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Comparison of eikonal-Born series and modified Glauber approach for the study of elastic electron-hydrogen scattering

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Abstract. Elastic electron-hydrogen scattering at medium and high energies has been analysed in eikonal-Born series and modified Glauber approach for $E_i \geqslant 50$ eV. We have used closed form expressions for evaluating the second Born term in the fixed scatterer approximation and for higher order terms of Glauber-eikonal series. The exchange effect has also been included via Glauber-Bonham-Ochkur type procedure. The fixed scatterer approximation calculations are compared with results obtained using other approximations for the second Born term available in the literature. The results compare fairly well with experimental data.

Keywords. Elastic scattering; eikonal-Born series; Glauber-eikonal series; Ochkur exchange.

1. Introduction

Recent efforts in obtaining an accurate description of electron-atom scattering in the intermediate energy region centre on Glauber and eikonal related methods (Glauber 1959; Gerjuoy and Thomas 1974). It is well known that the failure of the Glauber theory lies in neglecting completely the intermediate energy transfer. Attempts have been made to improve this method. In this direction eikonal-born series (EBS) method proposed by Byron and Joachain (1973) and two-potential approach by Ishihara and Chen (1975) have been quite successful in predicting the elastic differential cross-section for electron-atom collisions. The recent review articles by Bransden and McDowell (1977) and Byron and Joachain (1977) describe various approaches for electron-atom collisions and their applicability in different energy regions. Another approach very similar to EBS, referred to as the modified Glauber method (MG) suggested by Byron and Joachain (1975) and simultaneously proposed by Gein (1976, 1977) has been successful in analysing electron-hydrogen and helium elastic and inelastic scattering. In the modified Glauber approach, the second order term of Glauber eikonal expansion is replaced by the second order Born term in the conventional Glauber amplitude. This modification removes the singularity of the Glauber amplitude and at the same time retains the characteristics of the eikonal method in the rest of the amplitude.

The aim of the present paper is to make a detailed comparison of the EBS and MG results in the case of $\bar{e}$-H elastic scattering using the fixed scatterer approximation (FSA) results for the second Born term (Ghosh 1977; Srivastava and Tripathi 1978). The evaluation of the second Born term in this procedure is hardly any more difficult than the first Born term. Further, to account for the contribution of the first three terms of the Glauber-eikonal series (referred as f_{G1}, f_{G2}, f_{G3}) as needed in EBS and MG models, we have used the closed form expressions given by Yates (1974). Previous calculations based on EBS and MG model were made using double and triple numerical integration for the evaluation of f_{G2} and f_{G3} terms respectively. We have also taken into account the exchange contribution via Ochkur (1963) method in the EBS results and via Glauber-Bonham-Ochkur method (Dewangan 1976; Khayrallah 1976) in the MG model. In order to compare the present calculations, we have also calculated the differential cross-section in the EBS and MG approximations using the second Born term with the approximations employed by Byron and Joachain (1977) and by Gien (1977). In the following section we present a brief outline of the different expressions used in the present study and in § 3, we present the result and discussion.

2. Theory

The scattering amplitude in the eikonal-Born series approximation (f_{EBS}) considered through order k_i^{-2}, where k_i is the wave-number of the incident electron, is expressed as a combination of first and second Born terms (f_{B1} and f_{B2}) and third term of the Glauber-eikonal multiple series (f_{G3}) i.e.

$$f_{\text{EBS}} = f_{B1} + f_{B2} + f_{G3}. \tag{1}$$

Similarly, the scattering amplitude in modified Glauber approximation f_{GM} is given by

$$f_{\text{GM}} = f_G + f_{B2} - f_{G2}, \tag{2}$$

where f_G is the conventional Glauber amplitude and f_{G2} is the second order Glauber term. Equation (2) avoids the divergence in the forward direction by subtracting f_{G2} from the conventional Glauber amplitude and includes all the higher order terms of the Glauber amplitude.

Consider the ē-H elastic scattering. The second Born term occurring in (1) and (2) is evaluated from the analytic expression obtained by Ghosh (1977) and Srivastava and Tripathi (1978) (equation 2 of their paper) in the FSA, refrered here as f_{B2}^{FSA}. The closed form expressions for the second Born term has also been taken from the work of Byron and Joachain (1977) (equations A18 and A21); the first one is an improved second Born term ($f_{\bar{B}2}$) properly corrected for the 'on shell' and 'off shell' contributions arising from ground state and the second one is referred to as the simplified second Born term ($f_{\text{SB}2}$). Further, in order to calculate the contributions for f_{G2} and f_{G3}, we use the following closed form expressions of the Glaubeı-eikonal series given by Yates (1974)

$$f_{G1} = \frac{2+Z^2}{\lambda(1+Z^2)^2}, \; Z = (K/\lambda), \; \lambda = 2,$$

$$f_{G2} = \frac{i}{2k_i Z^3} (\partial/\partial Z) \frac{Z^4}{(1+Z^2)} \ln [(1+Z^2)/Z],$$

and
$$f_{G3} = \frac{i^2 \lambda}{16 k_i^2 Z^3} \frac{\partial}{\partial Z} \frac{Z^4}{1+Z^2} \left\{ 4 \left[\ln \left(\frac{1+Z^2}{Z} \right) \right]^2 + (\pi^2/3) - 2A(Z) \right\},$$

where
$$A(Z) = 2 (\ln Z)^2 + \frac{1}{6}\pi^2 + \sum_{n=1}^{\infty} \frac{(-Z^2)^n}{n^2}, \; Z \leqslant 1,$$

$$= - \sum_{n=1}^{\infty} [(-1/Z^2)^n/n^2], Z > 1$$

and finally, the conventional Glauber amplitude is calculated by using the closed form given by Thomas and Gerjuoy (1974) or by using the well-known expression given by Franco (1970). For exchange contribution we use the closed form expression for Glauber-Bonham-Ochkur (GBO) exchange amplitude as given by Dewangan (1976) and Khayrallah (1976)

$$g^{\mathrm{GBO}}(K) = (2^{2-2i\eta}/k_i^2)\, \Gamma(1-i\eta) \frac{-8+i\eta(4-K^2)}{(4+K^2)^{2-i\eta}},$$

where $\eta = 1/v_i$, v_i being the velocity of the incident electron (a.u.) and the Ochkur exchange given by

$$g^{\mathrm{Och}}(K) = -32/k_i^2(K^2+4)^2$$

3. Result and discussion

We have calculated the elastic differential cross-section (DCS) for electron-hydrogen at 50 and 100 eV as a function of scattering angle θ using (1) and (2). Figures 1 and 2 show the DCS calculated in the EBS and MG model along with the experimental data of Williams (1975) and VanWingerden *et al* (1977). Figure 1 shows the DCS for electron of energy 50 eV. It is clear from the figure that at small scattering angles i.e. upto 60°, the results of the EBS and MG model calculation including exchange are in reasonable good agreement but the disagreement between the two calculations starts as the scattering angle increases and the difference between the two calculations is as high as 20% at large scattering angles. The present calculation in EBS and MG models using closed form for the scattering amplitudes occurring in (1) and (2) compares well with the results of the calculation using a properly computed second Born term ($f_{\bar{B}2}$) in the region of small scattering angles but the results of the present calculation show a significant difference i.e. $\sim 50\%$ at large scattering angles as compared to the $f_{\bar{B}2}$ results. The $f_{\bar{B}2}$ results compare well with the experimental data than the present calculations (using f_{B2}^{FSA}) at large scattering angles. It is not surprising since the static interaction which dominates the large angle scattering has been

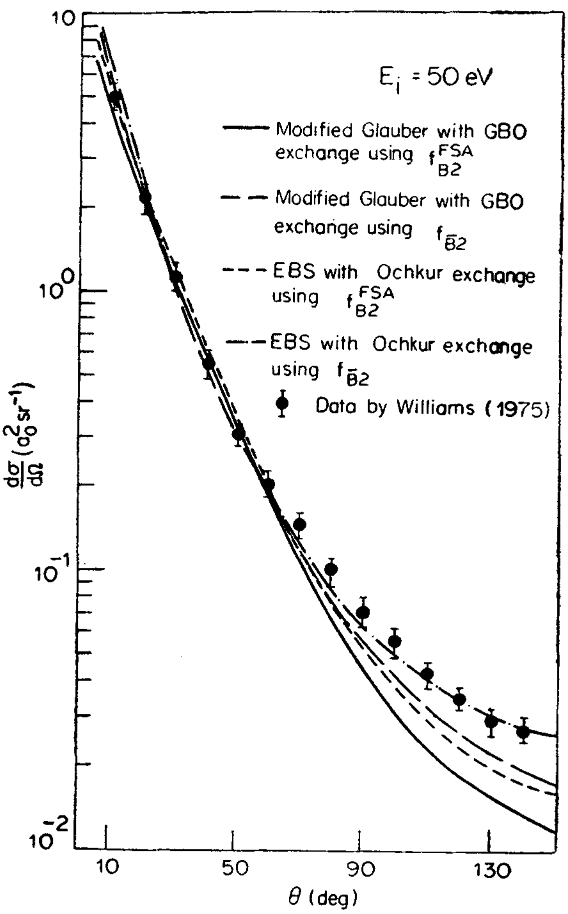

Figure 1. Differential cross-sections for elastic electron scattering by a hydrogen atom in its ground state at 50 eV.

very well accounted in the computation of $f_{\bar{B}2}$ but this feature is altogether absent in the present calculation. Apart from this disagreement another observed feature is that the results of the calculation obtained in MG and EBS models, using the same $f_{\bar{B}2}$ results always vary. This difference is more pronounced in the backward direction. It can be easily seen that the EBS model is well contained in the MG model and, therefore, the difference at the large scattering angles is caused by higher order Glauber terms f_{G4}, f_{G5} in the MG model. A similar comment can also be made about the inclusion of exchange both in EBS and MG models. The good agreement of EBS results with the Williams measurements is not an indication of the superiority of this model over MG model. Recently, Gien has also calculated DCS for $\bar{e}$-H in MG model using a simplified second Born term (f_{SB2}) and Glauber exchange through the two-dimensional formula of Foster and Williamson (1976). He has obtained a good agreement in the entire angular region with the experimental data. The use of this type of exchange in MG model is more consistent than the use of GBO type exchange as has been done in the present calculations. The use of GBO

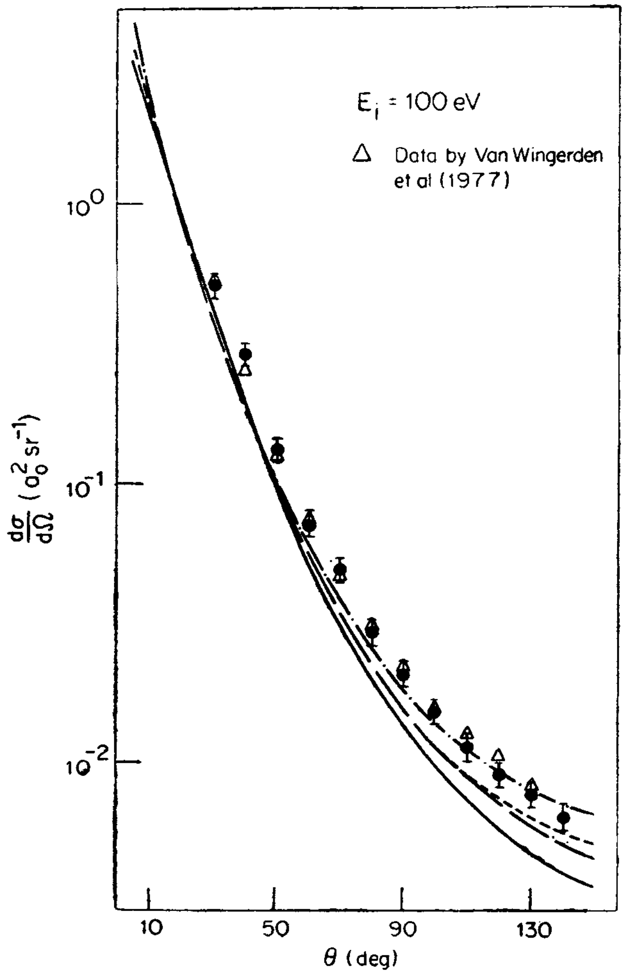

Figure 2. Differential cross-sections for elastic electron scattering by a hydrogen atom in its ground state at 100 eV. (For other details see figure 1).

exchange is not consistent because terms of the type $1/k_i^3$ which comes from the real part of the second Born exchange term are absent. The DCS obtained both in MG and EBS models using different types of second Born term are given in table 1 and compared with Gien's numerical results at 50 eV. The present calculations using GBO type exchange and f_{SB2} is within 5% of Gien's recent calculations using Glauber exchange in the region of small scattering angles. At large scattering angles the present calculation is off by 10–15%. It should also be pointed out that the use of f_{B2}^{FSA} in MG model reintroduces the infinity in the forward direction and in this way it does not correct the Glauber amplitude; however its use in EBS model only ignores the long range polarisation part of the target.

Figure 2 shows the situation at 100 eV. The general trend of the variation of DCS at 100 eV is the same as at 50 eV except that the difference in the results of the calculation using f_{B2}^{FSA} and $f_{\bar{B}2}$ both in MG and EBS model becomes narrower.

Table 1. Differential cross-section ($d\sigma/d\Omega$) in a_0^2 sr^{-1} for elastic scattering of electron by hydrogen atom at incident energies 50 and 100 eV.

Angles (deg)	Modified Glauber results with GBO exchange			EBS results with Ochkur exchange			Modified Glauber with Glauber exchange (Gien's results)
	$\left(\frac{d\sigma}{d\Omega}\right)^{FSA}_{f_{B2}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{f_{SB2}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{f_{\bar{B}2}}$	$\left(\frac{d\sigma}{d\Omega}\right)^{FSA}_{f_{B2}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{f_{SB2}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{f_{\bar{B}2}}$	
E_i = 50 eV							
5	6·67	8·71	8·52	7·80	9·67	9·49	8·47
10	5·52	5·68	5·49	4·69	6·39	6·20	5·60
20	1·99	2.39	2·27	2·26	2·67	2·55	2·38
30	1·08	1·31	1·04	1·17	1·23	1·14	1·11
50	3·24^{-1}	3·47^{-1}	3·02^{-1}	3·40^{-1}	3·65^{-1}	3·20^{-1}	3·22^{-1}
70	1·07^{-1}	1·37^{-1}	1·12^{-1}	1·17^{-1}	1·49^{-1}	1·25^{-1}	1·26^{-1}
90	4·53^{-2}	7·01^{-2}	5·44^{-2}	5·10^{-2}	7·82^{-2}	6·42^{-2}	6·44^{-2}
110	2·47^{-2}	4·38^{-2}	3·27^{-2}	2·85^{-2}	4·98^{-2}	4·08^{-2}	3·84^{-2}
130	1·55^{-2}	3·07^{-2}	2·22^{-2}	1·96^{-2}	3·69^{-2}	3·05^{-2}	2·44^{-2}
150	1·14^{-2}	2·42^{-2}	1·71^{-2}	1·58^{-2}	3·08^{-2}	2·56^{-2}	2·01^{-2}
E_i = 100 eV							
5	3·25	4·37	4·29	3·50	4.62	4·54	
10	1·96	2·33	2·75	2·08	2·47	2·41	
20	8·90^{-1}	8·96^{-1}	8·56^{-1}	9·18^{-1}	9·25^{-1}	8·86^{-1}	
30	4·14^{-1}	4·15^{-1}	3·89^{-1}	4·20^{-1}	4·21^{-1}	3·96^{-1}	
50	9·95^{-2}	1·11^{-1}	1·01^{-1}	1·02^{-1}	1.14^{-1}	1·04^{-1}	
70	3·10^{-2}	3·89^{-2}	3·41^{-2}	3·42^{-2}	4·28^{-2}	3·80^{-2}	
90	1·37^{-2}	1.86^{-2}	1·59^{-2}	1·57^{-2}	2·11^{-2}	1·86^{-2}	
110	7·44^{-3}	1·07^{-2}	9·04^{-3}	9·20^{-3}	1·29^{-2}	1·13^{-2}	
130	4·82^{-3}	7·19^{-3}	6·02^{-3}	6·46^{-3}	9·21^{-3}	8·12^{-3}	
150	3·64^{-3}	5·56^{-3}	4·62^{-3}	5·21^{-3}	7·47^{-3}	6·60^{-3}	

It is found that with increase of incident energy i.e. beyond 200 eV (not given here) there is hardly any difference in the results of different calculations using either f^{FSA}_{B2} or $f_{\bar{B}2}$ in MG and EBS models.

The present calculations hardly require any computational effort. MG model calculations should yield more satisfactory results at medium energy as compared to EBS model calculations. It is thus tempting to conclude that the present calculations provide a simple yet reliable approach to study electron hydrogen elastic scattering at medium energy.

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