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A model for multiparticle production in high energy hadronic collisions

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Abstract. A model for multiparticle production process in high-energy hadronic collisions is proposed. In the centre of mass (CM) system of colliding particles the target and the projectile are assumed to pass through each other sharing energies allowed by kinematical constraints. Thus in a *pp* collision the energy associated with each is $\sqrt{S}/2$ (S being the square of the CM energy) which is taken to be the real variable that governs the number of particles produced. In the case of hadron-nucleus collisions the projectile and the target of ν nucleons lying in a (Lorentz contracted) tube pass through each other sharing energies $\simeq \sqrt{S_A}/2$, where $S_A \simeq \nu S$. Before the final state particles emerge from these systems, the constituents of the target, i.e., ν nucleons share equally $(=\sqrt{S_A}/2\nu)$ the total energy associated with the target and become the centres from which final state particles stem out. Several results have been discussed.

Keywords. Multiparticle production; hadron-nucleus collision; mean normalised multiplicity; nuclear scaling.

1. Introduction

Interest in secondaries produced in high-energy hadron-nucleus collisions has considerably increased during recent years since they provide direct information about space-time development in elementary collision processes. Some results in this direction may be briefly mentioned. In the entire energy range from ~ 20 GeV and above, the average number of heavy prongs (particles with relative velocity $\beta \langle 0{\cdot}7$), $\langle N_h \rangle$, remains essentially constant. The mean normalised multiplicity defined as $R_A = \langle n_A \rangle / \langle n \rangle$, where $\langle n_A \rangle$ is the average number of particles created in collisions of hadron with nucleus of mass number A and $\langle n \rangle$ is the same quantity for nucleon target, remains essentially independent of energy. Target size dependence in the form of $R_A = A^{\alpha}$ gives $\alpha \sim 0{\cdot}13$–$0{\cdot}19$ for proton projectile with energies $\gtrsim 20$ GeV (Gurtu *et al* 1974; Hebert *et al* 1977; Azimov *et al* 1977; Aziz *et al* 1978 a, b). In both the hadron-nucleon and hadron-nucleus collisions the fraction of energy associated with secondaries (inelasticity $\langle K \rangle$) is found to be essentially constant in the entire energy range from a few GeV and above. Its dependence on target-size is very weak. Studies of the rapidity distributions show that excess particles in pA collisions appear essentially in lower rapidity regions (i.e., at larger angles). The shape of the rapidity distribution changes with target size—the centroid of the distribution of excess particles shifts towards the lower rapidity side with increasing target size.

The dispersion of the rapidity distribution remains essentially independent of target size (Florian *et al* 1976; Azimov *et al* 1977; Anzon *et al* 1977; Aziz *et al* 1978c). Although the data favour particle production through intermediate states, none of the existing models of multiparticle production is capable of explaining the results. Detailed information about the experimental results and theoretical concepts may be found in the reviews by Busza (1975, 1976), Dar *et al* (1976), Otterlund (1976), Nikitin *et al* (1977) and Azimov *et al* (1978).

2. A model for multiparticle production

We propose here a simple model for multiparticle production in high-energy hadron-nucleon and hadron-nucleus collisions. The following assumptions are made.

(i) The multiparticle final state is reached through the formation of intermediate states.

(ii) Subject to a hadron-nucleon collision, the projectile and the target pass through each other, excite each other and contribute independently towards the number of particles in the final state. The number of particles stemming out from each of the target and the projectile is governed by the energy associated with them in the CM system.

(iii) In a hadron-nucleus collision, at sufficiently high energies, the beam particle sees a very thin nucleus (due to Lorentz contraction) and thus interacts simultaneously with ν nucleons lying in a tube of the nucleus along its trajectory.

(iv) After the interaction the ν nucleons equally share the energy associated with the target and behave like ν excited objects to give independent contributions towards the number of final state particles. The number of particles stemming out from each is governed by the energy associated with them.

In the CM system of the colliding objects, the incident and the target particles share energy according to the kinematical constraints. The energies associated with the projectile and the target may be given by

$$E_p^* = (S+m_p^2-m_t^2)/2\sqrt{S},$$

and

$$E_t^* = (S+m_t^2-m_p^2)/2\sqrt{S},$$

where S is the square of the total CM energy and m_p and m_t are the masses of the projectile and the target. Thus for a proton-proton collision we have $E_p^* = E_t^* = \sqrt{S}/2$. The particles stemming out from each of the entities may be given by

$$n_p = n_t = f(\sqrt{S}/2),$$

and thus the total number of particles produced in proton-proton collision will be $n_p + n_t = n$ (say) given by

$$n = 2f(\sqrt{S}/2).$$

From the study of hadron-nucleon collisions it follows that the average multiplicities of the particles produced in these collisions vary as S^{α}, where α is to some extent energy-dependent (Albini *et al* 1976). At lower energies $\alpha \sim \frac{1}{4}$ whereas at sufficiently high energies $\alpha \sim \frac{1}{4}-\frac{1}{5}$. In the energy range from $\sim$20 GeV and above it is reasonable to take $\alpha = \frac{1}{4}$. Taking a quite general form of S^{α} for the variation of multiplicity with energy, the average number of particles in the final state may be given as

$$\langle n \rangle = K\, 2\, (\sqrt{S}/2)\, 2\alpha, \tag{1}$$

where K is a constant. For a value of $\alpha = \frac{1}{4}$ it is found that $K \sim 1$ in the energy range $\sim$20 GeV and above.

In the case of proton-nucleus collisions the energies associated with the projectile of mass m_p and the target of mass νm_p may be respectively given by

$$E_p^* = (S_A + m_p^2 - \nu^2 m_p^2)/2\sqrt{S_A},$$

and $$E_t^* = (S_A + \nu^2 m_p^2 - m_p^2)/2\sqrt{S_A}$$

where S_A is the square of CM energy in proton-nucleus collisions. Thus we can write

$$E_p^* = \frac{\sqrt{S_A}}{2}\left(1 - \frac{m_p^2\,(\nu^2-1)}{S_A}\right),$$

and $$E_t^* = \frac{\sqrt{S_A}}{2}\left(1 + \frac{m_p^2(\nu^2-1)}{S_A}\right).$$

The factor $m_p^2(\nu^2-1)/S_A$ is sufficiently small compared to 1. For example, taking $\nu=3$ the value of this factor is 0·058 at 20 GeV and 0·012 at 100 GeV which further decreases with increasing energy and may, therefore, be neglected as compared to 1. Thus, at sufficiently high energies it is reasonsable to take $E_p^* \simeq E_t^* \simeq \sqrt{S_A}/2$. Now it is assumed that energy E_t^* is equally shared by the ν nucleons and thus the energy associated with each of the ν nucleons shall be $\simeq \sqrt{S_A}/2\nu$. The number of particles in the final state of hadron-nucleus collisions may therefore be given as

$$n_A = f(\sqrt{S_A}/2) + \nu f(\sqrt{S_A}/2\nu).$$

Taking the same dependence of multiplicity on energy as in hadron nucleon collisions we have

$$\langle n_A \rangle = K(\sqrt{S_A}/2)^{2\alpha} + \nu K(\sqrt{S_A}/2\nu)^{2\alpha}. \tag{2}$$

2.1. *Mean normalised multiplicity*

The mean normalised multiplicity is given by

$$R_A = \langle n_A \rangle / \langle n \rangle = \frac{(\sqrt{S_A}/2)^{2\alpha} + \nu(\sqrt{S_A}/2\nu)^{2\alpha}}{2(\sqrt{S}/2)^{2\alpha}}$$

and since $S_A \simeq \nu S$ we have

$$R_A = (\nu^{\alpha} + \nu^{1-\alpha})/2. \tag{3}$$

Thus we find that R_A is essentially independent of energy which agrees with experiment. It may be noted that $R_A=1$ for $\nu=1$. The energy dependence in R_A through α is very weak and practically insignificant compared to the uncertainties involved in the experimental values of R_A. For $\nu=2$, 3, 4 and 5 the values of R_A have been calculated taking $\alpha=\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ and given in table 1. From the table it follows that the value of R_A remains essentially insensitive to the value of α. In order to compare with experimental results as well as for simplicity we take the average value of $\nu=A^{\frac{1}{3}}$ in proton-nucleus collisions (Meng Ta-Chung 1977; Aziz *et al* 1978b). The data on multiplicities in proton-proton and proton-nucleus collisions have been taken from the compilations by Albini *et al* (1976) and Aziz *et al* (1978b) respectively. The values of R_A have been estimated by taking the created charged particles in both the cases (Aziz *et al* 1978a, b). The multiplicity of particles in hadron-nucleon collisions is represented by $\langle n_{\mathrm{ch}} \rangle$, which is the average number of charged particles observed in the final state. This includes some contribution from initial channel hadrons (the projectile and the target) which also appear in the final channel as charged or neutral. Therefore, to estimate the actual number of charged particles created in the interaction the contributions of initial channel hadrons should be excluded from $\langle n_{\mathrm{ch}} \rangle$. In the case of hadron-nucleus collisions the multiplicity of particles is given by $\langle n_s \rangle$, which represents the average number of charged particles in the final state with relative velocity $\beta \geqslant 0{\cdot}7$. The target protons mostly appear as grey tracks ($0{\cdot}3 \leqslant \beta < 0{\cdot}7$) and do not contribute to $\langle n_s \rangle$. Only the incident hadron which appears as shower (charged or neutral) contributes to $\langle n_s \rangle$ and, therefore, if we exclude this contribution from $\langle n_s \rangle$ we get almost all the created charged particles in the final state of hadron-nucleus collisions. It may be mentioned here that a very small fraction of the created pions appears as grey tracks and are not counted into $\langle n_s \rangle$. The percentage of these slow pions is $\sim$5–8% of the number of grey tracks. For a proper analysis this fraction of slow pions should also be taken into account. However, since this fraction is too small to affect the results significantly we may neglect it. Partial justification to this also comes from the fact that almost a similar number of hit target protons appear as shower and are counted into $\langle n_s \rangle$. Thus excluding the contributions of the initial channel hadrons from $\langle n_{\mathrm{ch}} \rangle$ and $\langle n_s \rangle$ we get the number of truly created charged particles in the two cases. Since the created particles are mostly pions, using the observation of charge symmetry (i.e. $\langle n_{\pi^0} \rangle \simeq$

Table 1. Values of H_A for different values of ν with different values of α.

ν/α	2	3	4	5
1/3	1·42	1·76	2·05	2·32
1/4	1·43	1·80	2·12	2·42
1/5	1·44	1·83	2·17	2·50
1/6	1·45	1·85	2·22	2·56

$\frac{1}{2}\langle n_{\pi}\pm\rangle$) we may say that the estimation of the mean normalised multiplicity in terms of either created charged particles or all the created particles (including neutrals) shall be equivalent.

In the case of proton-proton collisions it is observed that the frequency of protons present in the final state which are counted into $\langle n_{ch}\rangle$ lies between 1·27 to 1·4 (e.g. Boggild *et al* 1971; Daniel *et al* 1973; Chadha *et al* 1974; Ammosov *et al* 1977). Therefore, this number should be subtracted from $\langle n_{ch}\rangle$ to find out the number of created charged particles. We may therefore take $\langle n_{ch}\rangle - 1{\cdot}33$ as the average number of charged particles created in *pp* collisions. In the case of proton-nucleus collisions the incident proton also appears in the final state as shower (charged or neutral) which is not a created particle. It appears as proton in about 60 to 70% of the cases (e.g. Pal *et al* 1963; Fredrikson 1975; Halliwell *et al* 1977) and the rest as neutron losing its charge. So we take the average number of created charged particles in the final state of *p*-nucleus collisions equal to $\langle n_s\rangle - 0{\cdot}67$. Thus the mean normalised multiplicity, R_A, may be estimated using the expression

$$R_A = \frac{\langle n_s\rangle - 0{\cdot}67}{\langle n_{ch}\rangle - 1{\cdot}33} = \frac{\langle n_A\rangle}{\langle n\rangle}. \tag{4}$$

The values of R_A so calculated at different energies are shown in figure 1 for emulsion nuclei. The solid lines represent equation (3) with $\alpha = \frac{1}{4}$. It may be seen from the figure that the experimental results are in excellent agreement with the estimated

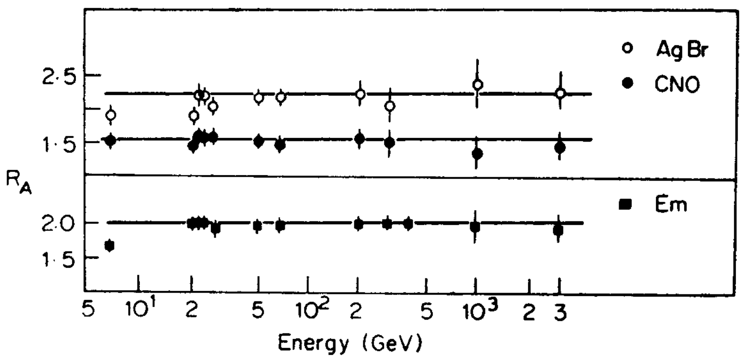

Figure 1. Variation of R_A with energy. The solid lines correspond to relation (3) with $\alpha = \frac{1}{4}$.

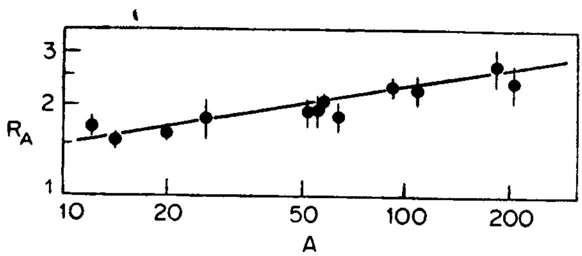

Figure 2. Variation of R_A with mass number A. The solid line corresponds to relation (3) with $\alpha = \frac{1}{4}$.

values of R_A in a very wide energy range for CNO ($A=14$), average emulsion ($\langle A^{\frac{1}{3}}\rangle = 3\cdot874$) and AgBr ($A=94$) nuclei. In figure 2 the target size dependence of R_A is shown. The solid line corresponding to equation (3) with a value $\alpha=\frac{1}{4}$ is in good agreement with the experimental results.

2.2. *Nuclear scaling*

An interesting prediction of the approach is that at sufficiently high energies we can have a variable

$$\eta(E_1/E_2) = \frac{\langle n\rangle_{E_1}}{\langle n\rangle_{E_2}} = \frac{\langle n_A\rangle_{E_1}}{\langle n_A\rangle_{E_2}} = \left(\frac{E_1}{E_2}\right)^{\alpha}, \tag{5}$$

where E_1 and E_2 are two lab. energies. This may be seen as follows. From relation (2) we can write

$$\langle n\rangle_{E_1}/\langle n\rangle_{E_2} = \frac{(\sqrt{S_1}/2)^{2\alpha}}{(\sqrt{S_2}/2)^{2\alpha}} = (E_1/E_2)^{\alpha},$$

since $S \simeq 2\, mE$, where m is the proton mass. Similarly from relation (2), we can write

$$\frac{\langle n_A\rangle_{E_1}}{\langle n_A\rangle_{E_2}} = \frac{(\sqrt{S_{A,\,1}}/2)^{2\alpha} + \nu\,(\sqrt{S_{A,\,1}}/2\nu)^{2\alpha}}{(\sqrt{S_{A,\,2}}/2)^{2\alpha} + \nu\,(\sqrt{S_{A,\,2}}/2\nu)^{2\alpha}} = (E_1/E_2)^{\alpha}$$

since $S_A \simeq \nu S \simeq 2m\,\nu E$.

Thus it follows that $\eta(E_1/E_2) = (E_1/E_2)^{\alpha}$ and the ratio of the average number of particles created at two different energies is independent of the size of the target (a kind of nuclear scaling) and is only a function of the ratio of the energies. It is worth mentioning that Friedlander *et al* (1974) have reported from the experimental results at 67,200 and 1000 GeV that $\eta(E_1/E_2)=(E_1/E_2)^{\alpha}$ and have obtained the value of $\alpha=0\cdot25\pm0\cdot003$ in excellent agreement with our choice of $\alpha=\frac{1}{4}$.

2.3. *Leading particle effect and inelasticity*

The Feynman scaling for leading particle inclusive distribution is well satisfied, i.e.,

$$(1/\sigma)\ (d\sigma^{\text{lp}}/dX) = \rho(X,\ \sqrt{S}) \to \rho(X) \text{ as } S \to \infty,$$

where $$X = E_{\text{lp}}^{\text{lab}} \big/ E_{\text{in}} \simeq 2E_{\text{lp}}^{\text{cm}} \big/ \sqrt{S}, \tag{6}$$

and lp stands for leading particle and E_{in} is the lab-energy of the incident particle.

The above expression gives

$$\int_{\text{lp}} \rho(X)\, dX = \langle N\rangle_{\text{lp}} = \text{constant}, \tag{7}$$

and $$\int_{\text{lp}} X\, \rho(X)\, dX = \langle X\rangle_{\text{lp}} = \text{constant}. \tag{8}$$

Applying the same hypothesis to hadron-nucleus collisions one can write

$$X_A = E_{\mathrm{lp}}^{\mathrm{lab}} \Big/ E_{\mathrm{in}} \simeq 2E_{\mathrm{lp}}^{\mathrm{cm}} \Big/ \sqrt{S_A}, \tag{9}$$

$$\int_{\mathrm{lp}} \rho(X_A)\, dX_A = \langle N_A \rangle_{\mathrm{lp}} = \text{constant}, \tag{10}$$

and
$$\int_{\mathrm{lp}} X_A\, \rho(X_A)\, dX_A = \langle X_A \rangle_{\mathrm{lp}} = \text{constant}. \tag{11}$$

Thus from equations (10) and (11) we have the average number of leading particles and the fraction of primary energy taken away by them as constant and essentially independent of energy. This, in effect, means the saturation of nuclear response at sufficiently high energies which is directly reflected from the constancy of $\langle N_h \rangle$ and the mean normalised multiplicity R_A. The inelasticities follow at once. For hadron-nucleon and hadron-nucleus collisions we have the average inelasticities as

$$\langle K \rangle = (1 - \langle X \rangle), \tag{12}$$

and
$$\langle K_A \rangle = (1 - \langle X_A \rangle). \tag{13}$$

And therefore, from relations (6), (9) and (11) we can write

$$\langle X_A \rangle \simeq \langle X \rangle / \sqrt{\nu}, \tag{14}$$

and
$$\langle K_A \rangle \simeq (1 - \langle X \rangle / \sqrt{\nu}). \tag{15}$$

It is found that around 50% of the energy goes with the leading particles in hadron-nucleon collisions and thus taking $\langle X \rangle = \frac{1}{2}$ we have

$$\langle K_A \rangle \simeq (1 - 1/2\, \sqrt{\nu}). \tag{16}$$

Equation (16) is true for hydrogen target also as for $\nu = 1$, $\langle K \rangle = \frac{1}{2}$ as required. The values of $\langle K_A \rangle$ given by (16) closely agree with the experimental results reported by Azaryan *et al* (1975) where $\langle K_A \rangle = 0{\cdot}51\ \mathrm{\AA}^{0{\cdot}09}$.

3. Concluding remarks

The model proposed here seems to account for most of the experimental facts. The mean normalised multiplicity has been estimated using power dependence of multiplicity on energy. However, a logarithmic dependence may also be considered. The numerical values of R_A remain essentially unaltered; only the form of the expression for R_A changes

$$R_A \sim \left(\frac{1}{2} + \frac{1}{2}\frac{\ln \nu}{\ln S}\right) + \left(\frac{1}{2} - \frac{1}{2}\frac{\ln \nu}{\ln S}\right) \nu.$$

The model can easily account for the observed facts about rapidity distribution of secondaries. The particles stemming out from the ν excited objects shall appear in the backward hemisphere of CM system (i.e. at larger angles in the lab.) Since the energy associated with each of the ν entities $= (\sqrt{S_A}/2\nu) = (1/\sqrt{\nu})(\sqrt{S}/2)$ depends upon ν, the shape of the rapidity distribution is expected to depend upon target size—the centroid of the excess particles shifting towards lower rapidities with increasing target size. The dispersion in the rapidity distributions is expected to remain essentially unchanged. Details regarding the rapidity distribution of particles in the framework of the present picture will be given elsewhere. It may be remarked that a collective contribution from all (or a few) of the ν entities is, however, not ruled out completely. In such cases fast fragments like deuterons, tritons, He-nuclei, etc. may be observed, associated with a deficiency of fast protons along with a deficiency of created pions compared to normal events. Finally, the picture may be easily extended to nucleus-nucleus collisions.

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