s. of (41b) = r.h.s. of (42b)

r.h.s. of (40c) = r.h.s. of (41c) = r.h.s. of (42c)

r.h.s. of (40d) = r.h.s. of (41d) = r.h.s. of (42d)

These 8 relations involve only the 27 observable parameters of the leptonic and semi-leptonic sectors and do not contain any of the nonleptonic parameters ξ, η, ζ, ρ whose experimental determination is beset with problems. Hence these may play a special role in further phenomenological work on neutral currents, if μ–e universality is violated.

3.3. *Models with three or more bosons*

Consider three-boson models. Since each current-parameter-vector has three components, there are now 30 current parameters. Out of these, there are only 28 scalar parameters since the orientation angle of one of these vectors can be chosen arbitrarily. Eliminating these 28 parameters from the 31 equations (26a) to (34d), we therefore have 3 relations among the observables, which may be derived in the future.

For $n \geqslant 4$, the number of current parameters exceed 31 and so there are in general no relations among observable parameters for gauge models with more than three gauge bosons.

4. Summary and discussion

We have deduced all the model-independent relations among the diverse neutral-current parameters that follow from the mediation of a single Z boson or two Z bosons, with or without μ-e universality. The number of relations that exist in the various cases is summed up in table 1.

Hopefully, this systematic presentation of all the general relations that exist among the neutral current parameters will help to enlarge the scope of neutral-current phenomenology and sharpen its confrontation with particular gauge models.

In particular, a general analysis of parity-violating nuclear forces on the basis of the single boson relations (14g) to (14j) or two-boson relations (15a) to (15d) can be made. It is important to see whether a consistent picture of all neutral-current phenomena emerges in this way. Application of these relations is reserved for a future publication.

If μ–e universality is violated, we then have a much larger number of observable parameters in which case there is a much richer set of relationships that would allow us to probe deeper into the nature of the neutral current interaction.

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Table 1. Enumeration of the number of relations among neutral-current parameters.

	Number of observables	Number of relations			
		$1Z$	$2Z$	$3Z$	$4Z$
With μ-e universality	17	10	4	0	0
Without μ-e universality	31	21	12	3	0

providing facilities for summer visit. The other (GR) thanks the University Grants Commission for financial support to the research project.

Appendix A

Charged current contribution to the parity-violating nuclear force

The charged-current interaction is (ignoring the Cabibbo rotation which is taken into account later)

$$\mathscr{L}_{cc} = -\frac{G}{\sqrt{2}}\bar{u}\,\gamma_\lambda\,(1+\gamma_5)\,d\,\bar{d}\,\gamma_\lambda\,(1+\gamma_5)\,u \tag{A.1}$$

By Fierz transformation this can be rewritten in the charge-retention form:*

$$\mathscr{L}_{cc} = -\frac{G}{\sqrt{2}}\bar{u}\,\gamma_\lambda\,(1+\gamma_5)\,u\,\bar{d}\,\gamma_\lambda\,(1+\gamma_5)\,d. \tag{A.2}$$

The parity-violating part of this is

$$\begin{aligned}\mathscr{L}^{pv}_{cc} &= -\frac{G}{\sqrt{2}}\{\bar{u}\,\gamma_\lambda\,u\,\bar{d}\,\gamma_\lambda\,\gamma_5\,d + \bar{u}\,\gamma_\lambda\,\gamma_5\,u\,\bar{d}\,\gamma_\lambda\,d\},\\ &= -\frac{G}{\sqrt{2}}\left\{-\frac{1}{2}(\bar{u}\,\gamma_\lambda\,u - \bar{d}\,\gamma_\lambda\,d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\,u - \bar{d}\,\gamma_\lambda\,\gamma_5\,d)\right.\\ &\quad\left.+\frac{1}{2}(\bar{u}\,\gamma_\lambda\,u + \bar{d}\,\gamma_\lambda\,d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\,u + \bar{d}\,\gamma_\lambda\,\gamma_5\,d)\right\}.\end{aligned} \tag{A.3}$$

By combining with the neutral-current contribution given in equation (5) of the text, we have the total parity-violating nuclear interaction:

$$\begin{aligned}\mathscr{L}^{pv}_{\text{total}} &= -\frac{G}{\sqrt{2}}\{\tfrac{1}{4}\,(\xi-2)\,(\bar{u}\,\gamma_\lambda\,u - \bar{d}\,\gamma_\lambda\,d)\,(\bar{u}\,\gamma_\lambda\,\gamma_5\,u - \bar{d}\,\gamma_\lambda\,\gamma_5\,d)\\ &\quad+\tfrac{1}{4}\,(\eta+2)\,(\bar{u}\gamma_\lambda\,u + \bar{d}\gamma_\lambda\,d)\,(\bar{u}\gamma_\lambda\,\gamma_5 u + \bar{d}\gamma_\lambda\,\gamma_5 d)\\ &\quad+\tfrac{1}{4}\,\zeta\,(\bar{u}\gamma_\lambda\,u - \bar{d}\gamma_\lambda\,d)\,(\bar{u}\gamma_\lambda\,\gamma_5 u + \bar{d}\gamma_\lambda\,\gamma_5 d)\\ &\quad+\tfrac{1}{4}\,\rho\,(\bar{u}\gamma_\lambda\,u + \bar{d}\gamma_\lambda\,d)\,(\bar{u}\gamma_\lambda\,\gamma_5 u - \bar{d}\gamma_\lambda\,\gamma_5 d)\}.\end{aligned} \tag{A.4}$$

*Because of the nonlocality implied by the intermediate boson, this is only of approximate validity, but it is a good approximation for low-energy experiments.

Thus, the effect of adding the charged current contribution is to change the parameters as follows:

$$\triangle I = 0, 2 \quad \begin{cases} \xi \to \xi - 2 \\ \eta \to \eta + 2 \end{cases}$$

$$\triangle I = 1 \quad \begin{cases} \zeta \to \zeta \\ \rho \to \rho \end{cases}$$

So, from an analysis of the observed parity-violating effects in nuclear physics, the pure neutral-current parameters ξ, η, ζ and ρ can be determined, at least in principle.

Let us next indicate the changes caused by Cabibbo rotation. In (A.1) and (A.2), d is replaced by

$$d' = d \cos \theta_c + s \sin \theta_c, \tag{A.5}$$

where θ_c is Cabibbo angle. Hence, $\mathscr{L}_{cc}$ becomes, on ignoring the strangeness-changing part,

$$\mathscr{L}_{cc} = -\frac{G}{\sqrt{2}} \bar{u}\gamma_\alpha (1+\gamma_5) u \{\cos^2 \theta_c \bar{d}\gamma_\alpha (1+\gamma_5) d + \sin^2 \theta_c \bar{s}\gamma_\alpha (1+\gamma_5) s\}. \tag{A.6}$$

The parity-violating part of this is

$$\begin{aligned} \mathscr{L}_{cc}^{pv} = -\frac{G}{\sqrt{2}} [\cos^2 \theta_c \{ & -\tfrac{1}{2} (\bar{u}\gamma_\alpha u - \bar{d}\gamma_\alpha d)(\bar{u}\gamma_\alpha \gamma_5 u - \bar{d}\gamma_\alpha \gamma_5 d) \\ & + \tfrac{1}{2} (\bar{u}\gamma_\alpha u + \bar{d}\gamma_\alpha d)(\bar{u}\gamma_\alpha \gamma_5 u + \bar{d}\gamma_\alpha \gamma_5 d)\} \\ & + \sin^2 \theta_c \{(\tfrac{1}{2} (\bar{u}\gamma_\alpha u - \bar{d}\gamma_\alpha d) + \tfrac{1}{2} (\bar{u}\gamma_\alpha u + \bar{d}\gamma_\alpha d)) \bar{s}\gamma_\alpha \gamma_5 s \\ & + (\tfrac{1}{2} (\bar{u}\gamma_\alpha \gamma_5 u - \bar{d}\gamma_\alpha \gamma_5 d) + \tfrac{1}{2} (\bar{u}\gamma_\alpha \gamma_5 u + \bar{d}\gamma_\alpha \gamma_5 d)) \bar{s}\gamma_\alpha s\}]. \end{aligned} \tag{A.7}$$

The $\triangle I = 1$ contributions arise from the terms $(\bar{u}\gamma_\alpha u - \bar{d}\gamma_\alpha d)\bar{s}\gamma_\alpha \gamma_5 s$ and $(\bar{u}\gamma_\alpha \gamma_5 u - \bar{d}\gamma_\alpha \gamma_5 d)\bar{s}\gamma_\alpha s$ and these contain the well-known suppression factor $\sin^2 \theta_c$. Further, as we already noted in the text, this term involves $\bar{s}s$ current whose strength will be small in nuclear states.

Let us now complete the picture, by including the charmed quarks via Glashow-Iliopoulos-Maiani rotation:

$$\mathscr{L}_{cc} = -\frac{G}{\sqrt{2}} \{\bar{u}\gamma_\alpha (1+\gamma_5) d' \bar{d}' \gamma_\alpha (1+\gamma_5) u + \bar{c} \gamma_\alpha (1+\gamma_5) s' \bar{s}' \gamma_\alpha (1+\gamma_5) c\},$$

$$= -\frac{G}{\sqrt{2}}\{\bar{u}\,\gamma_a\,(1+\gamma_5)\,u\,\bar{d'}\,\gamma_a\,(1+\gamma_5)\,d' \\ + \bar{c}\,\gamma_a\,(1+\gamma_5)\,c\,\bar{s}'\,\gamma_a\,(1+\gamma_5)\,s'\}, \tag{A.8}$$

where d' is given by (A.5) and

$$s' = -d\sin\theta_c + s\cos\theta_c. \tag{A.9}$$

Picking out the parity-violating and strangeness-and-charm conserving part, we have

$$\mathscr{L}_{cc}^{pv} = -\frac{G}{\sqrt{2}}[\bar{u}\,\gamma_a\,u\,\{\cos^2\theta_c\,\bar{d}\,\gamma_a\,\gamma_5\,d + \sin^2\theta_c\,\bar{s}\,\gamma_a\,\gamma_5\,s\} \\ + \bar{u}\,\gamma_a\,\gamma_5\,u\,\{\cos^2\theta_c\,\bar{d}\,\gamma_a\,d + \sin^2\theta_c\,\bar{s}\,\gamma_a\,s\} \\ + \bar{c}\,\gamma_\sigma\,c\,\{\sin^2\theta_c\,\bar{d}\gamma_a\,\gamma_5\,d + \cos^2\theta_c\,\bar{s}\,\gamma_a\,\gamma_5\,s\} \\ + \bar{c}\,\gamma_a\,\gamma_5\,c\,\{\sin^2\theta_c\,\bar{d}\,\gamma_a\,d + \cos^2\theta_c\,\bar{s}\,\gamma_a\,s\}] \tag{A.10}$$

Addition of further flavour-doublets will proceed in an analogous manner.

Correspondingly, we should include the $\bar{s}s$ and $\bar{c}c$ terms in the neutral-current Lagrangian also, which will bring in additional parameters. But such terms (both from the charged-current as well as neutral-current interaction) cannot be expected to play a significant role in low-energy nuclear physics and so may not contribute effectively to the measured parity-violating nuclear effects.

However, there may be other parity-violating phenomena in hadron physics. For instance, in hadron-hadron collisions at large transverse momenta, strong interactions may be damped out sufficiently for the weak parity-violating effects to be detectable. In these, the effect of the heavy-flavour-currents such as $\bar{s}s$ and $\bar{c}c$ may become measurable. Parity-violation in hypernuclear states is another place where the $\bar{s}s$ currents will show up.

Appendix B

Derivation of the two-boson relations

Although the procedure is straightforward, a systematic method has to be adopted because of the large number of parameters to be eliminated.

With μ-e university

First rewrite the 17 equations (8a) to (12d) explicitly in terms of the components of the vectors. We start with the two equations (10a) and (11a) and solve for $C_a^{(1)}$ and $C_a^{(2)}$. We get

$$C_\alpha^{(1)} = \frac{1}{2E}(\tilde{\alpha}\, C_0^{(2)} - \alpha\, C_A^{(2)}),\; C_\alpha^{(2)} = \frac{1}{2E}(\alpha\, C_A^{(1)} - \tilde{\alpha}\, C_0^{(1)}),$$

where $E = C_A^{(1)}\, C_0^{(2)} - C_A^{(2)}\, C_0^{(1)}$.

Similarly, using the pairs (10b) and (11b), (10c) and (11c) and (10d) and (11d), we get

$$C_\beta^{(1)} = \frac{1}{2F}(\tilde{\beta}\, C_0^{(2)} - \beta\, C_V^{(2)}),\; C_\beta^{(2)} = \frac{1}{2F}(\beta\, C_V^{(1)} - \tilde{\beta}\, C_0^{(1)}),$$

$$C_\gamma^{(1)} = \frac{1}{2E}(\tilde{\gamma}\, C_0^{(2)} - \gamma\, C_A^{(2)}),\; C_\gamma^{(2)} = \frac{1}{2E}(\gamma\, C_A^{(1)} - \tilde{\gamma}\, C_0^{(1)}),$$

$$C_\delta^{(1)} = \frac{1}{2F}(\tilde{\delta}\, C_0^{(2)} - \delta\, C_V^{(2)}),\; C_\delta^{(2)} = \frac{1}{2F}(\delta\, C_V^{(1)} - \tilde{\delta}\, C_0^{(1)}),$$

where $F = C_V^{(1)}\, C_0^{(2)} - C_V^{(2)}\, C_0^{(1)}$.

Using these in (12a) to (12d), we obtain

$$\xi = \frac{1}{f}\left[\tilde{\alpha}\,\tilde{\beta}\, L - \frac{1}{2}(\tilde{\alpha}\,\beta\, g_V + \alpha\,\tilde{\beta}\, g_A - 2\,\alpha\,\beta\, h_{VA})\right], \tag{B.1}$$

$$\eta = \frac{1}{f}\left[\tilde{\gamma}\,\tilde{\delta}\, L - \frac{1}{2}(\tilde{\gamma}\,\delta\, g_V + \gamma\,\tilde{\delta}\, g_A - 2\,\gamma\,\delta\, h_{VA})\right], \tag{B.2}$$

$$\zeta = \frac{1}{f}\left[\tilde{\alpha}\,\tilde{\delta}\, L - \frac{1}{2}(\tilde{\alpha}\,\delta\, g_V + \alpha\,\tilde{\delta}\, g_A - 2\,\alpha\,\delta\, h_{VA})\right], \tag{B.3}$$

$$\rho = \frac{1}{f}\left[\tilde{\gamma}\,\tilde{\beta}\, L - \frac{1}{2}(\tilde{\gamma}\,\beta\, g_V + \gamma\,\tilde{\beta}\, g_A - 2\,\gamma\,\beta\, h_{VA})\right], \tag{B.4}$$

where we have used the definitions:

$$L = \mathbf{C}_0^2 = (C_0^{(1)})^2 + (C_0^{(2)})^2, \tag{B.5}$$

$$f = 2\, EF = 2\,(C_A^{(1)} C_0^{(2)} - C_A^{(2)}\, C_0^{(1)})\;(C_V^{(1)}\, C_0^{(2)} - C_V^{(2)}\, C_0^{(1)}). \tag{B.6}$$

Now we have only to determine L and f in terms of the observable parameters.

Using (8a) and (9a), we solve for $C_V^{(1)}$ and $C_V^{(2)}$:

$$C_V^{(1)} = \frac{1}{2L}\{g_V C_0^{(1)} + \eta_V C_0^{(2)} (4Lh_{VV} - g_V^2)^{1/2}\}, \tag{B.7}$$

$$C_V^{(2)} = \frac{1}{2L}\{g_V C_0^{(2)} - \eta_V C_0^{(1)} (4L h_{VV} - g_V^2)^{1/2}\}, \tag{B.8}$$

where $\eta_V = \pm 1$. Similarly, from (8b) and (9b),

$$C_A^{(1)} = \frac{1}{2L}\{g_A C_0^{(1)} + \eta_A C_0^{(2)} (4L h_{AA} - g_A^2)^{1/2}\}, \tag{B.9}$$

$$C_A^{(2)} = \frac{1}{2L}\{g_A C_0^{(2)} - \eta_A C_0^{(1)} (4L h_{AA} - g_A^2)^{1/2}\}, \tag{B.10}$$

where $\eta_A = \pm 1$. Substituting (B.7) to (B.10) into (9c) we get an equation for L:

$$h_{VA} = \frac{1}{4L}[g_V g_A + \eta_V \eta_A \{(4L h_{VV} - g_V^2)(4L h_{AA} - g_A^2)\}^{1/2}]. \tag{B.11}$$

Solving this equation for L, we obtain

$$L = \frac{(h_{VV} g_A^2 + h_{AA} g_V^2 - 2h_{VA} g_V g_A)}{4(h_{VV} h_{AA} - h_{VA}^2)}. \tag{B.12}$$

We can now substitute this value of L into (B.7) to (B.10) and insert these into (B.6) to get

$$f = \pm \frac{(g_V h_{AA} - g_A h_{VA})(g_A h_{VV} - g_V h_{VA})}{2(h_{VV} h_{AA} - h_{VA}^2)}. \tag{B.13}$$

Finally, using (B.12) and (B.13) in (B.1) to (B.4), we obtain the desired relations (15a) to (15d).

Without μ-e universality

We consider the 31 equations given in (26a) to (34d). Out of these, the 17 equations given by (26), (28), (30), (32) and (34) correspond to the 17 equations (8a) to (12d) considered above (the case of μ-e universality), the only difference being that h_{VV}, h_{AA} and h_{VA} have been replaced by k_{VV}, k_{AA} and k_{VA}. Hence, our first group of relations (40a) to (40d) follow by the same method given above.

Further, the 17 equations contained in (27), (28), (31), (32) and (34) again are analogous to the original 17 equations and so the same procedure yields the second group of relations (41a) to (41d).

The derivation of the remaining 4 relations (42a) to (42d) requires further work which is even more tedious. We shall restrict ourselves to indicating the steps. We use the 21 equations contained in (27), (28), (29), (31,), (33) and (34). These 21 equations involve 17 current parameters and by eliminating them we arrive at our final four relations.

We start with (31a) and (33a) and solve for $C_\alpha^{(1)}$ and $C_\alpha^{(2)}$:

$$C_\alpha^{(1)} = \frac{1}{2E'}(\tilde{\alpha}'\, C_0'^{(2)} - \alpha'\, C_A'^{(2)}),\ C_\alpha^{(2)} = \frac{1}{2E'}(\alpha'\, C_A'^{(1)} - \tilde{\alpha}'\, C_0'^{(1)}),$$

where $\quad E' = C_A'^{(1)}\, C_0'^{(2)} - C_A'^{(2)}\, C_0'^{(1)}.$

Similarly using (31b), (33b), (31c), (33c), (31d), and (33d), we get

$$C_\beta^{(1)} = \frac{1}{2F'}(\tilde{\beta}'\, C_0'^{(2)} - \beta'\, C_V'^{(2)}),\ C_\beta^{(2)} = \frac{1}{2F'}(\beta'\, C_V'^{(1)} - \tilde{\beta}'\, C_0'^{(1)}),$$

$$C_\gamma^{(1)} = \frac{1}{2E'}(\tilde{\gamma}'\, C_0'^{(2)} - \gamma'\, C_A'^{(2)}),\ C_\gamma^{(2)} = \frac{1}{2E'}(\gamma'\, C_A'^{(1)} - \tilde{\gamma}'\, C_0'^{(1)}),$$

$$C_\delta^{(1)} = \frac{1}{2F'}(\tilde{\delta}'\, C_0'^{(2)} - \delta'\, C_V'^{(2)}),\ C_\delta^{(2)} = \frac{1}{2F'}(\delta'\, C_V'^{(1)} - \tilde{\delta}'\, C_0'^{(1)}),$$

where $\quad F' = C_V'^{(1)}\, C_0'^{(2)} - C_V'^{(2)}\, C_0'^{(1)}.$

Inserting these expressions for $C_\alpha^{(1)}$ .... $C_\delta^{(2)}$ into (34a)-(34d), we obtain

$$\begin{aligned}\xi = \frac{1}{f'}[&\tilde{\alpha}'\, \tilde{\beta}'\, L' - \tilde{\alpha}'\, \beta'\, \{C_0'^{(2)}\, C_V'^{(2)} + C_0'^{(1)}\, C_V'^{(1)}\} \\ &- \alpha'\, \tilde{\beta}'\, \{C_A'^{(2)}\, C_0'^{(2)} + C_A'^{(1)}\, C_0'^{(1)}\} \\ &+ \alpha'\, \beta'\, \{C_A'^{(2)}\, C_V'^{(2)} + C_A'^{(1)}\, C_V'^{(1)}\}],\end{aligned} \tag{B.14}$$

where $\quad L' = (C_0'^{(1)})^2 + (C_0'^{(2)})^2, \qquad$ (B.15)

$$f' = 2(C_A'^{(1)}\, C_0'^{(2)} - C_A'^{(2)}\, C_0'^{(1)})\, (C_V'^{(1)}\, C_0'^{(2)} - C_V'^{(2)}\, C_0'^{(2)}). \tag{B.16}$$

and similar expressions for η, ζ, ρ. Note that (B.14) involves $\mathbf{C}'_0 \cdot \mathbf{C}'_V$, $\mathbf{C}'_0 \cdot \mathbf{C}'_A$ and $\mathbf{C}'_V \cdot \mathbf{C}'_A$ which define the coupling constants in the $\nu_\mu\, \mu$ and $\mu\mu$ sectors. For the present it is better to eliminate them which we proceed to do.

We solve (29a) and (29d) for $C'^{(1)}_V$ and $C'^{(2)}_V$ and also solve (29b) and (29c) for $C'^{(1)}_A$ and $C'^{(2)}_A$.

$$C'^{(1)}_V = \frac{k'_{AV}\, C^{(2)}_V - k'_{VV}\, C^{(2)}_A}{2\,(C^{(1)}_A\, C^{(2)}_V - C^{(2)}_A\, C^{(1)}_V)} \tag{B.17}$$

$$C'^{(2)}_V = \frac{k'_{VV}\, C^{(1)}_A - k'_{AV}\, C^{(1)}_V}{2\,(C^{(1)}_A\, C^{(2)}_V - C^{(2)}_A\, C^{(1)}_V)} \tag{B.18}$$

$$C'^{(1)}_A = \frac{k'_{VA}\, C^{(2)}_A - k'_{AA}\, C^{(2)}_V}{2\,(C^{(1)}_V\, C^{(2)}_A - C^{(2)}_V\, C^{(1)}_A)} \tag{B.19}$$

$$C'^{(2)}_A = \frac{k'_{AA}\, C^{(1)}_V - k'_{VA}\, C^{(1)}_A}{2\,(C^{(1)}_V\, C^{(2)}_A - C^{(2)}_V\, C^{(1)}_A)} \tag{B.20}$$

Also, from (27) and (28), we get by the same procedure as followed with μ-e universality,

$$C^{(1)}_V = \frac{1}{2L'}\{g'_V\, C'^{(1)}_0 + \eta_V\, C'^{(2)}_0\, (4L'k_{VV} - g'^2_V)^{1/2}\}, \tag{B.21}$$

$$C^{(2)}_V = \frac{1}{2L'}\{g'_V\, C'^{(2)}_0 - \eta_V\, C'^{(1)}_0\, (4L'\, k_{VV} - g'^2_V)^{1/2}\}, \tag{B.22}$$

$$C^{(1)}_A = \frac{1}{2L'}\{g'_A\, C'^{(1)}_0 + \eta_A\, C'^{(2)}_0\, (4L'\, k_{AA} - g'^2_A)^{1/2}\}, \tag{B.23}$$

$$C^{(2)}_A = \frac{1}{2L'}\{g'_A\, C'^{(2)}_0 - \eta_A\, C'^{(1)}_0\, (4L'\, k_{AA} - g'^2_A)^{1/2}\}. \tag{B.24}$$

where $\eta_V = \pm 1$ and $\eta_A = \pm 1$ independently.

Substitution of (B.21) to (B.24) into (B.17) to (B.20) which are then substituted into (B.14) yields an equation for ξ involving current parameters only through L' and f':

$$\xi = (L', f'). \tag{B.25}$$

We shall not write this equation explicitly since it is rather unwieldy. We have now to calculate L' and f'.

We have already obtained $C^{(1)}_V$, $C^{(2)}_V$, $C^{(1)}_A$ and $C^{(2)}_A$ in terms of $C'^{(1)}_0$ and $C'^{(2)}_0$ and L' in (B.21) to (B.24). Using these in (28c), we get an equation involving L'. Solving this for L', we obtain

$$L' = \frac{k_{VV}\, g'^2_A + k_{AA}\, g'^2_V - 2\, k_{VA}\, g'_V\, g'_A}{4\,(k_{VV}\, k_{AA} - k^2_{VA})}. \tag{B.26}$$

This calculation is identical to that done with μ-e universality.

Insertion of equation (B.21) to (B.24) into (B.17) to (B.20) which are then used in the expression for f' given in (B.16), gives after simplification

$$f' = -\frac{2L'^2 (k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)}{\{\eta_A g'_V (4L' k_{AA} - g'^2_A)^{1/2} - \eta_V g'_A (4L' k_{VV} - g'^2_V)^{1/2}\}^{1/2}},$$

$$= \pm \frac{1}{8} \frac{(k'_{VA} g'_A - k'_{AA} g'_V)(k'_{AV} g'_V - k'_{VV} g'_A)}{(k_{VV} k_{AA} - k^2_{VA})}. \qquad \text{(B.27)}$$

Substitution of (B.26) and (B.27) in (B.25) yields the desired relation (42a). The other relations (42b) to (42d) can be written down by exact analogy.

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