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Alpha particle scattering in the rigid projectile approximation

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Abstract. We study elastic α-particle scattering off p,α-particle and ^{12}C targets at 17·9 GeV/c incident momentum in the rigid projectile approximation of the Glauber model. Differential and total cross-sections are computed and compared with the data. Reasonable agreement with the observed differential cross-sections is found for small momentum transfers but short-range dynamical correlations in the target will probably have to be taken into account to get better agreement at larger momentum transfers, particularly in the case of α–^{12}C scattering.

Keywords. Alpha particle scattering; Glauber model; rigid projectile approximation.

1. Introduction

Over the years, the multiple scattering formalism developed by Glauber (1959) has provided a basis for calculations which have described quite successfully particle-nucleus collisions at high energies. The theory has been extended (Czyz and Maximon 1969; Franco and Varma 1975; Alexander and Rinat 1976; Varma and Franco 1977) to cover nucleus-nucleus collisions as more experimental data with composite particle projectiles have become available. However, as the projectile becomes more complex, the full Glauber series for such scattering processes become progressively more difficult to evaluate, even if one chooses simple forms for nuclear densities and spin-averaged two-particle amplitudes. Various approximations to the full Glauber series have therefore been considered in the literature. One approach is to introduce an expansion in terms of the optical phase shift function. This expansion has been shown to converge fairly rapidly for light and medium nuclei (Chaumeaux *et al* 1976; Franco 1974; Franco and Varma 1977). Another approach, which is simpler and does not entail assumptions regarding convergence based on an examination of the first few terms of a series, is the so-called rigid projectile approximation (Alkhazov *et al* 1977; Varma 1978). Both these techniques have been applied to the analysis of the recent measurements performed at Saclay with 1·37 GeV α-particle projectiles (Chaumeaux *et al* 1976; Alkhazov *et al* 1977) and have been successful in explaining these scattering processes in terms of simple forms of nuclear densities with spin averaged nucleon-nucleon scattering amplitudes as input.

In the rigid projectile approximation, one treats the projectile as an elementary object and assumes that it stays in its ground state throughout the scattering process. The effect of the polarisation of the projectile is thus ignored during scattering. This leads to a tractable form of the Glauber series which in addition to its simplicity commends itself because it does not assume, as the optical limit expressions of the

Glauber series do, that the target also remains in its ground state during the whole of the scattering process. Recent calculations by Varma (1978) and by Viollier and Turtschui (1978) have shown that the rigid projectile approximation is fairly accurate at small momentum transfers and gives, for α-particle scattering, results almost identical to the full Glauber series and may therefore be relied upon as a simple effective approximation for analysing α-particle interactions with light and medium nuclei at high energies.

The main advantage of studying interactions involving α-particles is that their spin and isospin are zero. One therefore has to deal with only two elementary α-nucleon amplitudes in contrast to the situation involving protons where in principle as many as 10 elementary nucleon-nucleon amplitudes may contribute. In addition, the α-particle may be considered as the lightest of the heavy ions, so that the effect of the composite nature of the α-particle on the scattering process may also be studied. Recently, experiments on the elastic scattering of 17·9 GeV/c α-particles off a variety of targets have been performed at Dubna (Ableev *et al* 1977) and in the present work we have tried to examine whether or not these can be understood in the framework of the rigid projectile approximation of the Glauber theory, particularly as this approximation has been applied with considerable success to the lower energy Saclay data.

In § 2 of this paper we set out the expressions for the Glauber series for α-nucleus scattering in the rigid projectile approximation. In § 3 we obtain simple spin-independent parametrisations of α–p and α–α elastic scattering amplitudes by χ^2 minimisation. The purpose is to obtain expressions for α–^{12}C scattering not only in terms of two-particle scattering data but also when the α–p and α–α interactions are treated as elementary. In § 4 we compare our theoretical predictions with the data and finally summarise our results in § 5.

2. Alpha-nucleus scattering-rigid projectile approximation

The rigid projectile approximation in the framework of the Glauber theory assumes that the projectile nucleus stays in its ground state during the whole of the multiple scattering process. In particular, one assumes that the projectile undergoes scattering off the target nucleus without being decomposed into its constituent nucleons. With this assumption one first constructs the scattering amplitude for the projectile nucleus and a target nucleon using nucleon-nucleon scattering amplitude as input. This is then used to obtain the expression for the nucleus-nucleus scattering amplitude.

The amplitude for the elastic scattering of α-particles off a nucleus A is given in the Glauber theory by

$$F_{\alpha A}(\triangle) = \frac{ik}{2\pi} K_A(\triangle) \int d^2\mathbf{b}\, \exp(i\,\mathbf{\Delta}\cdot\mathbf{b})\, \Gamma^{\text{total}}_{\alpha A}(b), \tag{1}$$

where $\Gamma^{\text{total}}_{\alpha A}(b)$ is the total profile function for α-nucleus scattering, $K_A(\triangle)$ is the centre of mass correction for the nucleus A, $\mathbf{\Delta}$ is the momentum transfer, k is the magnitude of the incident momentum and $\mathbf{b}$ is the impact parameter.

Assuming the additivity of phase shifts the total profile function for scattering between the α-particle and the nucleus A can be written as

$$\Gamma_{\alpha A}^{\text{total}}(b) = 1 - \exp\,[i\,\chi_{\alpha A}^{c}(b)] + \exp\,[i\,\chi_{\alpha A}^{c}(b)]\;\Gamma_{\alpha A}^{\text{strong}}(b), \tag{2}$$

where $\chi_{\alpha A}^{c}$ is the coulomb phase shift function. Equation (1) can now be written as

$$F_{\alpha A}(\triangle) = K_A(\triangle)\,F_{\alpha A}^{c}(\triangle) + \frac{ik}{2\pi}\,K_A(\triangle) \int \exp\,(i\,\mathbf{\Delta}\cdot\mathbf{b})\cdot$$

$$\exp\,[i\,\chi_{\alpha A}^{c}(b)]\;\Gamma_{\alpha A}^{\text{strong}}(b)\;d^2\,\mathbf{b}. \tag{3}$$

If we assume the coulomb phase shift to originate from the nucleus as a whole and treat both the projectile and target nucleus as point charges, then

$$F_{\alpha A}^{c}(\triangle) = -\frac{2\eta\,k}{\triangle^2}\exp\,[-2i\,\{\eta\,\ln\,(\triangle/2k) - \arg\,\Gamma\,(1+i\eta)\}], \tag{4}$$

where $\eta = Z_\alpha\, Z_A\, e^2/\hbar v$, v being the relative velocity between the projectile and the target. In the rigid projectile approximation we have

$$\Gamma_{\alpha A}^{\text{strong}}(b) = 1 - [1 - \Gamma_{\alpha N}(b)]^A,$$

with
$$\Gamma_{\alpha N}(b) = \frac{1}{2\pi i k_N}\int \exp\,(-i\,\mathbf{q}\cdot\mathbf{b})\;S_A(q)\;f_{\alpha N}(q)\;d^2\,\mathbf{q} \tag{5}$$

where k_N is one fourth the momentum of the incident α-particle and $f_{\alpha N}(q)$ is the amplitude for scattering between the α-particle and a nucleon in the nucleus A. Assuming the α-particle form factor to be purely Gaussian, viz.

$$S_\alpha(q) = \exp\,(-R_\alpha{}^2\,q^2/4)$$

and the high energy spin-independent parametrisation of the nucleon-nucleon amplitude to be

$$f_{pN}(q) = \frac{k_N\,\sigma\,(i+\rho)}{4\pi}\;\exp\,(-\beta q^2/2) \tag{6}$$

$f_{\alpha N}$ is given by

$$f_{\alpha N}(q) = -\frac{ik_N}{2}\,K_\alpha(q)\sum_{j=1}^{4}\binom{4}{j}\left[\frac{-\sigma\,(1-i\rho)}{2\pi Q}\right]^j\,\frac{Q}{j}\exp\,(-q^2 Q/4j) \tag{7}$$

where $Q = 2\beta + R_\alpha^2$.

Using (5), (7) and also assuming a gaussian form factor for the nucleus A, viz.

$$S_A(q) = \exp(-R_A^2 q^2/4),$$

equation (3) takes the following form:

$$F_{\alpha A}(\triangle) = K_A(\triangle) F^c_{\alpha A}(\triangle) - ik K_A(\triangle) \sum_{j=1}^{A} \binom{A}{j} \sum_{k=0}^{j} \binom{j}{k} C_1^{j-k}$$

$$\sum_{l=0}^{k} \binom{k}{l} C_2^{k-l} \sum_{m=0} \binom{l}{m} C_3^{l-m} C_4^m \left[\frac{k^2}{\chi_{jklm}}\right]^{i\eta} \frac{1}{\chi_{jklm}}$$

$$\exp(-\triangle^2/4\chi_{jklm})\, {}_1F_1(-i\eta, 1;\ \triangle^2/4\chi_{jklm}), \tag{8}$$

where $C_j = \binom{4}{j} \frac{Q}{jP_j} \left[\frac{-\sigma(1-i\rho)}{2\pi Q}\right]^j; \; j = 1, 2, 3, 4$

$$\chi_{jklm} = j/P_1 + k(1/P_2 - 1/P_1) + l(1/P_3 - 1/P_2) + m(1/P_4 - 1/P_3)$$

and $P_j = R_A^2 - R_\alpha^2/4 + Q/j$.

Equation (8) is the general expression of the amplitude for the scattering of α-particles off a nucleus A in the rigid projectile approximation.

3. Parametrisation of α–A amplitudes

One can parametrise the α–A strong interaction amplitudes in a high energy spin-independent form as a sum of gaussians:

$$f_{\alpha A}(q) = \frac{(i+\rho_{\alpha A})}{4\pi} k_{\alpha A}\, \sigma_{\alpha A} \sum_{j=1}^{N} \gamma_{\alpha A j} \exp(-\beta_{\alpha A j}\, q^2/2) \tag{9}$$

with $\sum_j \gamma_j = 1$.

If now we write the total α–A phase shift as

$$\chi^{\text{total}}_{\alpha A} = \chi^c_{\alpha A} + \chi^{\text{strong}}_{\alpha A}$$

and treat both the α-particle and the nucleus A as point charges, then the elastic scattering amplitude for α–A scattering is given by

$$F_{\alpha A}(\triangle) = F^{c}_{\alpha A}(\triangle) + \frac{k_{\alpha A}\,\sigma_{\alpha A}}{4\pi}(i+\rho_{\alpha A})\,\Gamma(1+i\eta)$$
$$\times \sum_{j=1}^{N} \gamma_{\alpha Aj}\,(2\beta_{\alpha Aj}\,k^{2}_{\alpha A})^{i\eta} \exp\,(-\triangle^{2}\beta_{\alpha Aj}/2)\;{}_1F_1\,(-i\eta,\,1;\;\triangle^{2}\beta_{\alpha Aj}/2) \quad (10)$$

To obtain a parametrisation of the α–A strong interaction amplitude, the constants occurring in (9) can be taken to be those which give the minimum value of χ^2 when the differential cross-section implied by (10) is compared with the experimental data. Strong interaction α–p and α–α amplitude parametrised in this manner can be used to obtain the elastic scattering amplitudes for α–α and α–^{12}C, the calculation being done both for an independent particle as well as the α-particle model for the ^{12}C nucleus.

4. Results

4.1. α-p scattering

We first compute the elastic differential cross-section in the Glauber theory using proton-proton data at 4·2 GeV/c (Jenni *et al* 1977) as input (this is close to the 4·5

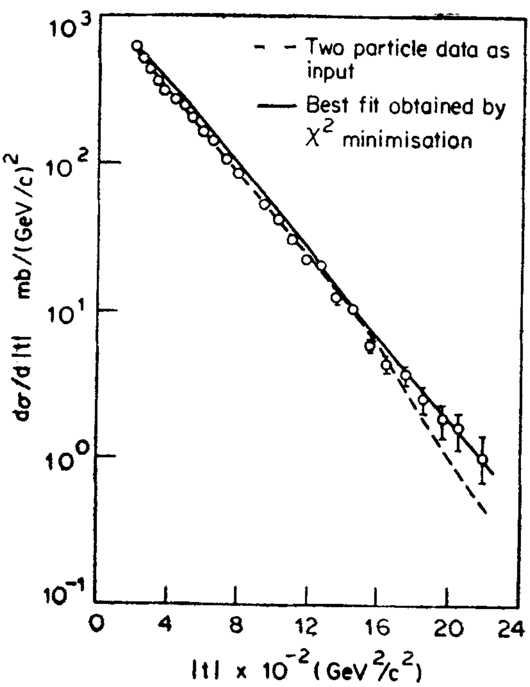

Figure 1. α-p elastic differential cross-section at 17·9 GeV/c as a function of momentum transfer.

GeV/c momentum per nucleon of the present measurements). We have assumed that the energy is sufficiently large to take proton-neutron parameters equal to the proton-proton values: $\rho_{pp}=-0{\cdot}39$, $\beta_{pp}=7{\cdot}5$ $(\mathrm{GeV/c})^{-2}$ and $\sigma_{pp}=42{\cdot}2$ mb. The expression for the α–p scattering amplitude with coulomb effects taken into account in the fashion described above is obtained by putting $A=1$, in (8).† The resulting fit to the data is exceedingly good as is evident from figure 1. The total cross-section predicted is 142 mb to be compared with the experimental value of $\sigma_{\alpha p}^{\mathrm{tot}}=147\pm1$ mb.

We would also like to determine the constants occurring in the spin-independent parametrisation of the α–p scattering amplitude given by (9) (with $N=1$), to enable their subsequent use in computing the α–α and α–^{12}C cross-sections at this energy. We take $\sigma_{\alpha p}=147$ mb, $\beta_{\alpha p}=32{\cdot}5$ $(\mathrm{GeV/c})^{-2}$ as given by Ableev *et al* (1977). We treat $\rho_{\alpha p}$ as a free parameter and find that the sum of the mean squared deviations of the differential cross-section predicted by (10) from the experimental data is minimum for $\rho_{\alpha p}=-0{\cdot}09$. The best fit corresponding to this value is also displayed in figure 1, and is practically indistinguishable from the fit with *pp* input, upto values of the square momentum transfer $|t|\approx 0{\cdot}15$ $(\mathrm{GeV/c})^2$.

4.2. *α-α scattering*

The expression for the elastic scattering amplitude with two-particle data as input in the rigid projectile approximation is given by (8) with $A=4$, $R_A=R_\alpha$. The

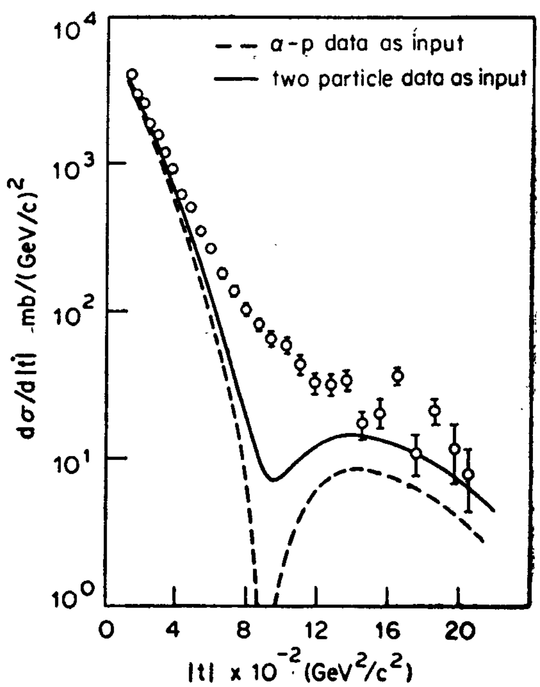

Figure 2. α-α elastic differential cross-section at 17·9 GeV/c as a function of momentum transfer.

† R_α for gaussian form factor of α-particle is 1·366 fm (Bassel and Wilkin 1968).

differential cross-section obtained by using the two-particle data of Jenni *et al* (1977) as input is displayed in figure 2. The agreement is reasonably good in the forward direction and again for 0·14 $(GeV/c)^2 \leqslant |t| \leqslant 0{\cdot}2\ (GeV/c)^2$. The theoretical curve shows a distinct minimum near $|t| = 0{\cdot}09\ (GeV/c)^2$ whereas the experimental data show only a gradual flattening in this region. The value of the total cross-section predicted is 390 mb which does not agree well with the experimentally measured value of $\sigma_{\alpha\alpha}^{tot}=450\pm20$ mb.

We now use the spin-independent parametrisation of the α–p scattering amplitude obtained in § 4.1 to obtain an expression for the α–α scattering amplitude. The expression for α-α scattering is then given by (7) with N replaced by α and the values of the parameters σ, β, ρ occurring in this equation are equal to the best fit values for α–p scattering listed in § 4.1. The corresponding differential cross-section is also displayed in figure 2 and the fit is worse than that obtained in the case of two-particle input, the minimum being much deeper. The total cross-section now predicted is however 400 mb.

Finally we determine the constants occurring in the spin-independent parametrisation of the α-α scattering amplitude given by (9). The use of $N=1$ (i.e. a single gaussian) in (9) gives a poor agreement with the experimental data the best fit, displayed in figure 3 being good only upto $|t| \leqslant 0{\cdot}06\ (GeV/c)^2$. Using the values $\sigma_{\alpha\alpha}=450$ mb and $\beta_{\alpha\alpha} = 72{\cdot}2\ (GeV/c)^{-2}$ given by Ableev *et al* (1977), we find that $\rho_{\alpha\alpha}=-0{\cdot}45$ gives the minimum mean squared deviation from the data. A reasonably good fit for values of $|t|$ extending to 0·2 $(GeV/c)^2$ requires the use of at least a sum of four

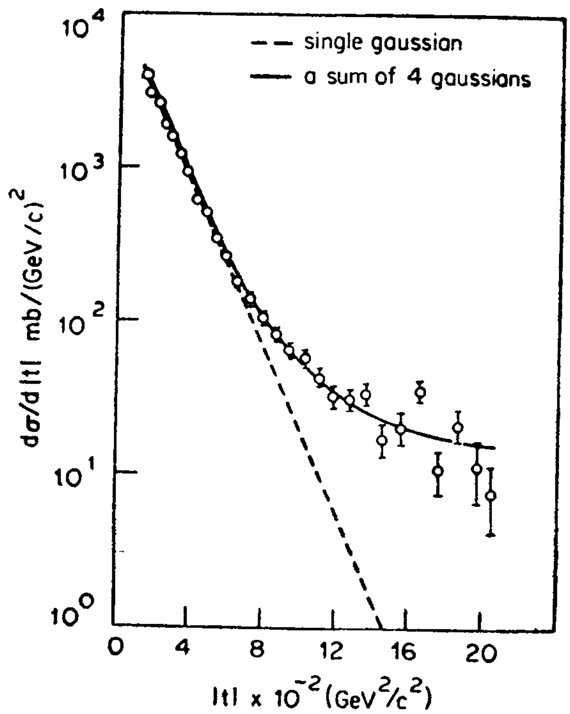

Figure 3. Differential cross-section for elastic α-α scattering, showing the best fit curves.

gaussians in (9). In this case we keep $\sigma_{\alpha\alpha}$ fixed at 450 mb and treat $\rho_{\alpha A}$, $\beta_{\alpha Aj}$ and $\gamma_{\alpha Aj}$ as free parameters. The best fit is obtained for values of these parameters listed in table 1 and this fit is also shown in figure 3.

4.3. α–^{12}C scattering

We first compute the α–^{12}C elastic differential cross-section with two-particle input in the rigid projectile approximation and with gaussian form factors† for both the α

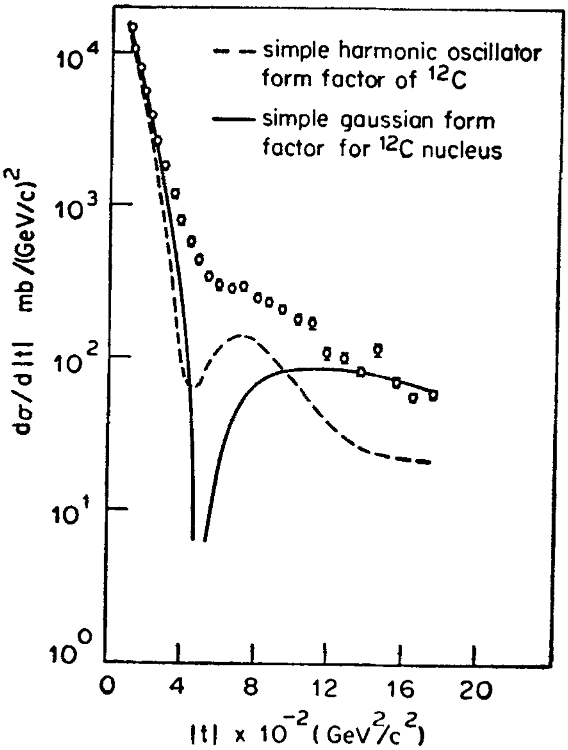

Figure 4. α-^{12}C elastic differential cross-section at 17·9 GeV/c as a function of momentum transfer for two particle data as input.

Table 1. The best fit parameters for α-α scattering amplitude (sum of four gaussians)

$\rho_{\alpha\alpha}$	$\beta_{\alpha\alpha j}$ (GeV/c)$^{-2}$	$a_{\alpha\alpha j}$	$\gamma_j = \frac{a_{\alpha\alpha j}}{\sum_j a_{\alpha\alpha j}}$
	77·5	−3·27	0·399
0·15	5·5	−0·31	0·038
	4·5	−0·20	0·025
	71·3	−4·41	0·538

†R_c for gaussian form factor of ^{12}C is 1·96 fm and for the simple harmonic oscillator form factor is 1·59 fm, both corresponding to a rms radius of 2·41 fm (Varma and Franco 1977).

and ^{12}C nucleus. We also take into account the coulomb scattering using (8) with $A=12$ and use appropriate two-particle values for ρ, β and σ. The results are plotted in figure 4. As in the case of α-α scattering, we find that the fit is reasonable both for small momentum transfers and in the region beyond the first minimum, but whereas the calculation predicts the existence of a well-defined minimum at $|t|=0{\cdot}05$ $(\mathrm{GeV/c})^2$ the data only show a gradual flattening around this value. The value of the total cross-section predicted is 853 mb to be compared with the experimentally measured value $\sigma^{\mathrm{tot}}_{\alpha c}=877\pm10$ mb.

One can attempt to study the effect of the structure of the ^{12}C nucleus by using a more realistic form factor† given by

$$S_c(q) = [1-R_c^2 q^2/9] \exp(-R_c^2 q^2/4).$$

The corresponding expression for the α–^{12}C scattering amplitude including coulomb interaction can be worked out as in the case of the gaussian form factor, and yields a differential cross-section which is also displayed in figure 4. The dip is now shallower but the fit both in the forward direction and in the region beyond the first minimum is not as good as before. The total cross-section now predicted is 811 mb.

We next calculate the α–^{12}C differential cross-section with α–p input with a gaussian form factor for the ^{12}C nucleus and coulomb interactions taken into account. This fit is shown in figure 5 and is worse than in the case of the two-particle input. The

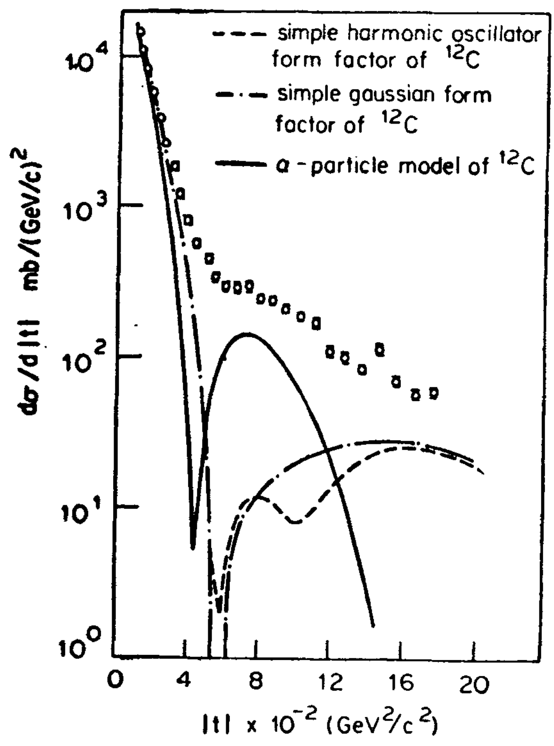

Figure 5. Differential cross-section for elastic α-^{12}C scattering with α-p data used as input.

minimum is sharper and the agreement with the experimental data away from the minimum is poorer. We have repeated these calculations for the realistic ^{12}C form factor and also for the α-particle model of the ^{12}C nucleus (for details please see Ghosh and Varma 1978). Of the three cases that we have studied using α-p input, the α-particle model seems to give the best fit, but this is in any case not nearly as good as the fit with two-particle input. The predicted total cross-sections in these three cases are 850 mb, 828 mb and 832 mb when the ^{12}C nucleus is described by an alpha particle model, an independent particle model and a simple harmonic oscillator model respectively.

Finally we use the parametrisation of the α-α amplitude given in § 4·2 as input to calculate the α-^{12}C scattering amplitude within an alpha particle model of the ^{12}C nucleus (for details see Ghosh and Varma 1978). In figure 6 we show the α-^{12}C differential cross-section calculated for both cases when the α-α amplitude is parametrised either as a single gaussian or as a sum of four gaussians. It can be seen from the plots that the interference minimum is quite shallow when a single gaussian parametrisation is used, but the fits in the extreme forward and higher momentum transfer regions are not as good as in the case of two-particle input. The sum of four gaussians also leads to poor fits. The total cross-section values predicted are 909 mb for the case of a single gaussian and 930 mb for the case of a sum of four gaussians.

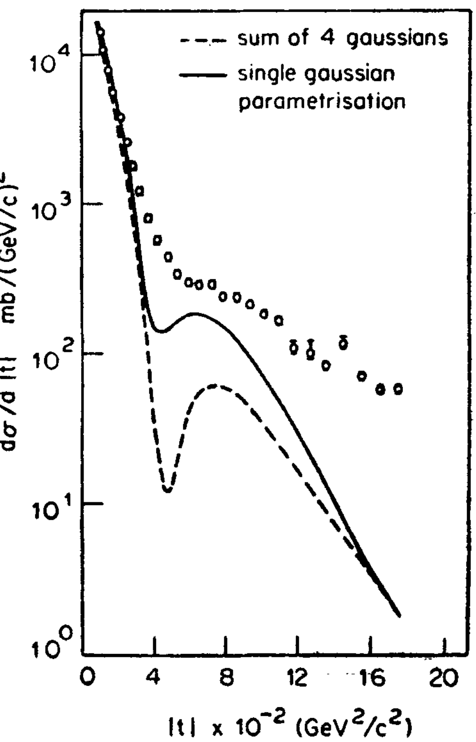

Figure 6. Differential cross-section for elastic α-^{12}C scattering with α-α data used as input.

5. Conclusions

In the preceding sections we have studied scattering of α-particles off proton, α–particle and ^{12}C targets using a rigid projectile approximation to the Glauber model of high energy scattering. We find that although the α-p data of Ableev *et al* (1977) are well described by our calculations, the rigid projectile approximation is not as successful in describing the α–α and the α–^{12}C data at the present energy as it was in the case of the lower energy Saclay data (Varma 1978; Viollier and Turtschi 1978). The rigid projectile approximation predicts deep interference minima in these cases at both energies. However, whereas at the lower energy the experimental data indicate the existence of such minima, the higher energy data show only a gradual flattening. One is therefore forced to conclude that the rigid projectile approximation leads to useful simplification only in the case of scattering off light nuclei at small momentum transfers and at relatively lower energies. It ceases to be as satisfactory at relatively higher energies possibly on account of the higher probability of dissociation of the projectile inside the target. Our calculations seem to suggest that the detailed structure of ^{12}C may play an important role in determining the ^{12}C differential cross-section even for low momentum transfers and to get better agreement in the case of the heavier nuclei at higher energies. It seems inevitable that one will have to take into account short-range dynamical correlations in the target. We are currently investigating this possibility.

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