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Chiral SU(4) × SU(4) breaking: masses and decay constants of charmed hadrons

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Abstract. The (4, 4*) ⊕ (4*, 4) model of broken chiral SU (4) × SU (4) symmetry has been used to calculate the third-order coupling constants involving charmed and ordinary pseudoscalar mesons. These coupling constants are exploited to derive some interesting new relations among the masses and decay constants of these charmed particles. Using the known masses and decay constants as inputs, we exploit these relations to predict: $F_D = -1{\cdot}41\ F_\pi$, $F_F = -1{\cdot}13\ F_\pi$, $F_D/F_F = 1{\cdot}25$, $m(D_s) = 1{\cdot}43$ GeV, $m(F_s) = 1{\cdot}39$ GeV and $m(K_s) = 1{\cdot}02$ GeV.

Keywords. SU(4) × SU(4) symmetry; chiral symmetry breaking; pseudoscalar mesons; charmed particles; masses and decay constants.

1. Introduction

Since early fifties, the concept of approximate symmetries and partially-conserved quantum numbers has played an increasingly important role in particle physics. In the sixties it was discovered by Gell-Mann (1962) and Néeman (1961) that the strong interactions are approximately invariant under the SU(3) symmetry. Later it was suggested by Gell-Mann (1964) that the strong interactions are approximately invariant also under the bigger chiral group SU(3) × SU(3), which is generated by the algebra of vector and axial-vector currents of the hadrons. Since the chiral SU(3) ×SU(3) is only an approximate symmetry, it is important to know the nature of the symmetry-breaking piece of the Hamiltonian. The most popular and successful model of chiral symmetry breaking is the (3, 3*) ⊕(3*, 3) representation of SU(3) ×SU(3), proposed almost a decade ago by Gell-Mann and others (Gell-Mann *et al* 1968; Glashow and Weinberg 1968).

The chiral SU(3)×SU(3) has been further generalised on theoretical grounds and it has been proposed that the strong interaction Hamiltonian is invariant under the larger group SU(4)×SU(4) except for a symmetry breaking term which turns out to be of the order of 20%. The form of this symmetry breaking term is assumed to be the (4, 4*)⊕(4*, 4) representation of SU(4) × SU(4). This generalised model was no more than a mere theoretical curiosity till a few years ago. But after the recent experimental discovery of the new particles Ψ and Ψ′ and the new quantum number ‘charm’, the chiral SU(4) × SU(4) symmetry and the (4, 4*) ⊕ (4*, 4) model of chiral symmetry breaking have assumed a new importance in hadron physics.

Very recently the present authors had exploited the $(4, 4^*) \oplus (4^*, 4)$ model to derive two new relations among the masses and decay constants of charmed mesons and to estimate the decay amplitude for the $\eta \rightarrow 3\pi^0$ decay (Tiwari *et al* 1978a, b). In the present paper we exploit this model to derive several new and important relations among the masses and decay constants of scalar and pseudoscalar charmed particles. These relations are utilized along with the known experimental masses and decay constants as inputs, to predict the same in the cases when they are unknown or not so well-established experimentally.

2. The $(4, 4^*) \oplus (4^*, 4)$ model

It is assumed that the strong interaction Hamiltonian H is given by

$$H = H_0 + H'. \tag{1}$$

Here H_0 is invariant under the full chiral group $SU(4) \times SU(4)$, while H', which breaks this symmetry, is assumed to transform as the $(4, 4^*) \oplus (4^*, 4)$ representation of $SU(4) \times SU(4)$ and is given by

$$H' = \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_{15} u_{15}. \tag{2}$$

This form of H' is based on the requirement that the isospin, hypercharge and charm is conserved. The symmetry breaking terms depend on the real constants (symmetry breaking parameters) ϵ_i and the well-known scalar densities u_i. There are 16 scalar densities u_i and 16 pseudoscalar densities $v_i (i=0, 1, 2, \ldots, 15)$, both of which transform according to the $(4, 4^*) \oplus (4^*, 4)$ representation of $SU(4) \times SU(4)$. These scalar and pseudoscalar densities obey the followingwell-known, equal-time commutation relations:

$$[Q_i(t),\ u_j(x)]_{x_0=t} = i f_{ijk}\, u_k(x), \tag{3a}$$

$$[Q_i(t),\ v_j(x)]_{x_0=t} = i f_{ijk}\, v_k(x), \tag{3b}$$

$$[Q_i^5(t),\ u_j(x)]_{x_0=t} = i d_{ijk}\, v_k(x), \tag{3c}$$

$$[Q_i^5(t),\ v_j(x)]_{x_0=t} = -\, i d_{ijk} u_k(x), \tag{3d}$$

where $\quad i=1, 2, \ldots, 15;\ j, k = 0, 1, 2, \ldots, 15.$

Here Q_i and Q_i^5 are the generators of the group, while f_{ijk} and d_{ijk} are the well-known SU(4) structure constants tabulated by many authors (see, for example, Maki *et al* 1972).

Some years ago Cicogna *et al* (1972) analysed the consequences of chiral $SU(3) \times SU(3)$ breaking and discussed in detail the results which follow simply from the $(3, 3^*) \oplus (3^*, 3)$ representation of the symmetry-breaking piece of the Hamiltonian. In this analysis the approach was as general as possible and no reference was made to a Lagrangian model, such as the σ-model, or to a semi-classical approximation.

Their analysis showed that some of the results obtained previously by using specific models were, in fact, model-independent and follows simply from the assumption of the (3, 3*) + (3*, 3) form of symmetry breaking. In this paper, our approach is similar to that of Cicogna *et al* (1972), but we assume that the strong interactions are invariant under the chiral SU(4)×SU(4) group except for a symmetry-breaking term which transforms as the (4, 4*) + (4*, 4) representation of SU(4)×SU(4).

3. Third-order coupling constants

The theoretical approach most suitable for the discussion of the spontaneous breakdown of symmetries in quantum field theory is the functional method. For simplicity we consider the case in which the field theory is described by a Lagrangian of the form

$$L(x)=L_{\text{free}}\,(x) + L_{\text{interaction}}\,(x) + \epsilon_i\,\phi_i\,(x)\,, \tag{4}$$

where $L_{\text{free}}\,(x)+L_{\text{int}}\,(x)\; L_{\text{inv}}\;(x)$ is invariant under the given group G, ϵ_i are real constants and $\phi_i\,(x)$ are the basic local fields in terms of which the Lagrangian is constructed. We note that a linear breaking in the Lagrangian or Hamiltonian density is the basic assumption of the Gell-Mann-Oakes-Ranner model (Gell-Mann *et al* 1968) as well as the Glashow-Weinberg model (Glashow and Weinberg 1968) of chiral SU(3) × SU(3) breaking.

Starting with the Lagrangian $L(x)$ of (4) and following the standard techniques of the functional method, Cicogna *et al* (1972) obtained the first-, second- and third-order Ward identities, on the basis in which the mass matrix is diagonal, as follows:

$$\epsilon_i[\mathrm{G}^a, \lambda]_i=0, \tag{5a}$$

$$m_i^2\,[\mathrm{G}^a, \lambda]_i = [\mathrm{G}^a, \epsilon]_i, \tag{5b}$$

$$g_{i,jk}[\mathrm{G}^a, \lambda]_k = (m_i^2 - m_j^2)\;[\mathrm{G}^a, \hat{\phi}_j]_{\bar{\imath}}, \tag{5c}$$

(no sum over i and j).

In these Ward identities, G^a are the generators; $m_i{}^2$ and $\bar{\imath}$ denote, respectively, the squared mass and charge conjugate of the particles i while g_{ijk} is the third-order coupling constant between the particles i, j and k. Moreover,

$$\lambda = \lambda_i\,\hat{\phi}_i,\;\; \epsilon = \epsilon_i\,\hat{\phi}_i, \tag{6}$$

where $\hat{\phi}_i$ is the unit vector in the direction i and λ_i is the vacuum expectation value of the local field $\phi_i(x)$:

$$\lambda_i = \langle 0\,|\,\phi_i(x)\,|\,0\rangle.$$

We note that the second-order Ward identity of (5b) is just the generalisation of the contents of Goldstone's theorem. In the limit $\epsilon \to 0$, the 'modes' i, for which $[G^{\alpha}, \lambda]_i$ is different from zero, correspond to Goldstone's bosons. The second-order Ward identity of (5b) has already been exploited recently by the present authors (Tiwari *et al* 1978a) to obtain two relations among the masses and decay constants of charmed particles and to predict the masses of the charmed mesons D and F, in good agreement with the experimental data; so it will not be discussed any further in this paper. Here we are concerned with the consequences of the third-order Ward identity of (5c).

Actual calculations show that when G^{α} is one of the generators

$$(Q_1^5+iQ_2^5)/\sqrt{2},\ Q_3^5,\ (Q_4+iQ_5)/\sqrt{2},\ (Q_4^5+iQ_5^5)/\sqrt{2},\ (Q_6+iQ_7)/\sqrt{2},$$

$$(Q_6^5+iQ_7^5)/\sqrt{2},\ (Q_9+iQ_{10})/\sqrt{2},\ (Q_9^5+iQ_{10}^5)/\sqrt{2},\ (Q_{11}+iQ_{12})/\sqrt{2},$$

$$(Q_{11}^5+iQ_{12}^5)/\sqrt{2},\ (Q_{13}+iQ_{14})/\sqrt{2} \text{ and } (Q_{13}^5+iQ_{14}^5)/\sqrt{2}$$

or their charge conjugates, (5c) assumes the simple form

$$g_{ijk} = (m_i^2-m_j^2)\, C_{ijk} \text{ (no sum over } i \text{ and } j). \tag{7}$$

The values of this related constant C_{ijk} for the above-mentioned generators are collected in table 1 for various combinations of the scalar and pseudoscalar particles i, j and k. In this table, the calculated values of the constants C_{ijk} have been written in terms of the decay constants F_{π}, F_K, F_D and F_F. The constants C_{ijk} can be related to these decay constants through the application of partial conservation of axial-vector currents (PCAC) hypothesis, which we take in the form

$$\partial^{\mu} A_{\mu}^{i}(x) = F_i\, m_i^2\, \phi_i(x), \tag{8a}$$

which leads to the relation

$$F_i = -i\, [G^{\alpha}, \lambda]_i, \tag{8b}$$

where F_i is the decay constant of the meson i. In these calculations the commutators of (5c) and (8b) are evaluated with the help of the well-known commutation relations given in (3a) to (3c). From (8b) we get the following expressions for the decay constants $F_i (i=\pi, K, D$ and $F)$ in terms of the parameters λ_i (Tiwari *et al* 1978a):

$$F_{\pi} = \frac{1}{\sqrt{2}}\lambda_0 + \frac{1}{\sqrt{3}}\lambda_8 + \frac{1}{\sqrt{6}}\lambda_{15}, \tag{9a}$$

$$F_K = \frac{1}{\sqrt{2}}\lambda_0 - \frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{\sqrt{6}}\lambda_{15}, \tag{9b}$$

Table 1. Third-order coupling constants $g_{ijk} = (m_i^2 - m_j^2)\, C_{ijk}$

G^a	i	j	k	C_{ijk}	Relation No.
$(Q_1^5 + iQ_2^5)/\sqrt{2}$	$\bar{k}^0$	K_s^+	π^-	$1/\sqrt{2}\, F_\pi$	(1)
	$\bar{D}^0$	D_s^+	π^-	$1/\sqrt{2}\, F_\pi$	(2)
Q_3^5	K_s^-	K^+	π^0	$-1/2\, F_\pi$	(3)
	K^0	$\bar{K}_s^0$	π^0	$-1/2\, F_\pi$	(4)
	D_s^0	$\bar{D}^0$	π^0	$-1/2\, F_\pi$	(5)
$(Q_4 + iQ_5)/\sqrt{2}$	K^0	π^+	K_s^-	$1/\sqrt{2}\, (F_\pi - F_k)$	(6)
	π^0	K^+	K_s^-	$1/2\, (F_\pi - F_k)$	(7)
	F^+	$\bar{D}^0$	K_s^-	$1/\sqrt{2}\, (F_\pi - F_k)$	(8)
$(Q_4^5 + iQ_5^5)/\sqrt{2}$	K^0	π_s^+	K^-	$1/\sqrt{2}\, F_k$	(9)
	K^+	π_s^0	K^-	$1/2\, F_k$	(10)
	$\bar{D}_s^0$	F^+	K^-	$1/\sqrt{2}\, F_k$	(11)
$(Q_6 + iQ_7)/\sqrt{2}$	F^+	D^-	$\bar{K}_s^0$	$1/\sqrt{2}(F_\pi - F_k)$	(12)
	K^+	π^-	$\bar{K}_s^0$	$1/\sqrt{2}(F_\pi - F_k)$	(13)
	π^0	K^0	$\bar{K}_s^0$	$1/2(F_\pi - F_k)$	(14)
$(Q_6^5 + iQ_7^5)/\sqrt{2}$	F_s^+	D^-	$\bar{K}^0$	$-1/\sqrt{2}F_k$	(15)
	F^+	D_s^-	$\bar{K}^0$	$-1/\sqrt{2}F_k$	(16)
	π^0	K_s^0	$\bar{K}^0$	$-1/2F_k$	(17)
	K_s^+	π^-	$\bar{K}^0$	$-1/\sqrt{2}F_k$	(18)
$(Q_9 + iQ_{10})\sqrt{2}$	D^-	$\bar{D}^0$	D_s^0	$-1/\sqrt{2}(F_D - F_\pi)$	(19)
	π^0	$\bar{D}^0$	D_s^0	$-1/\sqrt{2}(F_D - F_\pi)$	(20)
	F^-	K^+	D_s^0	$-1/\sqrt{2}(F_D - F_\pi)$	(21)
$(Q_9^5 + iQ^5_{10})/\sqrt{2}$	D_s^-	π^+	D^0	$-1/\sqrt{2}F_D$	(22)
	π^0	$\bar{D}_s^0$	D^0	$-1/2F_D$	(23)
	F_s^-	K^+	D^0	$-1/\sqrt{2}F_D$	(24)
$(Q_{11} + iQ_{12})/\sqrt{2}$	$\bar{D}^0$	π^-	D_s^+	$-1/\sqrt{2}(F_D - F_\pi)$	(25)
	π^0	D^-	D_s^+	$-1/2(F_D - F_\pi)$	(26)
	F^-	K^0	D_s^+	$-1/\sqrt{2}(F_D - F_\pi)$	(27)
$(Q^5_{11} + iQ^5_{12})/\sqrt{2}$	$\bar{D}^0$	π_s^-	D^+	$1/\sqrt{2}F_D$	(28)
	π_s^0	D^-	D^+	$1/\sqrt{2}F_D$	(29)
	F^-	K_s^0	D^+	$1/\sqrt{2}F_D$	(30)
	D_s^-	π^0	D^+	$1/2F_D$	(31)
$(Q_{13} + iQ_{14})/\sqrt{2}$	$\bar{D}^0$	K^-	F_s^+	$-1/\sqrt{2}(F_D - F_k)$	(32)
	D^-	$\bar{K}^0$	F_s^+	$-1/\sqrt{2}(F_D - F_k)$	(33)
$(Q^5_{13} + iQ^5_{14})/\sqrt{2}$	$\bar{D}^0$	K_s	F^+	$1/\sqrt{2}F_F$	(34)
	D^-	$\bar{K}_s^0$	F^+	$1/\sqrt{2}F_F$	(35)

$$F_D = \frac{1}{\sqrt{2}}\lambda_0 + \frac{1}{2\sqrt{3}}\lambda_8 - \frac{1}{\sqrt{6}}\lambda_{15}, \tag{9c}$$

$$F_F = \frac{1}{\sqrt{2}}\lambda_0 - \frac{1}{\sqrt{3}}\lambda_8 - \frac{1}{\sqrt{6}}\lambda_{15}. \tag{9d}$$

We note that these expressions for the decay constants lead directly to the relation

$$F_F - F_D = F_K - F_\pi, \tag{9e}$$

which is an interesting relation among the decay constants of charmed and ordinary pseudoscalar mesons.

4. Some interesting relations

The relations among the squared masses and decay constants of the ordinary and charmed mesons can be obtained by noticing in table 1 that the same particles i, j and k can occur in different order with several generators G^a in (5c) and then equating the third-order coupling constants g_{ijk} in such cases. For example, the particles D, F and K_s occur in the relations (8), (30) and (34) of table 1. Relating the constants C_{ijk} with the help of (7), and then equating g_{DFKs} in these three cases, we get the relation

$$\frac{D-F}{F_k - F_\pi} = \frac{F-K_s}{F_D} = \frac{D-K_s}{F_F}. \tag{10}$$

In this and the following relations, we use the particle symbol to denote the squared mass of the corresponding particle for simplicity, e.g., symbol D denotes the squared mass $m^2(D)$ of the charmed pseudoscalar meson D. Similarly, the subscript s attached to a particle symbol denotes the corresponding scalar particle. Other similar relations can be obtained in the same manner from table 1. For convenience, we give below all such independent relations (including those of (10)) among the masses and decay constants of charmed and ordinary particles, which can be obtained from table 1.

$$\frac{D-F}{F_K - F_\pi} = \frac{F-K_s}{F_D}, \tag{11a}$$

$$\frac{D_s-\pi}{F_D} = \frac{D_s-D}{F_\pi}, \tag{11b}$$

$$\frac{F_s-D}{F_K} = \frac{F_s-K}{F_D}, \tag{11c}$$

$$\frac{F-K_s}{F_D} = \frac{D-K_s}{F_F}, \tag{11d}$$

$$\frac{F-K}{F_D-F_\pi}=\frac{D_s-F}{F_K}, \tag{11e}$$

$$\frac{K_s-\pi}{F_K}=\frac{K_s-K}{F_\pi}. \tag{11f}$$

We note here that the last relation, viz. (11f) involves only uncharmed scalar and pseudoscalar particles and it was derived earlier by Cicogna *et al* (1972) in the (3, 3*) ⊕ (3*, 3) model and by Sastry and Tiwari (1976) in the (8, 8) model of broken SU (3) × SU (3) symmetry.

5. Results and discussion

In order to compare the relations given above in (11a) to (11f) with experimental data, we use the known experimental masses and decay constants as inputs in these relations to estimate the same quantities in the cases where they are not known at all or not so well-established experimentally. For this purpose we use the following numerical values for the average experimental masses of π, K, D and F mesons and the ratio F_K/F_π of the kaon and pion decay constants:

$$m(\pi)=0{\cdot}1373 \text{ GeV}, \qquad m(K)=0{\cdot}4958 \text{ GeV},$$

$$m(D)=1{\cdot}865 \text{ GeV}, \qquad m(F)=2{\cdot}020 \text{ GeV},$$

$$F_K/F_\pi=1{\cdot}28. \tag{12}$$

With these numerical values as inputs, we get from (11a) to (11f) the following estimates for the masses and decay constants of other particles,

$$F_D/F_\pi=-1{\cdot}41,\; F_F/F_\pi=-1{\cdot}13,\; F_D/F_F=1{\cdot}25,$$

$$m(D_s)=1{\cdot}43 \text{ GeV},\; m(F_s)=1{\cdot}39 \text{ GeV},\; m(K_s)=1{\cdot}02 \text{ GeV}, \tag{13}$$

which satisfy the relation among the decay constants, given earlier in (9e).

The decay constants of the charmed mesons D and F and the masses of the charmed and ordinary scalar mesons D_s, F_s and K_s are either not known at all or not so well established experimentally. So the estimates of these quantities given in (13) may be treated as the predictions of the (4, 4*) ⊕ (4*, 4) model and the PCAC hypothesis. For the mass of the scalar kaon K_s, the value calculated by Sastry and Tiwari (1976) for the (8, 8) model was 1·008 GeV, while Cicogna *et al* (1972) obtained two solutions, viz., $m(K_s)=0{\cdot}938$ GeV and 0·698 GeV, in the (3, 3*) ⊕ (3*, 3) model. The value of $m(K_s)$ is not well known experimentally, but the physical situation suggests that it lies in the range between 1·080 and 1·260 GeV. On the other hand, almost nothing is known experimentally about the masses of the charmed scalar mesons D_s and F_s.

Similarly, very little is known experimentally about the decay constants of the charmed particles D and F. However, it has been pointed out by many authors on theoretical grounds that while $F_K \approx F_\pi$ and $F_D \approx F_F$, which follow from only an approximate SU(3) symmetry, that F_D/F_π and F_F/F_π may not at all be close to unity. Consequently, the estimates of F_D/F_π and F_F/F_π by various authors differ widely from unity. For example, Quigg and Rosner (1977) estimated $F_D/F_\pi \approx 0{\cdot}3$ in their phase space model, while Singer's investigations yielded $F_D/F_\pi \approx 2{\cdot}79$ and $F_F/F_\pi \approx 2{\cdot}88$ (Singer 1976).

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