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Neutral currents in left-right symmetric gauge models

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Abstract. We analyse all the neutral-current phenomena following from the general class of gauge models based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$. It is found that the neutral-current couplings in these models bear a remarkable similarity to those in the standard Weinberg-Salam gauge model. The parameter which plays the role of $\sin^2\theta_W$ is found to lie between 0 and $\frac{1}{2}$. Comparison with experimental data shows that even a model with the ratio of the masses of the two Z bosons as small as 1·9 is not ruled out.

Keywords. Neutral currents; gauge models; $SU(2)_L \otimes SU(2)_R \otimes U(1)$; neutrino; parity-violation.

1. Introduction

In this work, we present a general analysis of all neutral current phenomena in the class of models based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ (Mohapatra and Sidhu 1977; Pati *et al* 1978; De Rujula *et al* 1977). Carrying out such an analysis at this stage when the standard model (Weinberg 1967; Salam 1968) based on the group $SU(2)_L \otimes U(1)$ seems so successful may require justification. But it must be remembered that the major part of the neutral current data, on which the spectacular success of the standard model is based, comes from the neutrino interactions. Neutrino is a very special object. Laboratory neutrinos are left-handed since they arise from the usual π and K decays. Hence, they are neutral under any right-handed gauge groups that may be present. Because of this special property, it is possible to envisage a gauge group larger than the $SU(2)_L \otimes U(1)$ such that only the $SU(2)_L \otimes U(1)$ part manifests itself in the neutrino-interactions. The success of the standard model in the neutrino sector does not in any way pin down the group, or close the doors upon the groups like $SU(2)_L \otimes SU(2)_R \otimes U(1)$.

Models based on $SU(2)_L \otimes SU(2)_R \otimes U(1)$—to be called left-right symmetric models—have attractive features. With the observation of parity-violation in the electron-deuteron scattering experiment at SLAC (Prescott *et al* 1978), the original motivation for these models, namely, that of arranging zero parity-violation for neutral current induced phenomena, does not exist any more. However, as has been often pointed out, the left-right symmetric models have claims to validity based on considerations beyond those of ensuring agreement with parity-violation data. In most other models, parity-violation is exalted to the level of a fundamental asymmetry of Nature. In contrast, the left-right symmetric models contain the attractive possibility of attributing the observed left-right asymmetry to a low-energy manifestation of a basically symmetric theory. As we shall show, the class of left-right symmetric models encompasses a wide range of possibilities, of which the $VV+AA$ model (e.g., Mohapatra and Sidhu 1977; Ma and Pakvasa 1978), designed to ensure zero parity violation foı neutral current phenomena, is only one singular limit. And the standard model based on $SU(2)_L \otimes U(1)$ is just another limit—namely, the limit of one of the two Z-bosons becoming infinitely heavy. Therefore, it is of interest to study this general class of left-right symmetric models. In particular, we wish to investigate the extent to which the data allows one to deviate from the limit of the standard model, and to analyse the consequences of such deviations for the future neutral current experiments.

We work with the most general neutral current Lagrangian based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$, without making any assumptions about the details of the Higgs sector. In conformity with current thinking, we require that the neutrino-hadron sector of the left-right symmetric models is identical with that of the standard model. We find that the models restricted by this requirement, contain only two free parameters. Further, the space of these two parameters is severely constrained by the positivity conditions inherent in the model. We analyse the consequences of these constraints and one remarkable result is that $\sin^2 \theta_W \leqslant \frac{1}{2}$, where θ_W can be identified with the mixing angle of the standard model.

We then determine the ranges of the two parameters allowed by the two pieces of experimental data: (a) the neutrino-hadron couplings as summarised by the value of $\sin^2 \theta_W$, and (b) the amount of parity violation observed in the *e-d* scattering. We find that these data allow a large region in the parameter-space. This region includes points which are very far away from the standard model limit, and at which the ratio of the masses of the two Z bosons is as small as 1·9. This analysis shows how premature it is to close down our options to a gauge model with one Z boson.

The above surprising result has a simple interpretation. Once the models have been restricted so as to agree exactly with the standard model in the neutrino-hadron sector, all the neutral-current couplings reduce to a form similar to that in the standard model, except for a few multiplicative factors. This close similarity of the couplings does not allow any spectacular differences to appear between the neutral-current phenomena predicted by the left-right symmetric models and the standard model, even when the former contain two Z bosons of comparable mass.

It is important to note that the above conclusion is not likely to be altered even when more neutral current data become available. For, the maximum possible deviations from the standard model are only of the order of 30%. Such deviation will be easily masked by the experimental errors and other uncertainties in the analysis.

We also consider the more general left-right symmetric models in which the neutrino-hadron couplings are not restricted to be identical to those in the standard model. We then have four parameters whose determination from experimental data has to be left for the future. In view of our conclusion in the case of the restricted models, it is clear that the general models are also quite viable at present.

In § 2, we obtain the effective Lagrangian in the neutral-current sector for the general class of left-right symmetric models. In § 3, the Lagrangian for the restricted models is derived by imposing the condition that the couplings in the neutrino-hadron sector agree exactly with the standard-model couplings. The neutral-current couplings in the various sectors following from the restricted models are enumerated in § 4.

The formal constraints on the parameters of the model are studied in § 5, while the implications of the experimental data on these parameters are analysed in § 6. The unrestricted model with four parameters is discussed in § 7. Section 8 is devoted to a summary and main conclusions. The appendix supplements the derivation in § 2.

2. $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models in a general framework

We shall present the neutral-current sector of the left-right symmetric models in a general framework, which does not depend on the details of the symmetry breaking mechanism.

Consider the most general model based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$. The neutral current sector of such a model will consist of three neutral currents* J_L, J_R and J_Y-which are the neutral component of the $SU(2)_L$ current, the neutral component of the $SU(2)_R$ current and the U(1) current, respectively - and their associated neutral gauge bosons W_L, W_R and W_Y. The interaction in this sector is

$$\mathscr{L} = g_L W_L J_L + g_R W_R J_R + g_Y W_Y J_Y \tag{1}$$

where g_L, g_R and g_Y are coupling constants for the gauge groups $SU(2)_L$, $SU(2)_R$ and U(1) respectively. Left-right symmetry implies

$$g_L = g_R = g. \tag{2}$$

After symmetry-breaking, whose details need not be specified, we shall get the physical currents which are linear combinations of J_L, J_R and J_Y. One of these combinations has to be the electromagnetic current given by the generalized Gell-Mann-Nishijima formula:

$$J_{\text{em}} = J_L + J_R + J_Y \tag{3}$$

The other two currents can be, in general, written as

$$J_i = J_L + \alpha_i J_R + \beta_i J_Y \quad (i = 1, 2) \tag{4}$$

where α_i and β_i are real constants. Denoting the corresponding physical vector bosons as A_{em}, Z_i $(i=1, 2)$ the interaction in (1) can be rewritten as

$$\mathscr{L} = e A_{\text{em}} J_{\text{em}} + g_1 Z_1 J_1 + g_2 Z_2 J_2 \tag{5}$$

where e is electromagnetic coupling constant and g_1 and g_2 are the coupling constants for the weak neutral-current interactions. Of course, the symmetry-breaking should be such that A_{em} remains massless while Z_1 and Z_2 gain masses m_1 and m_2.

How are A_{em}, Z_1 and Z_2 related to W_L, W_R, and W_Y? This can be answered without reference to any particular symmetry-breaking mechanism. Use of

*We shall suppress the Lorentz index, for brevity.

(1) and (5) along with the orthonormality relations among the physical fields allows us to read off the expressions for these fields directly as (see appendix):

$$A_{\text{em}} = e\left(\frac{W_L}{g} + \frac{W_R}{g} + \frac{W_Y}{g_Y}\right),$$

$$Z_i = g_i\left(\frac{W_L}{g} + \alpha_i \frac{W_R}{g} + \beta_i \frac{W_Y}{g_Y}\right). \tag{6}$$

Further, using the same orthonormality conditions (6 in number) among A_{em}, Z_1 and Z_2, we can reduce the 8 unknown parameters g^2, g_Y^2, α_1, α_2, β_1, β_2, g_1^2, and g_2^2 to 2 in number. Choosing these free parameters to be α_1 and α_2, we can express all the others in terms of them as:

$$\beta_1 = \frac{1+\alpha_1 \alpha_2}{1+\alpha_2}; \qquad \beta_2 = \frac{1+\alpha_1 \alpha_2}{1+\alpha_1};$$

$$g^2 = e^2 \frac{(1-\alpha_1)(1-\alpha_2)}{1+\alpha_1 \alpha_2}; \qquad g_Y^2 = -e^2 \frac{(1-\alpha_1)(1-\alpha_2)}{(1+\alpha_1)(1+\alpha_2)};$$

$$g_1^2 = e^2 \frac{1-\alpha_2^2}{(1+\alpha_1 \alpha_2)(\alpha_2-\alpha_1)}; \quad g_2^2 = e^2 \frac{1-\alpha_1^2}{(1+\alpha_1 \alpha_2)(\alpha_1-\alpha_2)}. \tag{7}$$

Only the squares of the coupling constants are determined, but these alone are needed for our purpose. The constraints on α_1 and α_2 implied by the positivity of the squares of the coupling constants and their consequences will be studied in § 5.

The observed weak neutral current phenomena arise through the exchange of the two bosons Z_1 and Z_2. The effective current × current form of the weak neutral Lagrangian for zero momentum transfer is

$$\mathscr{L}_{\text{eff}}^{NC} = -\left[\frac{g_1^2}{m_1^2} J_1 J_1 + \frac{g_2^2}{m_2^2} J_2 J_2\right]$$

$$= -\frac{e^2}{m_1^2} \frac{1-\alpha_2^2}{(1+\alpha_1 \alpha_2)(\alpha_2-\alpha_1)} \left[J_1 J_1 + \left(\frac{1-\alpha_1^2}{\alpha_2^2-1}\right) \frac{m_1^2}{m_2^2} J_2 J_2\right] \tag{8}$$

where we have used (7). It is useful to note that

$$\frac{g_2^2}{g_1^2} = \frac{1-\alpha_1^2}{\alpha_2^2-1}. \tag{9}$$

The neutral currents J_1 and J_2 given by (4) can be rewritten (eliminating β_i and J_Y) in the form:

$$J_1 = \frac{1-\alpha_1}{1+\alpha_2}(\alpha_2 J_L - J_R) + \frac{1+\alpha_1 \alpha_2}{1+\alpha_2} J_{\text{em}},$$

$$J_2 = \frac{1-\alpha_2}{1+\alpha_1}(\alpha_1 J_L - J_R) + \frac{1+\alpha_1 \alpha_2}{1+\alpha_1} J_{\text{em}}. \tag{10}$$

The currents J_L, J_R and J_{em} of the fermions can be written down in the usual way, once the classification of the fermions with respect to the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ is specified. We take all the left-handed fermions to be doublets under $SU(2)_L$ and singlets under $SU(2)_R$. A corresponding statement is valid for right-handed fermions. For the right-handed neutral lepton, one may envisage either a heavy lepton or a massless right-handed neutrino. We only keep in mind the fact that the laboratory neutrinos are left-handed since they arise from the usual π and K decays. So, we leave out the right-handed neutral lepton from our equations. Restricting the quarks to u and d, we therefore write

$$
\begin{aligned}
J_L &= \left\{\tfrac{1}{2}\,\bar{\nu}_e\,\gamma_\lambda \frac{1+\gamma_5}{2}\nu_e - \tfrac{1}{2}\,\bar{e}\,\gamma_\lambda \frac{1+\gamma_5}{2} e + (e\to\mu;\ \nu_e\to\nu_\mu)\right\} \\
&\quad + \left\{\tfrac{1}{2}\,\bar{u}\,\gamma_\lambda \frac{1+\gamma_5}{2} u - \tfrac{1}{2}\,\bar{d}\,\gamma_\lambda \frac{1+\gamma_5}{2} d\right\} \\
J_R &= \left\{-\tfrac{1}{2}\,\bar{e}\,\gamma_\lambda \frac{1-\gamma_5}{2} e - \tfrac{1}{2}\,\bar{\mu}\,\gamma_\lambda \frac{1-\gamma_5}{2}\mu\right\} \\
&\quad + \left\{\tfrac{1}{2}\,\bar{u}\,\gamma_\lambda \frac{1-\gamma_5}{2} u - \tfrac{1}{2}\,\bar{d}\,\gamma_\lambda \frac{1-\gamma_5}{2} d\right\} \\
J_{\text{em}} &= -\bar{e}\,\gamma_\lambda\, e - \bar{\mu}\,\gamma_\lambda\,\mu + \tfrac{2}{3}\,\bar{u}\,\gamma_\lambda\, u - \tfrac{1}{3}\,\bar{d}\,\gamma_\lambda\, d
\end{aligned} \tag{11}
$$

The neutral-current Lagrangian (8) with the currents given by (10) and (11) involves four free parameters α_1, α_2, m_1^2 and m_2^2. So, all the neutral-current phenomena in the general class of left-right symmetric models can be described in terms of these four parameters.

3. The neutrino-hadron sector and the restricted models

We now restrict the left-right symmetric models by the requirement that the neutral-current couplings in the neutrino-hadron sector be identical to those predicted by the standard Weinberg-Salam model. Experimentally, the neutrino-hadron sector is the only sector of neutral-current phenomena that has been well-studied. All the four coupling constants in this sector have been determined empirically (see the reviews of Sakurai 1978 and Sehgal 1978). The close agreement of these coupling constants with the values predicted by the standard model indicates that any alternative viable model will have similar structure in this sector. This provides the motivation for considering the restricted class of models.

The effective Lagrangian in the neutrino-hadron sector is simple because the neutrino is left-handed and electrically neutral. For the neutrino currents,

$$J_R^\nu = 0;\quad J_{\text{em}}^\nu = 0; \tag{12}$$

so that, the effective Lagrangian (8), in this sector becomes

$$\mathscr{L}_{\rm eff}^{NC}(\nu-h) = -\frac{e^2}{m_1^2}\frac{(1-a_1)(1-a_2)}{(1+a_1a_2)(a_2-a_1)} J_L^\nu\Bigg[\left\{a_2^2\left(\frac{1-a_1}{1+a_2}\right) - a_1^2\left(\frac{1-a_2}{1+a_1}\right)\frac{m_1^2}{m_2^2}\right\} J_L^h$$

$$+\left\{\frac{a_2(1+a_1a_2)}{1+a_2} - \frac{a_1(1+a_1a_2)}{1+a_1}\frac{m_1^2}{m_2^2}\right\} J_{\rm em}^h$$

$$-\left\{a_2\left(\frac{1-a_1}{1+a_2}\right) - a_1\left(\frac{1-a_2}{1+a_1}\right)\frac{m_1^2}{m_2^2}\right\} J_R^h\Bigg]. \tag{13}$$

On the other hand, the neutrino-hadron sector in the standard model is described by

$$\mathscr{L}_{\rm eff}^{NC}(\nu-h) = -\sqrt{2}\; 4\, G_F J_L^\nu (J_L^h - \sin^2\theta_{\rm W} J_{\rm em}^h), \tag{14}$$

where G_F is the Fermi coupling constant and $\theta_{\rm W}$ is the mixing angle. For the identification of (13) and (14), the parameters of the model have to satisfy the following two conditions:

(i) The coefficient of $J_L^\nu J_R^h$ in (13) should vanish:

$$\frac{m_1^2}{m_2^2} = \frac{a_2}{a_1}\left(\frac{1-a_1^2}{1-a_2^2}\right). \tag{15}$$

(ii) The strength of the coupling should be determined by G_F such that

$$\frac{e^2}{8m_1^2} \times \frac{(1-a_1)(1-a_2)}{1+a_1a_2} \times a_2\left(\frac{1-a_1}{1+a_2}\right) = \frac{G_F}{\sqrt{2}}. \tag{16}$$

The Weinberg-Salam parameter $\sin^2\theta_{\rm W}$ can now be identified with the following combination S, of parameters a_1 and a_2:

$$S = \frac{1+a_1a_2}{(1-a_1)(1-a_2)}. \tag{17}$$

Conditions (15) and (16) determine both m_1^2 and m_2^2 in terms of the parameters a_1 and a_2, which remain as the only two free parameters in the restricted models.

The condition (15) could have also been obtained by insisting that in the mass-matrix for the neutral vector-bosons there is no W_L-W_R mixing term. The absence of this term will be ensured if the symmetry-breaking mechanism is such that it does not cause any $L-R$ mixing, or in other words, there are no Higgs multiplets in the model which transform non-trivially under both $SU(2)_L$ and $SU(2)_R$. Georgi and Weinberg (1978) have shown that under the latter condition the neutirno interactions in a general $SU(2)_L \otimes U(1) \otimes G$ model become identical with those in the standard model. Thus the condition (15) obtained by identifying the neutrino hadron sector of the general model with the standard model, proves the converse of the Georgi-Weinberg theorem in the case of $SU(2)_L \otimes SU(2)_R \otimes U(1)$.

With these conditions, the complete neutral-current Lagrangian of (8) becomes

$$\mathscr{L}_{\text{eff}}^{NC} = -\frac{4}{\sqrt{2}}\, G_F \left[\{J_L - S\, J_{\text{em}}\}^2 - \frac{1}{\alpha_1 \alpha_2} \{J_R - S\, J_{\text{em}}\}^2 \right]. \tag{18}$$

This form of the interaction makes the connection with the standard *WS* model and the parity-conserving *VV+AA* model transparent. The standard model corresponds to $\alpha_1\alpha_2 \to -\infty$, while the *VV+AA* model is obtained by putting $\alpha_1\alpha_2 \to -1$. The continuum of values of $\alpha_1\alpha_2$ generates a variety of other models which we study here.

4. Neutral-current couplings in the restricted models

We are now ready to express the observable neutral current parameters for the various neutral-current sectors in terms of the two parameters α_1, α_2 of the restricted models. We substitute the currents given by (11) into the Lagrangian of (18) and identify the coupling constants in all the neutral-current sectors. For the various couplings, we employ the notation of Hung and Sakurai (1977) with an extension to the nuclear-parity-violation sector as given by Parida and Rajasekaran (1978).

4.1. *Neutrino-hadron sector*

The observables in this sector are α, β, γ and δ defined through the effective interaction

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}\, \gamma_\lambda\, (1+\gamma_5)\, \nu \left[\frac{\alpha}{2} (\bar{u}\, \gamma_\lambda\, u - \bar{d}\, \gamma_\lambda\, d) + \frac{\beta}{2} (\bar{u}\, \gamma_\lambda\, \gamma_5\, u - \bar{d}\, \gamma_\lambda\, \gamma_5\, d) + \frac{\gamma}{2} (\bar{u}\, \gamma_\lambda\, u + \bar{d}\, \gamma_\lambda\, d) + \frac{\delta}{2} (\bar{u}\, \gamma_\lambda\, \gamma_5\, u + \bar{d}\, \gamma_\lambda\, \gamma_5\, d) \right]. \tag{19}$$

As already explained in § 3, the restricted model has been tailored to give the same results for this sector as the standard model and the results are:

$$\alpha = 1 - 2S,\ \beta = 1,\ \gamma = -\tfrac{2}{3}S,\ \delta = 0, \tag{20}$$

where S is defined in (17).

4.2. *ν_μ—e and ν_e—e sector*

The interactions in this sector can be written in terms of two observables g_V and g_A:

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}} \bar{\nu}\ \gamma_\lambda (1+\gamma_5)\ \nu[\bar{e}(g_V \gamma_\lambda + g_A \gamma_\lambda \gamma_5)e]. \tag{21}$$

Since we have identified the ν-hadron sector with the standard model, and thereby satisfied the condition for the validity of the Georgi-Weinberg (1978) theorem, we expect the couplings in this sector also to be the same as in the standard model. We indeed obtain

$$g_V = -\tfrac{1}{2}(1-4S),\ g_A = -\tfrac{1}{2}. \tag{22}$$

4.3. *$e\bar{e} \to \mu\bar{\mu}$ sector*

The observables are h_{VV}, h_{AA} and h_{VA}, defined through

$$\begin{aligned}\mathscr{L} = -\frac{G_F}{\sqrt{2}} [&h_{VV} (\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda \mu) (\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda \mu) \\ &+ 2h_{VA} (\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda \mu) (\bar{e}\gamma_\lambda \gamma_5 e+\bar{\mu}\gamma_\lambda \gamma_5 \mu) \\ &+ h_{AA} (\bar{e}\gamma_\lambda \gamma_5 e+\bar{\mu}\gamma_\lambda \gamma_5 \mu) (\bar{e}\gamma_\lambda \gamma_5 e+\bar{\mu}\gamma_\lambda \gamma_5 \mu)].\end{aligned} \tag{23}$$

We get $\quad h_{VV} = f_{\text{pc}}\ \tfrac{1}{4}(4S-1)^2, \quad h_{AA} = f_{\text{pc}}\ \tfrac{1}{4},\ h_{VA} = f_{\text{pv}}\ \tfrac{1}{4}(1-4S),$

with $$f_{\text{pv}} = \frac{\alpha_1 \alpha_2 + 1}{\alpha_1 \alpha_2}\ ;\ f_{\text{pc}} = 2 - f_{\text{pv}}. \tag{24}$$

4.4. *Parity-violation in atoms and in e—N scattering*

The interaction Lagrangian in this sector is:

$$\begin{aligned}\mathscr{L} = -\frac{G_F}{\sqrt{2}}\Big[(\bar{e}\gamma_\lambda \gamma_5 e) \Big\{&\frac{\tilde{\alpha}}{2}(\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d) + \frac{\tilde{\gamma}}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d) \\ &+ \frac{\tilde{\beta}}{2}(\bar{u}\gamma_\lambda \gamma_5 u - \bar{d}\gamma_\lambda \gamma_5 d) + \frac{\tilde{\delta}}{2}(\bar{u}\gamma_\lambda \gamma_5 u + \bar{d}\gamma_\lambda \gamma_5 d)\Big\}\Big].\end{aligned} \tag{25}$$

For the coupling constants, we get

$$\tilde{\alpha} = f_{\text{pv}}\ (2S-1),\ \tilde{\beta} = f_{\text{pv}}\ (4S-1),\ \tilde{\gamma} = f_{\text{pv}}\ (\tfrac{2}{3}S),\ \delta = 0; \tag{26}$$

where f_{pv} has been defined in (24).

4.5. *Nuclear parity violation*

The interaction can be written in the form

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}}\Big[\frac{\xi}{2}(\bar{u}\gamma_\lambda u-\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda \gamma_5 u-\bar{d}\gamma_\lambda \gamma_5 d)$$

$$+\frac{\eta}{2}(\bar{u}\gamma_\lambda u+\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda \gamma_5 u+\bar{d}\gamma_\lambda \gamma_5 d)$$

$$+\frac{\xi}{2}(\bar{u}\gamma_\lambda u-\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda \gamma_5 u+\bar{d}\gamma_\lambda \gamma_5 d)$$

$$+\frac{\rho}{2}(\bar{u}\gamma_\lambda u+\bar{d}\gamma_\lambda d)(\bar{u}\gamma_\lambda \gamma_5 u-\bar{d}\gamma_\lambda \gamma_5 d)\Big]. \tag{27}$$

We get $\xi = f_{PV}(2-4S),\ \eta=0,\ \xi=0,\ \rho=f_{PV}(-\frac{4}{3}S).$ (28)

We thus find that all the neutral-current couplings in the restricted left-right symmetric models, reduce to a form similar to that in the standard model. In particular, we can draw the following conclusions:

(i) All neutrino-hadron neutral current couplings are the same as in the standard model.

(ii) In all other neutral-current sectors, the parity-violating couplings are the same as in the standard model, but multiplied by the factor f_{PV}.

This conclusion remains valid even when we relax the requirement that the neutrino-hadron couplings be identical to those in the standard model (see § 7). Hence, no model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)$ can explain the discrepancy between the non-vanishing parity-violation observed in the SLAC *e-d* scattering experiment and the negligible parity-violation in bismuth atoms as observed by the Oxford and Washington groups (Lewis *et al* 1977; Baird *et al* 1977).

(iii) The parity-conserving couplings in $e\bar{e}\to\mu\bar{\mu}$ sector are also given by the standard-model forms multiplied by the factor f_{PC}.

Before we proceed to confront the results of this section with experiment, we have to analyse the formal constraints on the parameters α_1 and α_2 following from the positivity conditions inherent in the model. This will also enable us to obtain the allowed range of values for S and f_{PV}.

5. Constraints on the parameters

The parameters α_1 and α_2 which we have introduced cannot in fact assume arbitrary values. The parameter-space is constrained by the positivity conditions imposed by the reality of the coupling constants e, g_1 and g_2 defined in (7). The three independent conditions are*:

$$\frac{1+\alpha_1\alpha_2}{(1-\alpha_1)(1-\alpha_2)} \geqslant 0,\ \frac{1+\alpha_2}{(1-\alpha_1)(\alpha_2-\alpha_1)} \geqslant 0,\ \frac{1+\alpha_1}{(1-\alpha_2)(\alpha_1-\alpha_2)} \geqslant 0. \tag{29}$$

*The positivity of g_Y^2/g^2 does not impose any new condition.

These are the only conditions for the general left-right symmetric models. For the restricted models having the neutrino-hadron sector identical with the standard model, the positivity of m_1^2/m_2^2 (eq. (15)) imposes the additional condition

$$\frac{\alpha_2}{\alpha_1}\left(\frac{1-\alpha_1^2}{1-\alpha_2^2}\right) \geqslant 0. \tag{30}$$

These four conditions constrain the allowed parameter space for the restricted models to the following four regions:

$1+\alpha_1\alpha_2>0$;	$0\leqslant\alpha_1<1$;	$\alpha_2<-1$	I
$1+\alpha_1\alpha_2>0$;	$0\leqslant\alpha_2<1$;	$\alpha_1<-1$	II
$1+\alpha_1\alpha_2<0$;	$-1<\alpha_1\leqslant 0$;	$\alpha_2>1$	III
$1+\alpha_1\alpha_2<0$;	$-1<\alpha_2\leqslant 0$;	$\alpha_1>1$	IV

These four regions are depicted as dark-shaded regions in figure 1. The regions are bounded by the rectangular hyperbola $1+\alpha_1\alpha_2=0$ as well as by the straight lines $\alpha_1=0$; $\alpha_2=0$; $\alpha_1=-1$ and $\alpha_2=-1$.

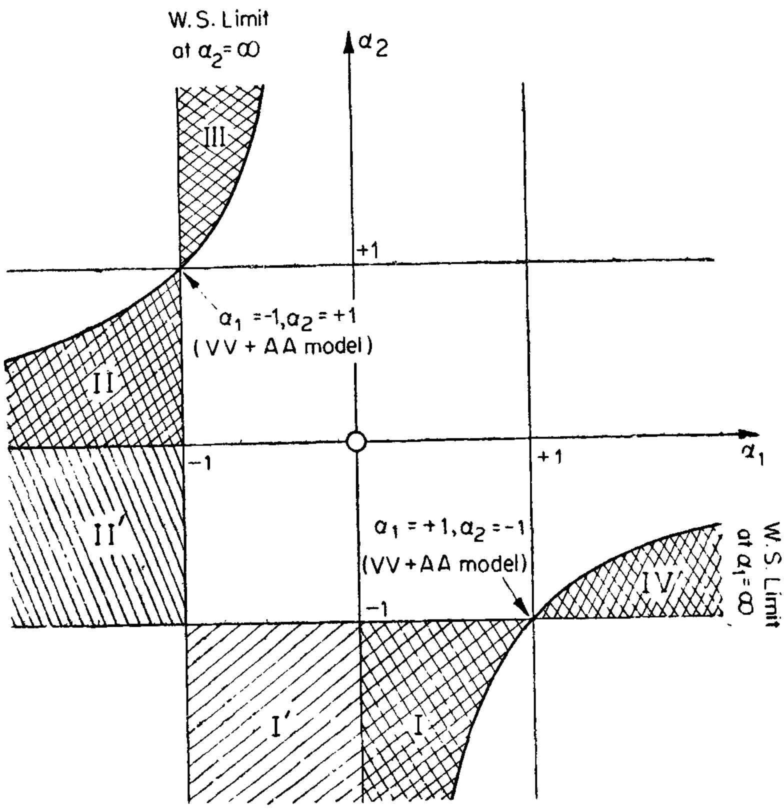

Figure 1. Allowed region in the $\alpha_1-\alpha_2$ plane. I, II, III and IV are allowed in the restricted models. I′ and II′ are also allowed in the general models.

For the general models in which no identification with the standard model is made, the condition (30) is not applicable. In these models, the allowed region gets enlarged by the following additions:

$$-1<\alpha_1<0; \quad \alpha_2<-1, \qquad \text{I}'$$

$$-1<\alpha_2<0; \quad \alpha_1<-1. \qquad \text{II}'$$

These enlargements are shown as the light-shaded regions I′ and II′ in figure 1.

What are the consequences of these constraints on the parameters of physical interest S, f_{pv} and f_{pc}? Using the expressions

$$S = \frac{1+\alpha_1\alpha_2}{(1-\alpha_1)(1-\alpha_2)}; \quad f_{\text{pv}} = \frac{\alpha_1\alpha_2+1}{\alpha_1\alpha_2}; \quad f_{\text{pc}} = \frac{\alpha_1\alpha_2-1}{\alpha_1\alpha_2};$$

it is easy to get the following bounds:

$$\left.\begin{array}{l} 0 \leqslant S < \frac{1}{2} \\ -\infty \leqslant f_{\text{pv}} < 0 \\ 2 < f_{\text{pc}} \leqslant \infty \end{array}\right\}, \qquad \text{in regions I and II}$$

$$\left.\begin{array}{l} 0 \leqslant S < \frac{1}{2} \\ 0 < f_{\text{pv}} \leqslant 1 \\ 1 \leqslant f_{\text{pc}} < 2 \end{array}\right\}, \qquad \text{in regions III and IV}$$

$$0 < S < \tfrac{1}{2}. \qquad \text{in regions I}' \text{ and II}'$$

We thus obtain the important result that the parameters S which plays the role of $\sin^2\theta_{\text{W}}$ in the left-right symmetric models is bounded to lie between 0 and $\frac{1}{2}$ in both the general as well as the restricted version of these models. It is to be stressed that this restriction on $\sin^2\theta_{\text{W}}$ is a consequence of the general group-structure of the model alone. The fact that the empirical value of $\sin^2\theta_{\text{W}}$ is known to be smaller than $\frac{1}{2}$ is an indication that the extension of the gauge group to $SU(2)_L \otimes SU(2)_R \otimes U(1)$ is a step in the right direction.

For the restricted models, the parity-violating parameter f_{pv} is bounded by $-\infty$ and 1, while the parity-conserving parameter f_{pc} is bounded by 1 and ∞. (For the general models, these parameters do not have any significance.) So, for positive f_{pv}, we cannot get more parity-violation than in the standard Weinberg-Salam model.

It is useful to have some idea of the physical significance of the various locations in the $\alpha_1-\alpha_2$ plane. The point $\alpha_1=\alpha_2=1$ would make all the three currents purely vector which is disallowed. The point $\alpha_1=\alpha_2=-1$ is not allowed in the restricted models, but is allowed as a limiting case in the general models.

The lines $\alpha_1=-1$ and $\alpha_2=-1$ are allowed as limiting cases. But the lines $\alpha_1=+1$ and $\alpha_2=+1$ which correspond to one of the weak neutral currents being pure vector are in general not allowed, except for one point on each of these lines—namely

$\alpha_1=+1$, $\alpha_2=-1$, and $\alpha_2=+1$, $\alpha_1=-1$, which correspond to the other current becoming a pure axial vector. These are precisely the $VV+AA$ models in which the neutral-current interaction is pure parity-conserving. We thus see again that the $VV+AA$ models belong to just one specific limiting case of the general class of models studied here.

Where does the Weinberg-Salam model come in this picture? As is clear from (18), this model corresponds to the limit $\alpha_1\alpha_2\to-\infty$. In this limit, the effective Lagrangian becomes

$$\mathscr{L}_{\text{eff}}^{NC}=-\frac{4}{\sqrt{2}}G_F\{J_L-SJ_{\text{em}}\}^2$$

which is identical with the standard model. In the $\alpha_1-\alpha_2$ plane (figure 1), this limit is reached on the lines

$$\alpha_2\to\infty;\qquad -1\leqslant\alpha_1\leqslant 0;$$

and

$$\alpha_1\to\infty;\qquad -1\leqslant\alpha_2\leqslant 0.$$

which are the lines bounding regions III and IV respectively, at infinity. The mass-ratio m_2^2/m_1^2 (eq. (15)) becomes infinite on the $\alpha_2\to\infty$ line and it vanishes on the $\alpha_1\to\infty$ line, so that in this limit one of the two neutral bosons is necessarily of infinite mass, as expected. The physical parameters S, f_{pv} and f_{pc} on these lines are given by

$$0\leqslant S\leqslant\tfrac{1}{2};\quad f_{\text{pv}}=1;\quad f_{\text{pc}}=1.$$

Thus, in this limit, the model reduces to the standard model in all its aspects, except that S is constrained to be less than $\frac{1}{2}$.

6. Implications of the experimental data

So far we have not used any experimental data. The constraints on α_1, α_2 discussed in the previous section follow from formal requirements alone. In this section we study the further limitations imposed on the parameters α_1 and α_2 by the experimental data. There are essentially two independent pieces of data available so far. The neutrino-hadron data fixes one combination S of the parameters, and the SLAC result (Prescott *et al* 1978) on the polarisation asymmetry in *e-d* scattering can be used to determine another combination, f_{pv}.

Extensive measurements have been carried out in the ν-hadron sector, and a fairly accurate value of $\sin^2\theta_W$ has been obtained. We use the recent BEBC number, 0·22±0·05 (Bosetti *et al* 1978). This fixes

$$S=\frac{1+\alpha_1\alpha_2}{(1-\alpha_1)(1-\alpha_2)}=0{\cdot}22\pm0{\cdot}05. \tag{31}$$

The asymmetry-parameter A measured in the SLAC experiment is defined by

$$A = -\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \tag{32}$$

where σ_L and σ_R are the cross-sections for the scattering of positive and negative helicity electrons, respectively, off unpolarised deuteron target. For the electron-hadron interaction specified in (25), quark-parton model gives (Cahn and Gilman 1978; Sakurai 1978):

$$A = \frac{G_F}{\sqrt{2}e^2}\frac{9}{5}q^2\left[\left(\tilde{\alpha}+\frac{\tilde{\gamma}}{3}\right)+\left(\tilde{\beta}+\frac{\tilde{\delta}}{3}\right)\frac{1-(1-y)^2}{1+(1-y)^2}\right] \tag{33}$$

where q^2 is the square of the momentum transfer and y is the ratio of the energy-loss of the electron to its initial energy. In the context of our models (see eq. (26)) this becomes

$$A = \frac{G_F}{\sqrt{2}e^2}\frac{9}{5}f_{\text{pv}}\,q^2\left[\left(\frac{20}{9}S-1\right)+(4S-1)\frac{1-(1-y)^2}{1+(1-y)^2}\right]. \tag{34}$$

The experimental result measured at $y \approx 0{\cdot}2$ is

$$A = -(9{\cdot}5 \pm 1{\cdot}6)\times 10^{-5}\,(q/\text{GeV})^2. \tag{35}$$

Comparison of (34) and (35) gives the numerical value of another combination of our basic parameters:

$$f_{\text{pv}}(3{\cdot}10S - 1{\cdot}22) = -0{\cdot}59 \pm 0{\cdot}10. \tag{36}$$

By using (31) in (36), we can extract f_{pv}. This is done by the graphical method in figure 2. In the S—f_{pv} plane, (31) would represent a vertical straight line while (36) would correspond to a hyperbola. The experimental errors convert the former into a region bounded by two vertical straight lines and the latter into a region bounded by two hyperbolae shown in figure 2. The overlap of these two regions gives the allowed values of S and f_{pv}. As shown in the previous section there is a further restriction $f_{\text{pv}} \leqslant 1$ required by the model. Therefore, the allowed region is actually bounded by three lines, two of them being the straight lines $S{=}0{\cdot}17$ and $f_{\text{pv}} = 1$, and the third being the hyperbola:

$$f_{\text{pv}}(-1{\cdot}22+3{\cdot}10\,S) = -0{\cdot}49, \tag{37}$$

and this is shown as the shaded region in figure 2. The allowed values thus obtained are seen to be

$$0{\cdot}17 \leqslant S \leqslant 0{\cdot}23;\; 0{\cdot}7 \leqslant f_{\text{pv}} \leqslant 1, \tag{38}$$

but the region is far from a rectangle.

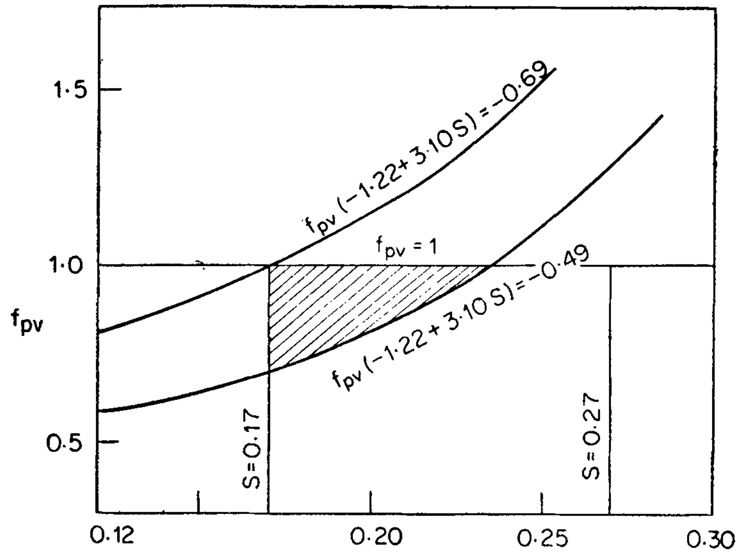

Figure 2. Allowed region in the S—f_{pv} plane, using neutrino hadron and electron-deuteron data.

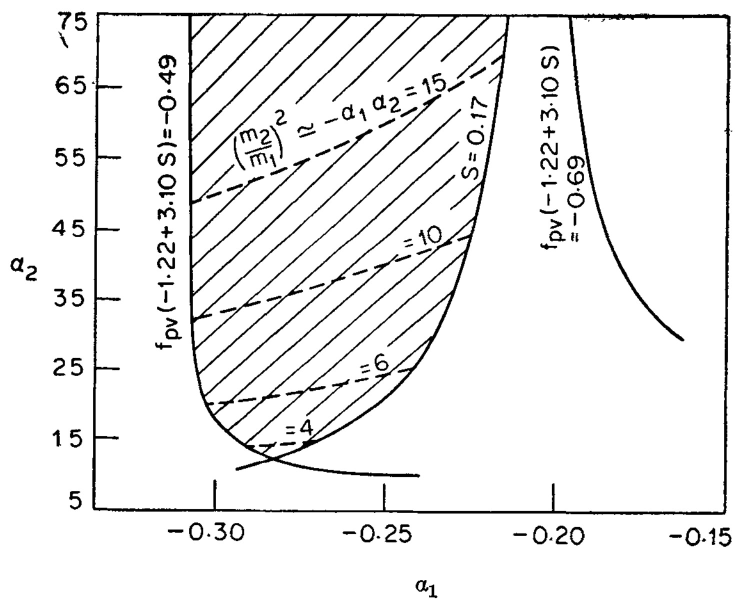

Figure 3. Experimentally allowed region in the α_1—α_2 plane. This allowed region lies in region III of figure 1. Dotted lines denote contours of equal $\alpha_1\alpha_2 \simeq -\left(\frac{m_2}{m_1}\right)^2$.

Finally, the allowed region in the S—f_{pv} plane can be converted into the allowed region in the α_1—α_2 plane. This is shown as the shaded region in figure 3. We have chosen $\alpha_2 \geqslant \alpha_1$ which does not involve any loss of generality. The experimentally allowed region shown in figure 3 lies in region III of figure 1. The allowed region is bounded by the two curves given by $S=0{\cdot}17$ and (37), and the third boundary line, $f_{pv}=1$, is at $\alpha_2 \to \infty$. An approximate representation of this region is given by the numerical values:

$$-0{\cdot}31 \leqslant \alpha_1 \leqslant -0{\cdot}21$$
$$12 \leqslant \alpha_2 \leqslant \infty. \tag{39}$$

The values of α_1 and α_2 determine the restricted model completely. In particular, the masses m_1 and m_2 of the neutral vector bosons are determined by α_1 and α_2. Contours of equal m_2^2/m_1^2 are shown as dotted lines in figure 3. It is seen that present data allow m_2/m_1 to be as small as 1·9.

Maximal deviation from the standard model allowed by the experimental data occurs at the lowest point in the allowed region of figure 3 and at that point we have

$$\alpha_1 \approx -0{\cdot}28; \quad \alpha_2 \approx 12. \tag{40}$$

This corresponds to a model with two Z bosons whose masses are given by

$$m_1 \approx 98 \text{ GeV}; \quad m_2 \approx 186 \text{ GeV}, \tag{41}$$

where we have used (15) and (16).

For the above masses, the model is physically very different from the standard model in which one of the bosons must be infinitely heavy. Nevertheless, we find that even at this extreme point, the observable neutral-current couplings deviate from the standard model by only about 30%. At this point

$$f_{pv} \approx 0{\cdot}7; \quad f_{pc} \approx 1{\cdot}3. \tag{42}$$

So, all the parity-violating coupling constants given in §4 are decreased by 30% while the parity-conserving coupling constants h_{VV} and h_{AA} are increased by 30%, compared to their standard model values. If the accuracy of the presently available neutral-current data is any indication, it may not be possible to discriminate between the left-right symmetric models and the standard model by any future neutral-current data.

7. Neutral currents in the unrestricted models

So far we have restricted the models by requiring the ν-hadron sector of these models to be identical with the standard model. We found that this restriction was sufficient to make the models closely mimic the standard model in all sectors of neutral current induced phenomena. However, the experimental values of the couplings in the ν-hadron sector being accurate upto only $\pm 20\%$, it is not essential to insist upon the identification with the standard model in this sector. One can expect a richer behaviour from the general models in which this identification is not made. These more general models will depend on four parameters m_1^2, m_2^2, α_1 and α_2. Experimentally, in effect 4 pieces of data are already available—the three nonvanishing ν-hadron couplings, and one number from the parity violation in e—d scattering.

ing. Hence, in principle, all the four parameters of the model can be determined. But, the large errors in the data do not seem to warrant such an analysis at the present stage. We shall present a brief account of the neutral-current couplings in this generalized model here.

It is convenient to use the four parameters $\triangle$, R, α_1, and α_2, where $\triangle$ and R are defined through the equations

$$\triangle = \frac{m_1^2}{m_2^2} - \frac{\alpha_2}{\alpha_1}\left(\frac{1-\alpha_1^2}{1-\alpha_2^2}\right), \tag{43}$$

$$R = \frac{\sqrt{2}}{G_F}\frac{e^2}{8m_1^2}\frac{(1-\alpha_1)(1-\alpha_2)}{1+\alpha_1\alpha_2} \times \alpha_2\left(\frac{1-\alpha_1}{1+\alpha_2}\right). \tag{44}$$

Comparison with (15) and (16) shows that $\triangle \neq 0$ and $R \neq 1$ are measures of the deviation of the ν-induced phenomena from the standard model. In addition we use

$$S = \frac{1+\alpha_1\alpha_2}{(1-\alpha_1)(1-\alpha_2)}, \tag{45}$$

although S can no longer be identified with the $\sin^2\theta_W$ of the standard model. Note that S is restricted to lie between 0 and $\frac{1}{2}$ even in the unrestricted models (see §5).

We now substitute the currents given by (10) and (11) into the Lagrangian of the unrestricted models given by (8) and identify the coupling constants in all the neutral current sectors. The results are the following:

(i) *ν-hadron sector*

$$\alpha = \rho_V\,[1-2S] \qquad \text{where} \qquad \rho_V = R\left(1-\frac{1-\alpha_1}{\alpha_1-\alpha_2}\triangle\right),$$

$$\beta = \rho_A$$

$$\gamma = \rho_V\,[-\tfrac{4}{3}\,S], \qquad \rho_A = R\left(1+\frac{1+\alpha_1}{\alpha_1-\alpha_2}\triangle\right). \tag{46}$$

$$\delta = 0.$$

(ii) *$\nu_\mu - e$ and $\nu_e - e$ sector*

$$g_V = -\tfrac{1}{2}\,\rho_V(4S-1), \text{ and } g_A = -\tfrac{1}{2}\,\rho_A \tag{47}$$

with ρ_V and ρ_A as defined in (46)

(iii) *$e\bar{e} \to \mu\bar{\mu}$ sector*

$$h_{VV} = f_{VV}\,\tfrac{1}{4}(4S-1)^2, \qquad \text{where} \qquad f_{VV} = R\,\frac{\alpha_1\alpha_2-1}{\alpha_1\alpha_2}\left[1-\frac{\alpha_2(1-\alpha_1)^2}{(\alpha_1-\alpha_2)(1-\alpha_1\alpha_2)}\triangle\right]$$

$$h_{AA} = f_{AA}\,\tfrac{1}{4} \quad , \quad \text{where} \quad f_{AA} = R\,\frac{\alpha_1\alpha_2 - 1}{\alpha_1\alpha_2}\left[1 - \frac{\alpha_2(1+\alpha_1)^2}{(\alpha_1-\alpha_2)(1-\alpha_1\alpha_2)}\,\Delta\right]$$

$$h_{VA} = f'_{\text{pv}}\,\tfrac{1}{4}\,(1-4S), \quad \text{where} \quad f'_{\text{pv}} = R\,\frac{\alpha_1\alpha_2+1}{\alpha_1\,\alpha_2}\left[1 - \frac{\alpha_2(1-\alpha_1^2)}{(\alpha_1-\alpha_2)(1+\alpha_1\alpha_2)}\,\Delta\right] \tag{48}$$

(iv) *Parity violation in atoms and in e—N scattering*

$$\tilde{\alpha} = f'_{\text{pv}}\,(2S-1), \tilde{\beta} = f'_{\text{pv}}\,(4S-1), \tilde{\gamma} = f'_{\text{pv}}\,\tfrac{2}{3}\,S, \tilde{\delta} = 0 \tag{49}$$

where f'_{pv} is as defined in (48)

(v) *Nuclear parity-violation sector*

$$\xi = f'_{\text{pv}}\,(2-4S),\ \eta = 0,\ \zeta = 0, \rho = f'_{\text{pv}}\,(-\tfrac{4}{3}S) \tag{50}$$

where f'_{pv} is again as defined in (48).

The following observations are of interest:

(i) In the neutrino-induced neutral-current phenomena, all couplings can be written in the standard model form except for a factor ρ_V multiplying all vector couplings and ρ_A multiplying all axial vector couplings.

(ii) Parity-violating couplings in all other sectors can be written as the standard model couplings multiplied by a scale factor f'_{pv}. This is the same result as in the restricted models, but the factor f'_{pv} is different from f_{pv}.

As already mentioned, in view of (ii), no model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)$ can explain the discrepancy between the zero parity violation in bismuth and the non-zero asymmetry in $e—d$ scattering. It is important to note that the restriction, $0 \leqslant S \leqslant \frac{1}{2}$, which is valid even in the unrestricted left-right symmetric models, is essential for reaching this conclusion. This conclusion will not hold if S could acquire negative values. In that case, the ' weak charge ' (Bouchiat and Bouchiat 1974),

$$Q_{\text{w}}\,(\text{Bi}) = -\,f'_{\text{pv}}\,(332\,S + 43), \tag{51}$$

could become zero without f'_{pv} being zero.

In view of results (i) and (ii), a better procedure for determining the parameter S is to use the ratio of two-vector or two axial-vector couplings in the neutrino sectors, or the ratio of any two pieces of data in the parity-violation sectors. From the values of α and γ in the neutrino-hadron sector;

$$\alpha = 0{\cdot}58 \pm 0{\cdot}14; \gamma = -\,0{\cdot}28 + 0{\cdot}14 \tag{52}$$

(Solution A of Sakurai 1978); we have, using (46)

$$S = \frac{3\gamma}{6\gamma - 2\alpha} = 0{\cdot}28 \pm 0{\cdot}09. \tag{53}$$

Hopefully, more data on e—d scattering for different values of y will soon be available. This will enable us to determine S from this sector also, by using

$$A = \frac{G_F}{\sqrt{2}e^2} \frac{9}{5} f_{\rm pv}' q^2 \left[\left(\frac{20}{9} S - 1 \right) + (4S-1) \frac{1-(1-y)^2}{1+(1-y)^2} \right]. \tag{54}$$

Consistency between the S values determined in the two different sectors would be a very good check on the viability of this whole class of models which includes the standard model also.

8. Summary and conclusions

We have investigated the consequences of the gauge models based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ for the neutral-current phenomena in various sectors. If we restrict the models by the requirement that the neutral-current couplings in the neutrino-hadron sector agree exactly with those in the standard Weinberg-Salam model, then we have only two free parameters.

An analysis of these two-parameters models yields the following results:

(i) The parameter S which is analogous to the $\sin^2 \theta_w$ of the standard model is restricted to lie between 0 and $\frac{1}{2}$. This is found to be a general consequence of the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ group.

(ii) All the parity-violating couplings in the $e\bar{e} \rightarrow \mu\bar{\mu}$, $eh \rightarrow eh$ and $hh \rightarrow hh$ sectors (where h refers to hadron) are the same as in the standard model, but multiplied by the factor $f_{\rm pv}$.

(iii) The parity-conserving couplings in the $e\bar{e} \rightarrow \mu\bar{\mu}$ sector are also given by the standard-model forms multiplied by the factor $f_{\rm pc} = 2 - f_{\rm pv}$.

These quantities, S and $f_{\rm pv}$, can be regarded as the two parameters of the model and comparison with experimental data in the neutrino-hadron sector and the parity-violation measurement in e-d scattering determines these two parameters to be the following:

$$S = 0{\cdot}20 \pm 0{\cdot}03,$$

$$f_{\rm pv} = 0{\cdot}85 \pm 0{\cdot}15.$$

Reinterpreting these numbers in terms of the masses of the two Z bosons which mediate the neutral-current interactions in these models, we find that the ratio of the masses of the heavier to the lighter boson lies in the range:

$$1{\cdot}9 \leqslant (m_2/m_1) \leqslant \infty.$$

For the minimum value 1·9 of the ratio (m_2/m_1) allowed by the present experimental data, the model is physically very different from the standard model for which this ratio is infinite. Nevertheless, because of the results (ii) and (iii) mentioned above even for this extreme model, the observable neutral-current couplings deviate from the standard model by only about 30%. So, these models are not likely to be ruled out by experimental data on the various neutral-current sectors.

In the more general $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models in which the neutrino-hadron couplings are not restricted to be equal to those in the standard model, there are four parameters to be determined from experiment. However, the conclusions to be drawn for these models are qualitatively the same as in the case of the restricted models.

How then are the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models to be established or ruled out by experiment? Short of discovering the two Z bosons, there may be two ways:

(i) Looking for deviations from the $V-A$ form in the charged-current sector and studying them.

(ii) Studying neutral-current phenomena at higher q^2, i.e., at q^2 not negligible compared to the squares of the Z boson masses.*

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Appendix

The expressions for the physical A and Z fields given in (6) are special cases of the following simple result (Rajasekaran 1978).

Given an interaction Lagrangian of the form:

$$\mathscr{L} = \sum_i g_i j_i W_i, \tag{A.1}$$

where g_i are coupling constants, j_i are currents and W_i are boson fields. After symmetry breaking, let the physical boson fields $\tilde{W}_i$ and the associated currents be given by some linear combinations:

$$\tilde{j}_k = \sum_i a_{ki} j_i, \tag{A.2}$$

$$\tilde{W}_k = \sum_i b_{ki} W_i, \tag{A.3}$$

*After a major part of this work was completed we received the preprints by Pati and Rajpoot 1978), Costa *et al* (1978) and Leide *et al* (1978) whose results have a partial overlap with ours.

such that the same Lagrangian can be rewritten as

$$\mathscr{L} = \sum_k \tilde{g}_k \tilde{j}_k \tilde{W}_k \tag{A.4}$$

where $\tilde{g}_k$ are the physical coupling constants. One can then show that b_{ki} are related to a_{ki} as follows:

$$b_{ki} = \frac{\tilde{g}_k}{g_i} a_{ki} \tag{A.5}$$

The dependance on the inverse of the coupling constants is specially to be noted. The expressions in (6) in the text follow from (A.5).

Although it is easy to give a direct proof of (A.5) based on the orthogonality relation

$$\sum_l b_{kl} b_{ml} = \delta_{km}, \tag{A.6}$$

the following argument is more appealing. The equality of (A.1) and (A.4) can be interpreted as the invariance of the scalar product of the two vectors $\{W_i\}$ and $\{g_i j_i\}$. The transformation of the field-vector $\{W_i\}$ given by (A.3) is orthogonal. For invariance of the scalar product, or in other words, for the equality of the Lagrangian in (A.1) and (A.4), the current-vector $\{g_i j_i\}$ should also transform by the same orthogonal matrix $\{b_{ki}\}$; i.e.,

$$g_k \tilde{j}_k = \sum_l b_{kl} g_l j_l \tag{A.7}$$

Comparison with (A.2) leads to the desired result (A.5).

References

Baird P E G *et al* 1977 *Phys. Rev. Lett.* **39** 798
Bosetti P C *et al* 1978 *Phys. Lett.* **B76** 505
Bouchiat M A and Bouchiat C C 1974 *Phys. Lett.* **B48** 111
Cahn R N and Gilman F J 1978 *Phys. Rev.* **D17** 1313
Costa G, D'Anna M and Marcolungo P 1978 University di Padova Preprint IFPD 5/78
De Rujula A, Georgi H and Glashow S L 1977 *Ann. Phys.* (*N.Y.*) **109** 242
Georgi H and Weinberg S 1978 *Phys. Rev.* **D17** 275
Hung P Q and Sakurai J J 1977 *Phys. Lett.* **B69** 323
Leide I, Maalampi J and Roos M 1978 University of Helsinki Preprint HU-TFT-78-11
Lewis L L *et al* 1977 *Phys. Rev. Lett.* **39** 795
Ma E and Pakvasa S 1978 *Phys. Rev.* **D17** 1881
Mohapatra R N and Sidhu D P 1977 *Phys. Rev. Lett.* **38** 667
Parida M K and Rajasekaran G 1978 Madras University Preprint MUTP/78/5
Pati J C, Rajpoot S and Salam A 1978 *Phys. Rev.* **D17** 131
Pati J C and Rajpoot S 1978 ICTP Preprint IC/78/71

Prescott C Y *et al* 1978 Stanford Preprint SLAC-PUB-2148

Rajasekaran G 1978 Madras University Preprint MUTP/78/10 (Lectures given at the TIFR Winter School, Panchagani, December 1977)

Sakurai J J 1978 University of California, Los Angeles Preprint UCLA/78/TEP/18 (Invited paper in the Conference on 'Neutrino Physics at Accelerators', Oxford, July 3-7, 1978)

Salam A 1968 Elementary particle theory ed N Svartholm (Stockholm: Almquist and Wiksells) p 367

Sehgal L M 1978 Aachen Preprint PITHA-NR 102 (1978) (Invited talk at the 'Neutrino-78', Purdue University, April 28-May 2, 1978)

Weinberg S 1967 *Phys. Rev. Lett.* **19** 1264