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Negative parity states in ^{144}Nd and multipolarity of gamma-transitions from 1$^-$ to 2$^+$ states by beta-gamma-gamma angular correlation method

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Abstract. Some of the low-lying states in many isotopes ^{144}Nd, ^{148}Sm, ^{152}Gd and ^{156}Gd show a similar typical behaviour. The first 2$^+$ is regarded as a single quadrupole phonon state and 3$^-$ as a single octupole phonon state. The levels with the spins and parities 1$^-$, 5$^-$, 3$^-$, 4$^-$, etc. are considered due to the simultaneous excitation of quadrupole and octupole phonons. If this consideration is correct, then the transition from J^- to 2$^+$ states must contain an appreciable $E3$ content. The β-γ-γ angular correlation coefficients for the cascade of β-rays of $E_{\max}$ 800 keV$\rightarrow\gamma$-rays of 1489 keV$\rightarrow\gamma$-rays of 696 keV are used to estimate $E3$ content in $E1$ transition in ^{144}Nd.

Keywords. Negative parity states; radioactivity of ^{144}Nd; gamma transitions; multipolarity.

1. Introduction

Some of the low lying states in many isotopes (^{144}Nd, ^{148}Sm, ^{152}Sm, ^{152}Gd and ^{156}Gd) show a similar typical behaviour. The first 2$^+$ is regarded as a single quadrupole phonon state and 3$^-$ as a single octupole phonon state. The levels with the spins and parities of 1$^-$, 5$^-$, 3$^-$, 4$^-$ are around an energy which is very close to the sum of energies of 2$^+$ and 3$^-$ states. Therefore, these negative parity states are considered due to simultaneous excitation of quadrupole and octupole phonon (Bhatt 1965; Raman 1968 and Behar *et al* 1974). Bhatt (1965) has calculated the spectrum considering the coupling of quadrupole to octupole phonon by giving the wave-function of this state as

$$|J^-\rangle = |[2^+, 3^-]\, J^-\rangle, \quad J = 1, 2, 3, 4, 5,$$

where the bracket [2$^+$, 3$^-$] indicates vector coupling of the angular momenta 2 and 3 to give the resultant J. The energy splitting of these negative parity states is given by

$$\triangle E_J = -\,CW\,(J232;\,32) \tag{1}$$

where $W(J232;\,32)$ is the Racah coefficient and C is a constant which determines the strength of interaction. This was compared with the experimental result and found

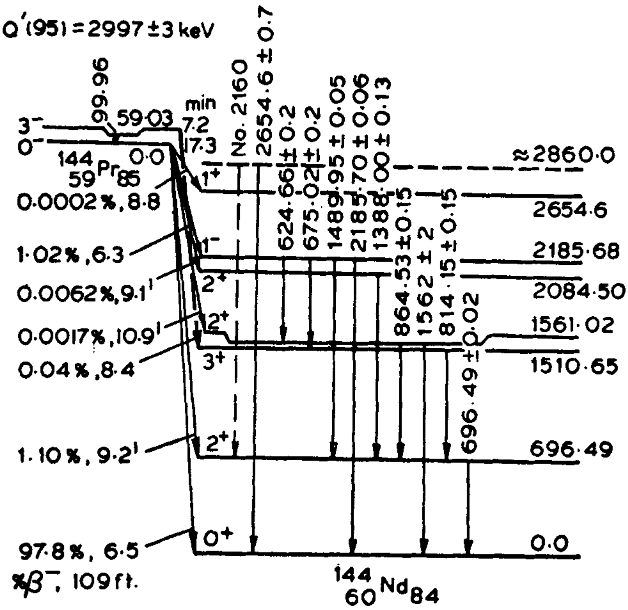

Figure 1. Level scheme for ^{144}Nd from ^{144}P decay (all energies in keV)

to be valid in ^{148}Sm. One of the important predictions made by Bhatt (1965) was that the transition from any member $|J^-\rangle$ of quadrupole-octupole multiplet to the quadrupole $|2^+\rangle$ state would occur through the collapse of the octupole phonon. Therefore, the corresponding transition should have an appreciable $E3$ content and the measurement of $E3$ will possibly provide a clue for this type of coupling and formation of the negative parity states.

One of the interesting cases is in ^{144}Nd from the decay of ^{144}Pr formed from the decay of ^{144}Ce. The decay scheme and level sequence, shown in figure 1, are well established by many investigators (Raman 1967; Raman 1968; Behar *et al* 1974; Rao 1976). The 1^- level at 2185·68 keV lies very near the sum of energies of 2^+ state at 696·49 keV and 3^- state at 1510·65 keV and therefore, the study of γ-transition from 1^- to 2^+ states may provide the necessary information.

There are indirect ways of estimating the higher multipole content in γ-transition. The more prevalent methods are based on

(i) internal conversion coefficient data,

(ii) half life measurements i.e. considering the transition probabilities,

(iii) angular correlation data.

The decay being complicated, the methods (i) and (ii) are difficult to be tried. The γ-γ angular correlation method is also not clear due to interference of unwanted γ-γ cascades. But these unwanted cascades can be avoided by β-γ-γ angular correlation. With this in view, the present study is undertaken.

2. Experimental set-up and results

The three detectors—one plastic scintillator (3 cm in diameter and 0·3 cm in length) and two NaI(Tl) detectors (3·8 cm in diameter and 3·8 cm in length)—along with spectrometers are used. These detectors are optically coupled with RCA 6810 A photomultiplier tubes. Conventional slow-fast coincidence circuits have been used for making a gate of coincidences of β- and γ-rays. This gate is used as one of the inputs of the mixer type coincidence unit with a resolving time of the order of 5×10^{-8} sec. The second input to this unit is from the third γ-ray movable spectrometer.

The detectors are mounted in the plane of the table such that the plastic scintillator in vacuum and one of thε NaI(Tl) detectors are perpendicular to each other and placed at distances 3·5 cm and 6 cm respectively from the source (also in vacuum) while the third movable NaI(Tl) detector is placed at 6 cm from the source. The source in the form of $CeCl_3$ in dilute HCl solution was obtained from the Bhabha Atomic Research Centre, Bombay. ^{144}Ce decays through β^- emission with a half life of 284 days to ^{144}Pr which in turn decays to ^{144}Nd. Few drops of the source were dried on the cellotape and is mounted on the perspex stand at the centre of rotation of the three detectors. The source on cellotape along with the stand is kept in the vacuum chamber.

β-γ ray (selecting γ-rays at the photopeak of 1489 keV in a fixed detector and using β-ray spectrometer as integral above 108 keV energy) coincidences are obtained using slow-fast coincidence set-up. Then the β-γ coincidences form the gate for one of the inputs of the other coincidence (mixer-type) and the second input is from the movable γ-ray spectrometer detecting γ-rays in the photopeak of 696 keV. The output of this coincidence unit gives the β-γ-γ coincidence spectrum which is shown in figure 2. The angular correlation study is done selecting 696 keV γ-ray in $4V$ channel width ($1V=30{\cdot}9$ keV). The movable detector is kept at several angles between 90° to 180° at the intervals of 22·5°.

The angular correlation function $W(\theta)$ obtained by the method of least square fit (without applying solid angle correction considered in the theoretical calculations) for the cascade of β-rays of E_{max} 800 keB → γ-rays of 1489 keV→ γ-rays 696 keV, is

$$W(\theta) = 1 + (0{\cdot}219 \pm 0{\cdot}019)\, P_2 (\cos\theta) + (0{\cdot}069 \pm 0{\cdot}023)\, P_4 (\cos\theta).$$

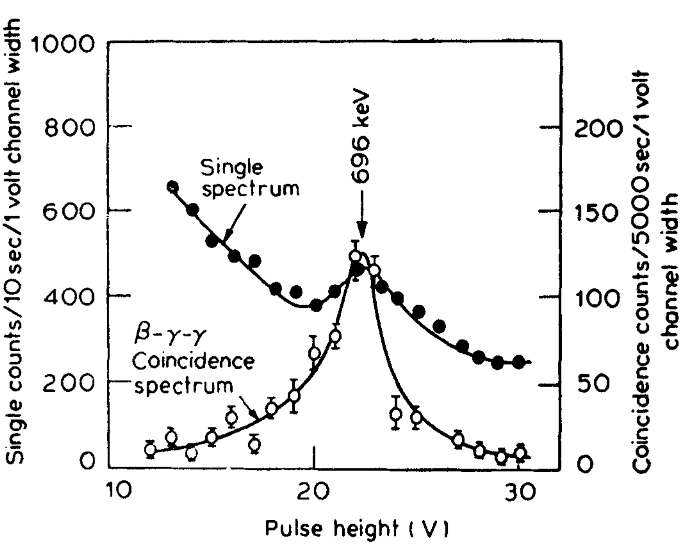

Figure 2. β-γ-γ coincidence spectrum along with the single spectrum

3. Theoretical consideration

The triple β (particle)$\rightarrow\gamma\rightarrow\gamma$ correlation function $W(\theta)$ was given by Ferguson (1965) for the cascade $a(l_1)\, b(L_2)\, c(L_3)d$ where a, b, c, d are the spin quantum numbers, l_1 is the angular momentum and l'_1 is the higher angular momentum carried by β-particles. L_2 and L_3 are the multipolarities of the second and third γ-transitions with the mixture of L_2' and L'_3 respectively. The function $W(\theta)$ is,

$$W(\theta) = \left(\frac{1}{4\pi}\right)^{3/2} \sum (-)^{a+d} (-1)^{l_1-l'_1+k_1} Z_1(l_1bl'_1b\colon ak_1),$$

$$\times G_\gamma \begin{Bmatrix} c & L_2 & b \\ c & L'_2 & b \\ k_3 & k_2 & k_1 \end{Bmatrix} Z_1\,(L_3\, cL'_3\, c\colon dk_3)\, \delta_1^{r_1}\, \delta_2^{r_2}\, \delta_3^{r_3}\, Q_{k_1}\, Q_{k_2}\, Q_{k_3},$$

$$\times\ a^i_{k_1k_2k_3k}\, P_k\,(\cos\theta),$$

with the summation over

$$l_1l'_1,\ L_2L'_2,\ L_3L'_3,\quad k_1k_2,\ k_3,k,$$

where r is the exponent having the values 0, 1 and 2. δ_1, δ_2 and δ_3 are the multipole mixing ratios. Q_{k_1}, Q_{k_2}, and Q_{k_3} are the attenuation coefficients for the radiations. $a^i_{k_1k_2k_3k}$ are the coefficients for the different geometries of the detectors (where $i\rightarrow s$ stands for geometry). $W(\theta)$ can be written in terms of the experimental A_2 and A_4. The solid angle correction has been considered in the theortical calculations of A_2 and A_4.

4. Discussion

Taking 696 keV γ-ray transition to be pure quadrupole, one can consider 1489 keV γ-ray transition to be mixture of dipole and quadrupole. Using the method of Arns and Wiedenbeck (1958) and extending it to β-γ-γ angular correlation, the usual plots are obtained for A_2 versus $Q = \delta^2/(1+\delta^2)$ where

$$\delta = \frac{\langle f \| L+l \| i\rangle}{\langle f \| \ L\ \| i\rangle}$$

as shown in figures 3 and 4. We thus obtain

$$\delta_{21} = +(0{\cdot}034\pm0{\cdot}015),\quad Q_{21} = 0{\cdot}001\pm0{\cdot}001,$$

if the $E3$ admixture is not present and considering the mixture of $E1+M2$ and

$$\delta_{31} = -(0{\cdot}026\pm0{\cdot}011),\quad Q_{31} = 0{\cdot}007^{+0{\cdot}0007}_{-0.0005},$$

if M_2 admixture is not present and considering the mixture of E_1+E_3. But if one wishes to include the contribution of E_3 together with M_2, the method of Arns and Wiedenbeck (1958) cannot be applied directly. However, this can be modified and the contribution of E_3 content is determined as given below. The directional correlation function $W(\theta)$ for the cascade of β(allowed) $\rightarrow \gamma \rightarrow \gamma$ can be written as

$$W(\theta) = \sum_{k_1 k_2 k_3 k} a_{k_1 k_2 k_3} \, a^{i}_{k_1 k_2 k_3 k} \, P_k(\cos\theta), \tag{2}$$

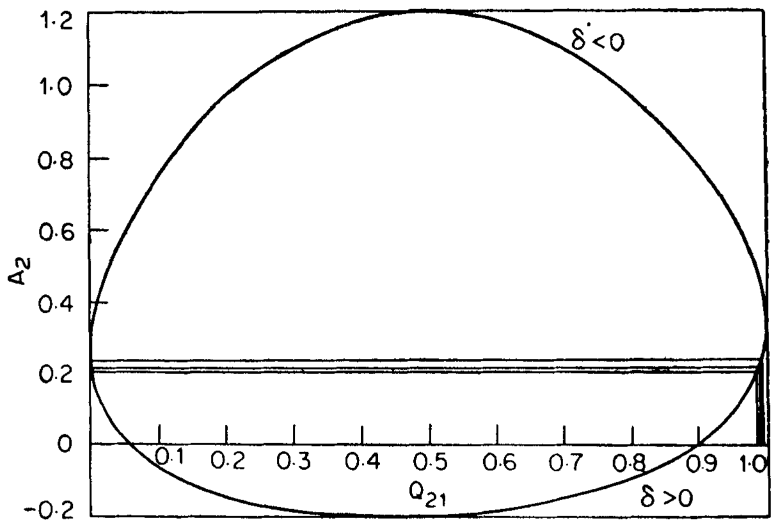

Figure 3. Theoretical plot of A_2 vs Q_{21}, the quadrupole content in dipole for the cascade 0^- (β-allowed) 1^- (M_2+E_1) 2^+ $(E_2)0^+$.

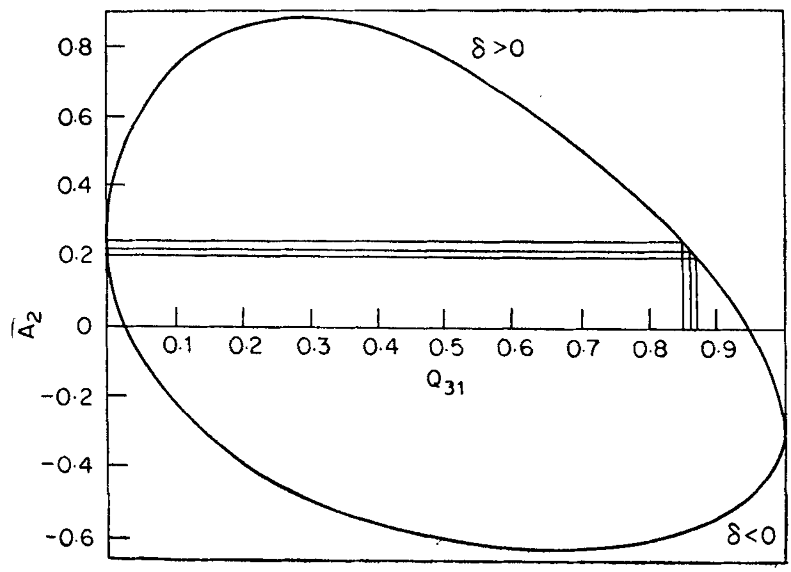

Figure 4. Theoretical plot of A_2 vs Q_{31}, the octupole content in dipole for the cascade 0^- (β-allowed) 1^- (E_1+M_3) 2^+ (E_2) 0^+.

where $$a_{k_1k_2k_3} = \left(\frac{1}{4\pi}\right)^{3/2} (-1)^{a+d}\, \Sigma\, (-1)^{k_1}\, Z_1(l_1b\, l_1b : a0) \times$$

$$G\gamma \begin{Bmatrix} c & L_2 & b \\ c & L'_2 & b \\ k_3 & k_2 & k_1 \end{Bmatrix} Z(L_3cL'_3c : dk_3)\, \delta_2^{r_2}\, \delta_3^{r_3}\, Qk_1{=}0\, Q_{k_2}\, Q_{k_3}. \tag{3}$$

Now writing

$$a_k = \sum a k_1k_2k_3 \;\; a^i_{k_1k_2k_3k}$$

and $$W(\theta) = \sum_k a_k P_k (\cos\theta). \tag{4}$$

Let us consider the case when one of the transitions (say the second) is the mixture of dipole + quadrupole + octupole and taking the β-transition to be allowed.

Therefore $l_1 = l_1'$ and $L_3 = L_3'$ and $G\gamma \begin{Bmatrix} c & L_2 & b \\ c & L_2' & b \\ k_3 & k_2 & k_1 \end{Bmatrix}$ is expanded as

$$G_\gamma \begin{Bmatrix} c & L_2 & b \\ c & L_2 & b \\ k_3 & k_2 & k_1 \end{Bmatrix} + 2\delta_{21}\, G_\gamma \begin{Bmatrix} c & L_2 & b \\ c & L_2+1 & b \\ k & k_2 & k_1 \end{Bmatrix}$$

$$+ \delta_{21}^2\, G_\gamma \begin{Bmatrix} c & L_2+1 & b \\ c & L_2+1 & b \\ k_3 & k_2 & k_1 \end{Bmatrix} + 2\delta_{21}\, \delta_{31}\, G_\gamma \begin{Bmatrix} c & L_2+1 & b \\ c & L_2+2 & b \\ k_3 & k_2 & k_1 \end{Bmatrix}$$

$$+ 2\delta_{31}\, G_\gamma \begin{Bmatrix} c & L_2 & b \\ c & L_2+2 & b \\ k_3 & k_2 & k_1 \end{Bmatrix} + \delta_{31}^2\, G_\gamma \begin{Bmatrix} c & L_2+2 & b \\ c & L_2+2 & b \\ k_1 & k_2 & k_3 \end{Bmatrix} \tag{5}$$

where δ_{21} and δ_{31}, the multipole mixing ratios, are defined as

$$\delta_{21} = \frac{\langle f \| M2 \| i \rangle}{\langle f \| E1 \| i \rangle} \text{ and } \delta_{31} = \frac{\langle f \| E3 \| i \rangle}{\langle f \| E1 \| i \rangle}.$$

Considering the first term of the expression (5), the equation (4) can be written in the form $W(\theta) = a_0 + a_2P_2(\cos\theta) + a_4P_4(\cos\theta)$. The values of the coefficients a_0, a_2 and a_4 are calculated. Similarly putting second, third, fourth, fifth and sixth terms respectively in (3), the values of the coefficients a'_0, a_2', a'_4; a'_0, a_2'', a_4''; a_0''', a_2''', a_3'''; a_0^{iv}, a_2^{iv} a_2^{iv}, a_4^{iv} and a_0^v, a_2^u, a_4^v are obtained. Therefore $W(\theta)$ is written as

$$\begin{aligned} W(\theta) = {} & [a_0 + a_2P_2(\cos\theta) + a_4P_4(\cos\theta)] + 2\delta_{21}\, [(a'_0 + a'_2P_2(\cos\theta) + \\ & + a'_4 P_4(\cos\theta)] + \delta^2_{21}\, [a_0'' + a_2''\, P_2(\cos\theta) + a_4''\, P_4(\cos\theta)] \\ & + 2\delta_{21}\, \delta_{31}\, [a_0''' + a_2'''P_2(\cos\theta) + a_4'''P_4(\cos\theta)] + 2\delta_{31} \\ & [a_0^{iv} + a_2^{iv}\, P_2(\cos\theta) + a_4^{iv}\, P_4(\cos\theta)] + \delta^2_{31}\, [a_0^v + a_2^v\, P_2(\cos\theta) \\ & + a_4^v\, P_4(\cos\theta)]. \end{aligned} \tag{6}$$

Rewriting this in the form $W(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$, the values of A_2 and A_4 are thus

$$A_2 = \frac{a_2+2\delta_{21}a_2'+\delta^2_{21}a_2''+2\delta_{21}\delta_{31}a_2'''+2\delta_{31}a_2^{iv}+\delta^2_{31}a_2^{v}}{a_0+2\delta_{21}a_0'+\delta^2_{21}a_0''+2\delta_{21}\delta_{31}a_0'''+2\delta_{31}a_0^{iv}+\delta^2_{31}a_0^{v}} \tag{7}$$

and
$$A_4 = \frac{a_4+2\delta_{21}a_4'+\delta^2_{21}a_4''+2\delta_{21}\delta_{31}a_4'''+2\delta_{31}a_4^{iv}+\delta^2_{31}a_4^{v}.}{a0+2\delta_{21}a_0'+\delta^2_{21}a_0''+2\delta_{21}\delta_{31}a_0'''+2\delta_{31}a_0^{iv}+\delta^2_{31}a_0^{v}} \tag{8}$$

Now putting the values of angular correlation coefficients a_k in (7) and (8), we have

$$A_2^{\text{expt}} = \frac{0{\cdot}4022-1{\cdot}7888\delta_{21}+0{\cdot}5708\delta^2_{21}+1{\cdot}3706\delta_{21}\delta_{31}+2{\cdot}390\delta_{31}-0{\cdot}5823\delta^2_{31}}{1{\cdot}5310+0{\cdot}8994\delta_{21}+1{\cdot}6612\delta^2_{21}-0{\cdot}0856\delta_{21}\delta_{31}-0{\cdot}7124\delta_{31}+2{\cdot}050\delta^2_{31}} \tag{9}$$

and
$$A_4^{\text{expt}} = \frac{0+0\times\delta_{21}+1{\cdot}0295\delta^2_{21}+2{\cdot}8779\delta_{21}\delta_{31}+2{\cdot}3164\delta_{31}+0{\cdot}1287\delta^2_{31}}{-1{\cdot}5310-0{\cdot}8994\delta_{21}-1{\cdot}6612\delta^2_{21}+0{\cdot}0856\delta_{21}\delta_{31}+0{\cdot}7124\delta_{31}-2{\cdot}050\delta^2_{31}} \tag{10}$$

In (9), taking a certain value of δ_{21}, δ_{31} is obtained. The plot of δ_{21} versus δ_{31} is given in figure 5. Similarly, the values of δ_{31} are obtained using (10) and the plot between δ_{21} versus δ_{31} is drawn. The interception of these two plots gives the values of δ_{21} and δ_{31} i.e.

$$\delta_{21}(M2:E1)=-(0{\cdot}025^{+0.010}_{-0.010}),\ \delta_{31}(E3:E1)=-(0{\cdot}045^{+0.020}_{-0.020}),$$

and
$$Q_{21}(M2:E1)=0{\cdot}0006^{+0.0006}_{-0.0004},\ Q_{31}(E3:E1)=0{\cdot}002^{+0.0002}_{-0.0001}$$

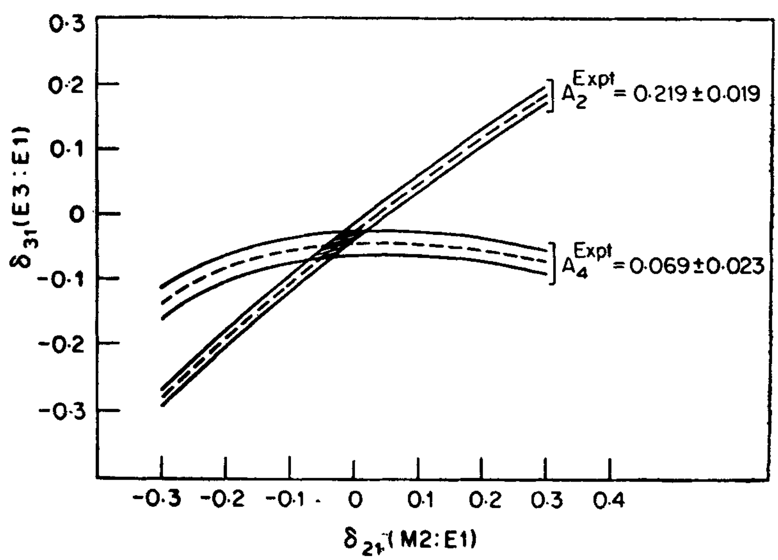

Figure 5. Quadratic plot of δ_{21} $(M2:E1)$ vs $\delta_{31}(E:E1)$ using experimental values of A_2 and A_4.

Therefore this clearly indicates $\sim$0·2% contribution of $E3$ transition. If this contribution is appreciable, then the model suggested by Bhatt (1965) can be used for further calculations. But the $E3$ transition is much slower compared to $E1$ and this $E3$ contribution in this respect is appreciable in the present measurement.

If we regard that there can be no appreciable contribution of $E3$ and $E1$ alongwith $M2$ and there is only $M2$ component, then the present study reveals that $M2/E1$, multipole mixing ratio is given by $\delta_{21}=0{\cdot}034\pm0{\cdot}015$ which is comparable to the one given by Raman (1967), Behar *et al* (1974) and other references given therein.

Apart from ^{144}Nd, we have four more isotopes i.e. ^{148}Sm, ^{152}Sm, ^{152}Gd and ^{156}Gd, where we find similar negative parity states, having the cascade $1^- \to 2^+ \to 0^+$. All these nuclei lie in rare earth region.

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References

Arns R G and Wiedenbeck M L 1958 *Phys. Rev.* **111** 1631
Bhatt K H 1965 *Phys. Lett.* **17** 282
Behar M, Grabowski Z W and Raman S 1974 *Nucl. Phys.* **A219** 516
Ferguson A J 1965 *Angular Correlation Methods in Gamma-Ray Spectroscopy* (Amsterdam: North-Holland Publishing Co.)
Raman S 1967 *Nuclear Data Sheets* **B2-1** 57
Raman S 1968 *Nucl. Phys.* **A107** 402, 407
Rao B V N 1976 *J. Phys. Soc. Jpn.* **40** 1