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^{4}He and soft-core radial function

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Abstract. The suitability of a soft-core radial wave function in ^{4}He calculations is tested by computing the charge form factor of the alpha particle. The free parameter in the radial function is fixed in a photo-disintegration calculation. The results show that the soft-core function is suitable and the ground state of ^{4}He may be described reasonably well if a suitable potential model is used with it.

Keywords. Helium-4; soft-core function; integrated photo disintegration cross-section; charge form factor.

1. Introduction

Raghavan and Srivastava (1972) have shown that for the 1*s* shell nuclei the integrated photo-disintegration cross-section (σ_{int}) calculations fail to distinguish the inter-nucleon potentials because of the range-depth relationship, $V_0 a^2$=constant. In this paper we exploit this property of σ_{int} calculations in ^{4}He, to extract the free parameter in a soft-core radial function and then we test whether this function will be suitable for describing the ground state of ^{4}He. The form of the soft-core function is given by (Bijedic and Maric 1969):

$$\psi = f(r_{ij}) \prod_{i<j} r_{ij}. \tag{1}$$

For the function $f(r_{ij})$ a variety of forms like Gaussian, Irving, etc. can be chosen. In this investigation we choose $f(r_{ij})$ to be the Irving form viz.,

$$f(r_{ij}) = N \exp\left[-\beta\left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right]; \quad i=j=1, 2, 3, 4, \tag{2}$$

in which N is the normalisation constant and β is a free parameter. The choice of (2) was motivated by the fact that the results of Jain and Srivastava (1968) and Mahanti *et al* (1971) show that this function is analytically tractable and give reasonably good values for the properties of ^{4}He. The parameter β is usually obtained by fitting the r.m.s. radius or by performing a binding energy calculation of ^{4}He. But in this paper we do a σ_{int} calculation by using the soft-core radial function and Malfliet and Tjon (1969) two-body potential and then extract the parameter β by fitting to the experimental value of σ_{int}. Next we calculate the charge form factor of ^{4}He [F_{ch} (^{4}He)] by using this value of β.

2. Calculations

In the electric dipole approximation, the integrated photo-disintegration cross-section is given by (Levinger 1960)

$$\sigma_{\text{int}} = (2\pi^2 e^2 \hbar/mc) \sum_n f_{\text{on}}, \tag{3}$$

in which the summed oscillator strength $\sum_n f_{\text{on}}$ is given by

$$\sum_n f_{\text{on}} = \Big(\sum_n f_{\text{on}}\Big)_{\text{kinetic}} + \Big(\sum_n f_{\text{on}}\Big)_{\text{potential}}, \tag{4}$$

$\left(\sum_n f_{\text{on}}\right)_{\text{kinetic}}$, the model-independent term is equal to NZ/A ($=1$ for ^{4}He). The potential term is evaluated to be (Yadav *et al* 1971):

$$\Big(\sum_n f_{\text{on}}\Big)_{\text{potential}} = -\left(\frac{m}{3\hbar^2}\right)\left(x + \frac{1}{2}y\right) \langle \Sigma V(r_{kl})\, r_{kl}^2 \rangle_{00}, \tag{5}$$

where k and l denote the protons and the neutrons and x and y are the fractions of Majorana and Heisenberg exchange forces respectively. Even though (5) contains the two-body potential term it has been shown (Raghavan and Srivastava 1972) that for the 1*s* shell nuclei σ_{int} calculations fail to distinguish the two-body potentials. *Because of this conclusion, in our* σ_{int} *calculations we use a simple interaction due to Malfliet and Tjon* (1969) which is of the form:

$$V(r) = -A \exp(-\mu r)/r + B \exp(-2\mu r)/r. \tag{6}$$

In the above equation $\mu=1{\cdot}55$ fm^{-1}; $A=3{\cdot}22$ and $B=7{\cdot}39$ for the triplet-even state and $\mu=1{\cdot}55$ fm^{-1}; $A=2{\cdot}64$ and $B=7{\cdot}39$ for the singlet-even state. We calculate the potential term $\left(\sum_n f_{\text{on}}\right)_{\text{potential}}$ using the above parameters and the soft-core function (given by (1) and (2)), by assuming the exchange forces to be Serber in character ($x+\frac{1}{2}y=0{\cdot}5$).

Next we use (3) to evaluate σ_{int} for various values of β. By comparing the calculated values of σ_{int} with the experimental value (Gorbunov and Spiridonov 1958) we find that $\beta=2{\cdot}34\pm0{\cdot}23$ fm^{-1}. (The error bars are due to fitting the deviations in the experimental value). This value of β is considerably higher than those obtained by other authors for the single parameter Irving function without the core. For example, Jain and Srivastava (1968) obtain β to be 0·867 fm^{-1} and 0·70 fm^{-1}, when fitted to the r.m.s. radius and when minimising the binding energy of ^{4}He, respectively. Irving (1951) gets β to be 0·92 fm^{-1} in the binding energy calculation of ^{4}He with a purely central potential. Thus the large value of β obtained in our calculation is due to the core in the wave function.

Next we evaluate the r.m.s. radius of ^{4}He with our value of β and find it to be 4/3 times the experimental value. This shows that either the single parameter in the radial function is not satisfactory or the parameter adjustment against σ_{int} is not very good. Inclusion of additional parameters may improve (e.g. see Jain and Srivastava 1968) our result for the r.m.s. radius, but the analytical and numerical work involved are enormous (particularly due to the nature of the core function) and so we have not made an attempt in this direction. Similarly, a value for the parameter obtained from an analytical calculation (say, a variational calculation) instead of fitting σ_{int} may give a reasonable value for the r.m.s. radius. But, work along this line must be pursued with a ' realistic ' two-body potential.

However, to find out whether the soft-core function gives a reasonable result in the charge form factor calculations of ^{4}He, we calculate F_{ch} (^{4}He) using the relation,

$$F_{ch}\ (^4\text{He}) = (F^p_{ch} + F^n_{ch}) \int \exp\,(i\mathbf{q}\cdot\mathbf{r}_i)\,|\psi|^2\, d\tau. \tag{7}$$

In the above equation F^p_{ch} and F^n_{ch} are the proton and neutron form factors and q is the momentum transfer. The calculated values of F^2_{ch} (^{4}He) with our soft-core function are shown in figure 1 along with the experimental results and other theoretical calculations (Singh *et al* 1969; Mahanti *et al* 1971). From the figure we find that the softcore function produces a diffraction minimum around q^2=7·5 fm^{-2} and a secondary

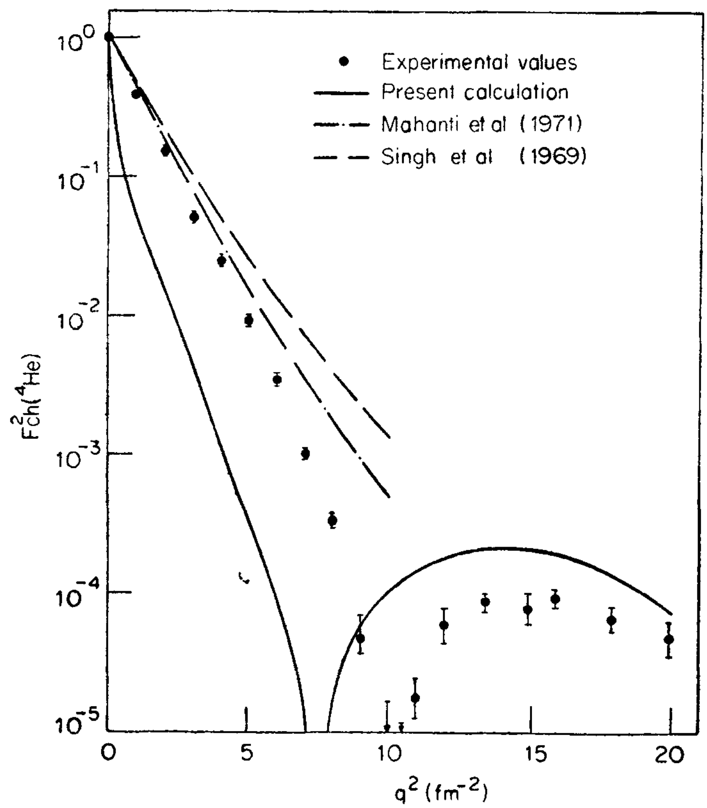

Figure 1. F^2_{ch} (^{4}He) versus q^2

maximum at $q^2=14\ \mathrm{fm}^{-2}$. These are to be compared with the experimental values (Frosch *et al* 1966) of 10 fm^{-2} and 16 fm^{-2} respectively. As mentioned earlier it might be possible to improve our form factor results also, if one pays attention to fixing the single parameter or adds additional parameters in the radial part of the wave function.

3. Conclusion

This investigation shows that the soft-core function given by (1) is tractable in ^{4}He calculations. Further, this function reproduces the essential features in the charge form factor of ^{4}He, namely, the diffraction minimum and the secondary maximum. On the other hand, the Irving functions without a core used by Jain and Srivastava (1968), Singh *et al* (1969) and Mahanti *et al* (1971) do not exhibit these characteristics.

Finally, we wish to point out that it might be possible to use the soft-core function with a repulsive soft-core potential, such as the momemum-dependent potential of Green (1962) and this model might describe the ground state of ^{4}He reasonably well.

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