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Colour effects in Drell-Yan process

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Abstract. We examine the predictions of gauge theories with colour excitation for the process $pp \to \mu^+ \mu^- X$. Relative to the predictions of quark parton model (with three colours) we find enhancements as large as a factor 3 – 4 for the cross-section $M^3 d^2 \sigma/dMdy \big|_{y=0}$ in the region $0{\cdot}03 \lesssim M/\sqrt{s} \lesssim 0{\cdot}2$ at $\sqrt{s} = 62$ GeV, M being the invariant mass and y the rapidity of the muon pair. We study the sensitivity of this result to the colour gluon mass and the underlying parametrisation of the quark and gluon distribution functions.

Keywords. Colour gauge theory; integral and fractional charge quarks; spontaneous symmetry breaking; colour gluons; Drell-Yan process.

1. Introduction

Within a unified gauge theory of strong, weak and electromagnetic interactions, there are two main approaches to the strong interactions. The one in which the strong interactions are generated by an exact colour SU(3) gauge symmetry involves fractionally charged coloured quarks and neutral massless colour gluons (Bardeen *et al* 1973; Fritzsch *et al* 1973; Gross and Wilczek 1973; Politzer 1973; Weinberg 1973). Colour is supposed to be permanently confined and the colour gauge group commutes with the gauge group of weak and electromagnetic interactions. The theory is asymptotically free and the Bjorken scaling is violated by logarithmic terms (Politzer 1974).

The second approach is based on integer charge Han-Nambu quark model (Han and Nambu 1965; Pati and Salam 1973, 1974). The electromagnetic current in such a model contains a colour singlet as well as a colour octet piece. By design the Han-Nambu model is equivalent to the one based on fractionally charged quarks in the colour singlet sector (Lipkin 1972; Chanowitz 1977). However, above the threshold for the production of colour nonsinglet states we expect dramatic differences to appear (Greenberg and Nelson 1977). Since no such differences have been observed in the presently accessible energy range, we are led to the conclusion that either the colour threshold is arbitrarily large, or if it is in the presently available energy range then there must be some mechanism which suppresses the colour quantum numbers, and quarks behave as if they have fractional charge. The class of gauge theories based on Han-Nambu quarks precisely achieve the second alternative. Here the strong gauge group is again colour SU(3) which is spontaneously broken. As a consequence the colour gluons acquire mass and also mix with vector bosons of the weak and electromagnetic gauge group. The colour gauge group does not commute with the physical generators of the weak and electromagnetic interactions and hence some of the colour gluons participate in weak and electromagnetic interactions.* The electromagnetic current contains both a colour singlet and a colour octet part, but because of a general property of such a gauge theory the colour octet part does not contribute with its full strength in lepton-hadron processes (Pati and Salam 1976; Rajasekaran and Roy 1975, 1976). This is because in a spontaneously broken colour gauge theory the colour octet contribution to electron- or muon-hadron processes arises through the exchange of the colour photon and its orthogonal colour gauge partner with the same strength. However, due to the opposite sign between these two contributions the colour effects are suppressed by a factor $m_g^4/(q^2-m_g^2)^2$, where q is the four momentum carried by the photon and m_g the colour gluon mass. This factor vanishes effectively for high momentum transfers, $|q^2| \gg m_g^2$. Thus, in electron- or muon-hadron interactions far above the colour threshold with $|q^2| \gg m_g^2$, the integer charge quarks in a spontaneously broken colour gauge theory behave as if they carry fractional charge. Nevertheless, colour may manifest itself through nonvanishing gluon contribution in deep inelastic electron- or muon scattering in the asymptotic region (Pati and Salam 1976; Rajasekaran and Roy 1975, 1976). Therefore, if we treat partons as quarks and colour gluons, the colour contributions appear as small scaling violations $\sim$10–20% (Pati 1976) which is not inconsistent with the present experimental data on deep inelastic lepton hadron scattering processes. Due to the same cancelling mechanism, the colour contributions from the spontaneously broken colour gauge theory in all lepton hadron interactions are similar enough to the small corrections due to the asymptotically free theory of exact colour symmetry that one cannot find any distinctive differences between them in the present energy range.

In view of all this, it is very important to choose appropriate physical processes and kinematic regions in which the colour suppression of the spontaneously broken colour gauge theory is not operative, i.e. the contribution of the colour octet part (of both quarks and gluons) is not suppressed as compared to that of the colour singlet part. Recently it has been shown (Lee and Kim 1978) that deep inelastic Compton scattering is one such process in which the colour effects of the spontaneously broken colour gauge theory are dominant and, indeed, are necessary to obtain an agreement of the parton model results with the experimental data. Two features of inelastic Compton scattering make it possible for the colour effects to contribute sizeably to the process. The first is that real photons are involved ($q^2=0$) and as such the suppression of the colour octet part of the electromagnetic current due to photon-gluon mixing does not occur and the quarks behave as if they have integral charges. However, this is not enough because if one does calculations with integer charges the results are somewhat higher than those in the fractional charge model but still too low to be in agreement with the experimental data. The second and the most important reason is, therefore, the inclusion of charged colour gluons which make the dominant contribution. Since gluons, which carry about 50% of the momentum of the proton, are supposed to have a sea-like distribution, they can make a sizeable contribution only in those kinematic regions where the sea of the nucleon is probed (small x, $x \lesssim 0{\cdot}1$). This is precisely the region which is probed in the present day deep inelastic Compton scattering experiments (Caldwell *et al* 1974), and that is why colour gluons make such a quantitative difference and are necessary to describe the data.

*Global colour symmetry is maintained approximately to $O(\alpha)$.

Although there is, thus, an evidence for the colour effects of the spontaneously broken colour gauge theory in inelastic Compton scattering, the doubt remains as to the applicability of parton model to such a process (Gilman 1974). We must, therefore, look for similar effects in other processes, particularly those where we can apply parton model results. Since the charged colour gluons make such a difference in Compton scattering process, and because they constitute the nucleon sea, we are led to the study of colour effects in the massive muon pair production in hadron-hadron collisions, the Drell-Yan process (Drell and Yan 1971),

$$p + p \rightarrow \mu^+\mu^- + \text{anything}. \tag{1}$$

In this process one can probe the nucleon sea with the present machines, and as we shall see the colour effects do contribute sizeably in an appropriate kinematical region in spite of the fact that the colour suppression due to photon-gluon mixing is operative in this process (the photon is massive). Since the quantitative predictions for Drell-Yan process depend crucially on the quark distribution functions which are extracted from deep inelastic lepton scattering, we shall study the sensitivity of our results on the assumed form of these distribution functions. The choice of the quark distribution functions, particularly those of the sea quarks, will be dictated by as to whether or not colour is excited in muon scattering experiments at Fermilab. Since in broken colour gauge theories there is one unknown parameter, the colour gluon mass m_g, we shall also study the effect of varying m_g on our results.

2. Calculations

In this section we shall calculate the cross-section for the production of massive muon pairs in hadron-hadron collisions in the broken colour gauge theory of Pati and Salam (1973, 1974). The general expression for the cross-section to form a lepton pair of invariant mass $M=\sqrt{Q^2}$ in process (1) is (Drell and Yan 1971)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2}\left(1-\frac{4m^2}{Q^2}\right)^{1/2}\left(1+\frac{2m^2}{Q^2}\right)$$

$$\times \frac{1}{\{[s-(M_1+M_2)^2)]\,[s-(M_1-M_2)^2]\}^{1/2}}\, W(Q^2, s). \tag{2}$$

Here s is the square of centre of mass energy, M_1 and M_2 are the masses of colliding hadrons, m is the lepton mass and

$$W(Q^2, s) = (-16\pi^2 E_1 E_2) \int d^4q\, \delta(q^2 - Q^2)$$

$$\times \sum_n (2\pi)^4\, \delta^4(P_1 + P_2 - q - P_n)\, \langle P_1 P_2 | J_\mu | n\rangle \langle n | J^\mu | P_1 P_2\rangle, \tag{3}$$

where J_μ is the hadronic electromagnetic current. In (3) a spin average over initial hadrons is understood. E_1, P_1 and E_2, P_2 are the energies, momenta of the two initial hadrons. The sum over n is for all unobserved hadron states.

We shall assume the validity of Drell-Yan mechanism (Drell and Yan 1971), according to which the massive lepton pair is produced via the parton-antiparton annihilation. In the centre of mass frame a parton from one of the hadrons annihilates an antiparton from the other (or vice versa) and the resulting system is very massive since the parton-antiparton energies add while their momenta subtract. If we take the partons to be fractionally charged quarks (with three colours) as in the exact colour SU(3) gauge theory, then in the limit of large Q^2 and s, with the ratio $\tau \equiv Q^2/s$ fixed, the cross-section can be written as (Drell and Yan 1971; Yan 1977)

$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{4\pi\alpha}{3Q^2}\,\frac{1}{Q^2}\,\frac{\tau}{(x^2+4\tau)^{1/2}} \times \tfrac{1}{3}\Big\{\sum_q e_q^2\,[q(x_1)\,\bar{q}(x_2) + \bar{q}(x_1)\,q(x_2)]\Big\}, \tag{4}$$

where $x_{1,2}$ are the fractions of the longitudinal momenta of their respective hadrons carried by the annihilating parton-antiparton pair, and $x = x_1 - x_2$. The functions $q(x)$ are the usual quark momentum distribution functions for the quark flavour q and the sum is over all flavours. The factor $\frac{1}{3}$ is the colour factor.

In a spontaneously broken colour gauge theory the effective quark electromagnetic current has both colour singlet as well as colour octet pieces and can be written as (Pati and Salam 1976; Rajasekaran and Roy 1975).

$$J_\mu\,(\text{quarks}) = J^0_\mu + \left(\frac{-m_g^2}{Q^2 - m_g^2}\right) J^8_\mu\,, \tag{5}$$

where $$J^0_\mu = \sum_i \left[\tfrac{2}{3}\,(\bar{u}_i\,\gamma_\mu\,u_i + \bar{c}_i\,\gamma_\mu\,c_i) - \tfrac{1}{3}\,(\bar{d}_i\,\gamma_\mu\,d_i + \bar{s}_i\,\gamma_\mu\,s_i\right],$$

the sum being over all colours, is a colour singlet piece, and

$$J^8_\mu = \sum_q \left(-\tfrac{2}{3}\,\bar{q}_1\,\gamma_\mu\,q_1 + \tfrac{1}{3}\,\bar{q}_2\,\gamma_\mu\,q_2 + \tfrac{1}{3}\,\bar{q}_3\,\gamma_\mu\,q_3\right),$$

where the summation is over all flavours, is the colour octet piece. In (5) Q is the four-momentum carried by the current. The Q^2 dependence of the effective current in (5) arises due to the phenomenon of photon-gluon mixing and is a general property of a spontaneously broken colour gauge theory. In addition there are four charged gluons in the theory which also contribute to the hadronic electromagnetic current (Pati and Salam 1976; Rajasekaran and Roy 1975). The electromagnetic current of these gluons can be written as (ρ^+, ρ^-, etc. refer to charged gluons)

$$\begin{aligned} J_\mu\,(\text{gluons}) = \left(\frac{m_g^2}{Q^2 - m_g^2}\right)\Big\{&[\epsilon'^\beta\,\epsilon^\alpha\,V_{\mu\alpha\beta}]_{\rho^+} - [\epsilon'^\beta\,\epsilon^\alpha\,V_{\mu\alpha\beta}]_{\rho^-} \\ &- \tfrac{1}{2}\,[\epsilon'^\beta\,\epsilon^\alpha\,V_{\mu\alpha\beta}\,\bar{\xi}_f\,(\tau'_3 + 1)\,\xi_i]_{K^+} \\ &+ \tfrac{1}{2}\,[\epsilon'^\beta\,\epsilon^\alpha\,V_{\mu\alpha\beta}\,\xi_f\,(\tau_3' + 1)\,\xi_i]_{K^-}\Big\}, \end{aligned} \tag{6}$$

where Q refers to the momentum carried by the current and $V_{\mu\alpha\beta}$ is the symmetric Yang-Mills vertex which in the case of two annihilating gluons can be written as $(p_1+p_2=Q)$

$$V_{\mu\alpha\beta} = [(-p_1 + Q)_\mu \, g_{\alpha\beta} - (p_2 + Q)_\alpha \, g_{\mu\beta} + (Q + p_1)_\beta \, g_{\mu\alpha}]. \qquad (7)$$

In (6) τ'_3 is a colour isospin matrix which is sandwiched between the final and the initial spinors, ξ_f and ξ_i, of the K-gluons which are defined to be $\begin{pmatrix}1\\0\end{pmatrix}$, and ϵ'^{β} and ϵ^{α} are the corresponding polarisation vectors. The Q^2 dependence in (5) is due to the same phenomenon of photon-gluon mixing which produced the Q^2 dependence in the quark electromagnetic current (5).

The cross-section for the Drell-Yan process in the spontaneously broken colour model can be calculated in a straightforward manner by using the currents (5) and (6) in the general cross-section formula (1) and (2). Since the initial hadrons are colour singlets, the cross-section can be written as a sum of three pieces:

$$\frac{d^2\sigma}{dQ^2dx} = \frac{d^2\sigma^0}{dQ^2dx} + \frac{d^2\sigma^8}{dQ^2dx} + \frac{d^2\sigma^G}{dQ^2dx}, \qquad (8)$$

where $d\sigma^0$ is the cross-section corresponding to the colour singlet piece of current (5) and is the same as in the fractional charge Gell Mann-Zweig model (with three colours) given by (4). The cross-sections $d\sigma^8$ and $d\sigma^G$ are the contributions of the colour octet quark current in (5) and the gluon current (6), respectively:

$$\frac{d^2\sigma^8}{dQ^2\,dx} = \frac{4\pi\,\alpha^2}{3Q^2}\,\frac{1}{Q^2}\,\frac{\tau}{(x^2+4\tau)^{1/2}}\left(\frac{m_g^2/s}{\tau - m_g^2/s}\right)$$

$$\times \frac{2}{27}\left\{\sum_q [q(x_1)\,\bar{q}(x_2) + \bar{q}(x_1)\,q(x_2)]\right\}, \qquad (9)$$

$$\frac{d^2\sigma^G}{dQ^2\,dx} = \frac{4\pi\,\alpha^2}{3Q^2}\,\frac{1}{Q^2}\,\frac{\tau}{(x^2+4\,\tau-8\,m_g^2/s)^{1/2}}\,\frac{(\tau-4\,m_g^2/s)}{(\tau-2\,m_g^2/s)}$$

$$\times\left(\frac{m_g^2/s}{\tau - m_g^2/s}\right)\left(12+20\,\frac{Q^2}{m_g^2}+\frac{Q^4}{m_g^4}\right)\left[\frac{1}{9}\,g(x_1)\,g(x_2)\right]. \qquad (10)$$

The sum over q in (9) is over all quark flavours, and in (10) $g(x)$ is the gluon distribution function which is the same for all types of gluons since the initial hadrons in (1) are supposed to be colour singlets. We note that in contrast to colour singlet contribution $d\sigma^0$, the colour octet quark contribution (9) does not scale and in fact vanishes for large s, whereas the gluonic contribution (10) is non-vanishing and in fact scales at large s and Q^2.

The experimental results are generally expressed in terms of rapidity cross-section

$$\left.\frac{d\sigma}{dQ^2\,dy}\right|_{y=0} \quad \text{versus } (\tau)^{1/2},$$

where y is the usual rapidity variable. The cross-section (8) can be transformed into a cross-section differential in the rapidity variable via the transformation

$$\left.\frac{d^2\sigma}{dQ^2\,dy}\right|_{y=0} = 2\,(\tau)^{1/2}\left.\frac{d^2\sigma}{dQ^2\,dx}\right|_{x=0}, \tag{11}$$

with $x_1 = x_2 = (\tau)^{1/2}$ (for quarks),

$x_1 = x_2 = (\tau - 2m_g^2/s)^{1/2}$ (for gluons).

From (8), (9), (10) and (11) we observe that $Q^4\, d\sigma/dQ^2\,dy\,|_{y=0}$ at large s is a function only of $(\tau)^{1/2}$.

3. Results and discussion

It is obvious from the cross-section formulae that any quantitative comparison of the theoretical predictions of the two models with the experiment will crucially depend on the knowledge of the parton distribution functions. These are generally obtained from the electron (muon) and neutrino scattering experiments. However, the anti-parton distributions are poorly determined because there is very little data for $x < 0{\cdot}1$, where the sea contribution is dominant. Moreover, the differences in the predictions of the two models will crucially depend on the gluon distribution function about which little is known at present. In addition, in the model with excitable colour there is a free parameter, the colour gluon mass m_g, about which also little is known. Recent estimates (Pati *et al* 1977) suggest that m_g can be as low as 1–2 GeV. In this section we shall study how sensitive the differences in predictions of the two models are to the quark and gluon distribution functions used, and to the value of the gluon mass m_g.

To study the sensitivity of the results on the parton distribution functions, we shall use the following two typical parametrisations of these distributions:

(1) Barger-Phillips (1974)

$$u(x) = u_v(x) + s(x),$$

$$d(x) = d_v(x) + s(x),$$

$$s(x) = \bar{s}(x) = \bar{u}(x) = \bar{d}(x), \tag{12}$$

$$u_v(x) = x^{-1/2}\,[0{\cdot}594\,(1-x^2)^3 + 0{\cdot}461\,(1-x^2)^5 + 0{\cdot}621\,(1-x^2)^7],$$

$$d_v(x) = x^{-1/2}\,[0{\cdot}072\,(1-x^2)^3 + 0{\cdot}206\,(1-x^2)^5 + 0{\cdot}621\,(1-x^2)^7],$$

$$s(x) = 0{\cdot}145\,x^{-1}\,(1-x)^9.$$

(2) Blankenbecler *et al* (1975)

$$u_v(x) = 1{\cdot}89\, x^{-1/2} (1-x)^7 + \{\theta\,(0{\cdot}35 - x)\, 90{\cdot}2\, x^{3/2} \exp(-7{\cdot}5\, x) + \theta\,(x-0{\cdot}35)\, 5\, (1-x)^3\}, \tag{13}$$

$$d_v(x) = 1{\cdot}03\, x^{-1/2} (1-x)^7 + 0{\cdot}7\, (1-x) \{\theta\,(0{\cdot}35-x)\, 90{\cdot}2\, x^{3/2} \exp(-7{\cdot}5x) + \theta\,(x-0{\cdot}35)\, 5\, (1-x)^3\},$$

$$s(x) = 0{\cdot}2\, x^{-1} (1-x)^7.$$

Keeping with the current practice and encouraged by the results on deep inelastic Compton scattering (Lee and Kim 1978), we shall assume gluons to have a sea quark-like distribution. With this assumption the gluon distribution function can be parametrised by the conventional sea function

$$x\, g(x) = A\, (1-x)^p. \tag{14}$$

The x dependence of the gluon distribution will be taken to be the same as that of the sea quark distribution used in a particular analysis, i.e. $p=9$ for the quark parametrisation (12), etc. The normalization A can then be fixed by the results of electron scattering experiments which imply that the neutral partons carry about 50% of the nucleon momentum. Note that we assume the energy momentum range in the SLAC experiments to be below colour threshold so that the charged colour gluons behave effectively as neutral gluons and do not contribute to the electron scattering structure functions measured at SLAC. Having specified the quark and gluon distribution functions we can study the sensitivity of the predictions of the excitable colour model on the colour gluon mass.

Most of the experimental data on the massive lepton pair production in hadron-hadron collisions is concentrated in the region $0{\cdot}2 \lesssim \sqrt{\tau} \lesssim 0{\cdot}5$ (Cobb *et al* 1977; Kaplan *et al* 1978). As noted earlier the differences between the two gauge models arise mainly due to the contribution of charged colour gluons of the excitable colour model. Since the gluons are supposed to have sea-like distribution, we do not expect any appreciable contributions from gluons in the region which has so far been explored in the Drell-Yan process. It is only in the low $\sqrt{\tau}$ region, $\sqrt{\tau}<0{\cdot}2$, that we expect marked differences to appear. However, in this region the cross-section will be very sensitive to the sea and not to the valence quark distributions used. Thus, from our point of view the main difference between the parametrisations (12) and (13) is in the sea quark distribution. In fact the difference in the predictions of the two models are insensitive to which of valence distributions in (12) or (13) is used.

3.1. *Dependence on m_g*

To study the sensitivity of results on the gluon mass, we shall use the parametrisation (13). In figure 1 we plot $M^3 d^2\sigma/dMdy|_{y=0}$, for pp collisions where $M = \sqrt{Q^2}$ is the invariant mass of the muon pair, versus $\sqrt{\tau}$ for different values of m_g. The solid

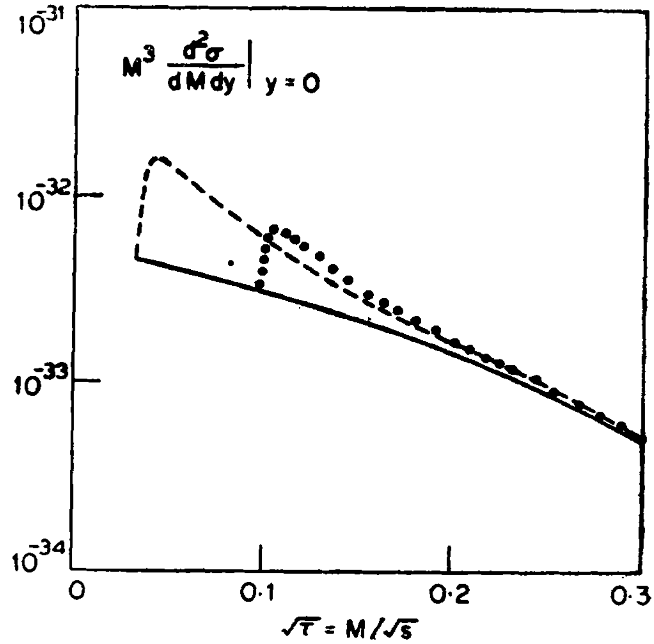

Figure 1. Enhancement in the Drell-Yan cross section for the spontaneously broken colour gauge theory at $\sqrt{s}$ = 62 GeV for two different values of m_g. The dashed curve is for m_g = 1 GeV and the dotted curve is for m_g = 3 GeV. The solid curve is the prediction of the simple quark parton model with three colours. For higher values of m_g or lower values of $\sqrt{s}$ the enhancement is pushed to the higher values of $\sqrt{\tau}$ and is consequently reduced.

curve is for the exact symmetry model without asymptotic freedom corrections (see (4)). The dashed curve is for the broken colour model (eqs (8), (9) and (10)) for $m_g=1$ GeV and $\sqrt{s}=62$ GeV, and the dotted curve for $m_g=3$ GeV but with the same value of $\sqrt{s}$. The spontaneously broken colour model predicts an enhancement over that of an unbroken model. For a given $\sqrt{s}$, the amount and the location of enhancement depends on m_g. A larger m_g predicts a smaller enhancement and shifts it to a higher $\sqrt{\tau}$ value. This should also be clear from (10) which shows that the colour gluons contribute only when $Q^2 > 4m_g^2/s$ (the contribution of the colour octet part of the quark current (9) is not significant at large s). Increasing m_g at a fixed s increases the value of Q^2 and hence $\sqrt{\tau} \equiv (Q^2/s)^{1/2}$ at which the gluons start contributing. But at larger $\sqrt{\tau}$ the gluon distribution function falls rapidly and, therefore, the enhancement is smaller. For the same reasons, at a given value of m_g but with a smaller s the enhancement is pushed to the higher values of $\sqrt{\tau}$ with a consequent decrease in the amount of enhancement. The enhancement is largest, being a factor $\simeq 4$ over the predictions of the exact symmetry model, at $\sqrt{\tau} \simeq 0{\cdot}04$ for the highest energy achieved so far, $\sqrt{s}=62$ GeV, at a value of $m_g=1$. For larger values of m_g or lower values of $\sqrt{s}$ the enhancement, being pushed to the higher values of $\sqrt{\tau}$, is smaller, and there is no significant difference between the predictions of two models for the Drell-Yan process.

The present experiments at $\sqrt{s}=62$ GeV (Cobb *et al* 1977) have explored the region $\sqrt{\tau} \gtrsim 0{\cdot}08$. In this region there is no significant difference in the predictions of the two models for $m_g > 1$. Since there are large errors in the present data (see figure 2),

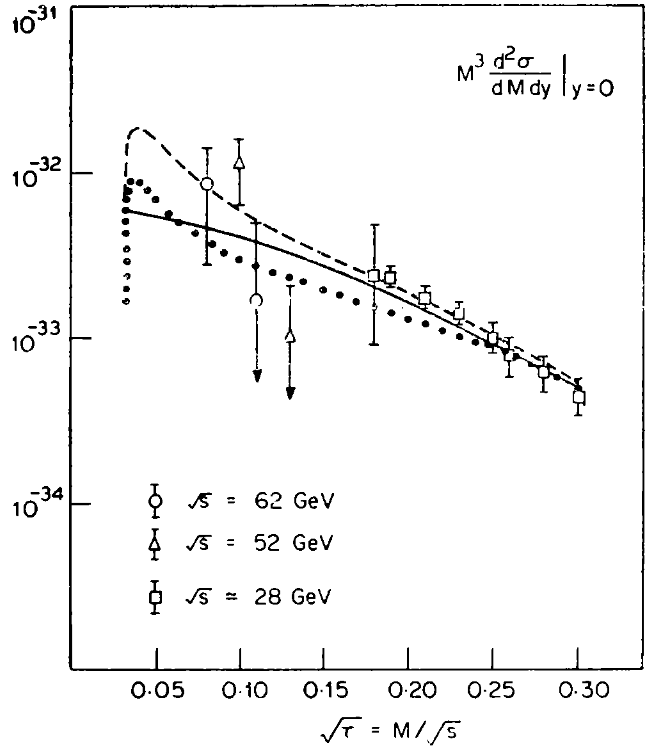

Figure 2. Sensitivity of the predictions to the parton distribution functions used. All the curves are with respect to the valence parametrization of (12). The dotted and the dashed curves are the predictions of the model with colour excitation for the sea parametrization (16) and (17), respectively. Both these curves are for $\sqrt{s} = 62$ GeV and $m_g = 1$. The parton model prediction for the sea parametrization (17) is shown by the solid curve. For $\sqrt{s} = 52$ GeV the prediction of the excitable colour model is not different from that at $\sqrt{s} = 62$ GeV. For $\sqrt{s} = 28$ GeV there is very little difference between the predictions of the two models, both of them effectively reproducing the solid curve. The data is from Cobb *et al* (1977) and Kaplan *et al* (1978).

it is very difficult to distinguish between the two models even for $m_g = 1$ GeV. With accurate data in the region $\sqrt{\tau} \lesssim 0{\cdot}08$ it may be possible to distinguish between the two models, in case $m_g = 1-2$ GeV. Since the present estimates of m_g put its value at 1–2 GeV (Pati *et al* 1977), it is important to obtain accurate data in this region. For lower values of $\sqrt{s}$ no meaningful comparison can be made between the two models in Drell-Yan process.

3.2. *Sensitivity to the parton distributions*

To study the sensitivity of the predictions on the assumed form of the parton distribution functions we shall take $m_g = 1$ GeV, because as discussed above it is only in this case that a meaningful distinction can be made between the two models in the Drell-Yan process, and because there is an independent estimate of m_g (Pati *et al* 1977) which puts its value at 1-2 GeV. Since the colour gluon effects are confined to the low $\sqrt{\tau}$ region, the results are not sensitive to the valence distribution functions used. For example the parametrisation (12) and (13) for the valence quarks gives the same shape for $M^3 d\sigma/dMdy|_{y=0}$, and produce an essentially same enhancement in the low $\sqrt{\tau}$ region for a particular sea distribution. However, the amount of enhancement is sensitive to the sea distribution used.

To study this sensitivity, we shall use the valence distributions given in (12) and consider sea distribution of the usual form

$$x\, s(x) = B(1-x)^n, \tag{15}$$

but vary both B as well as n. The results are not sensitive to n, different values leading to the same shape for the cross-section, but are sensitive to B. Unfortunately the normalisation of the sea cannot be fixed accurately by the present data on electron and muon scattering where there is an experimental limit on x. At SLAC the limit is $x \gtrsim 0{\cdot}1$, whereas at Fermilab one can go upto $x \gtrsim 0{\cdot}01$ at $Q^2 \simeq$ 2-5 GeV^2. In the context of gauge theories with colour excitation there are two ways to fix up the normalisation of the sea distribution, determined essentially by the location of the threshold for producing coloured states. If, as commonly assumed (Pati 1976), colour has indeed been excited in Fermilab μp experiments, but not in SLAC ep experiments, then we have $F_2^{\mu p}(x\to 0) \gg F_2^{ep}(x\to 0)$ because of the added contributions in the former due to the colour octet electromagnetic current of quarks and gluons. If, on the other hand, coloured states are not produced either in SLAC or Fermilab deep inelastic lepton-hadron experiments but can only be produced in the ISR energy range, which to date is the highest energy available, then we have $F_2^{\mu p}(x\to 0) = F_2^{ep}(x\to 0)$. We thus study the sensitivity of the present phenomenon to the above assumptions for $F_2^{\mu p}$: (i) $F_2^{\mu p} \gg F_2^{ep}$ for $x\to 0$, and (ii) $F_2^{\mu p} \simeq F_2^{ep}$ for $x \to 0$ both in the region $Q^2 \simeq 2-5$ GeV^2. As mentioned earlier, for definiteness we shall use the parametrisation (12) for the valence distributions and consider sea distributions consistent with two alternatives mentioned above.

Recently Barger and Phillips (1978) have extracted sea distributions from the Drell-Yan data in the region $0{\cdot}2 \lesssim \sqrt{\tau} \lesssim 0{\cdot}5$. It turns out that the two viable alternatives (at $Q^2 = Q_0^2 \sim 2$ GeV^2)

$$x\, s(x) = 0{\cdot}08\,(1-x)^5 \tag{16}$$

and

$$x\, s(x) = 0{\cdot}25\,(1-x)^9, \tag{17}$$

which they find, essentially meet our requirements (i) and (ii), respectively. In the following we shall take the x dependence of the gluon distribution function (14) to be the same as sea distribution (16) or (17) as the case may be.

In figure 2 we show the predictions of the exact colour symmetry model (without asymptotic freedom corrections) for $M^3 d^2\sigma/dMdy|_{y=0}$ in pp collisions using parametrisation (17) by the solid curve. Predictions of the excitable colour model at $\sqrt{s}=62$ GeV for the parametrisation (17) (dashed curve) and (16) (dotted curve) are also shown and compared with data. Predictions of the excitable colour model at $\sqrt{s}=52$ GeV do not differ much from those at $\sqrt{s}=62$ GeV and are not shown separately. The unbroken colour model prediction for parametrisation (16) lies considerably below the data points and is not shown here. For $\sqrt{s}=28$ GeV there is no significant difference between the predictions of the two models, and both of them essentially reproduce the solid curve. From figure 2 we clearly see that it is difficult to distinguish between the two models with the present accuracy of the data.

Finally, above colour threshold with $s(x)$ given by (16), theories with colour excitation predict

$$F_2^{\mu p}\,(x = 0{\cdot}01) \simeq 0{\cdot}52, \text{ at } Q^2/m_g^2 = 2, \tag{18}$$

$$F_2^{\mu p}\,(x = 0{\cdot}01) \simeq 0{\cdot}34, \text{ at } Q^2/m_g^2 = 10,$$

i.e. $F_2^{\mu p}$ is a rapidly decreasing function of Q^2. In fact $F_2^{\mu p}$ in such a model is a decreasing function of Q^2 for all x. While the available data (Anderson *et al* 1976, 1977) is not inconsistent with such a behaviour at large x, the predicted sharp fall off with Q^2 at small x (equation 18) seems to run counter to the observed behaviour and the pattern of scaling violation predicted by an asymptotically free theory. However, as emphasised by Pati and Salam (1976), above colour threshold $F_2^{\mu p}$ in the excitable colour model will be a decreasing function of Q^2 for all x only when $Q^2 \gtrsim 2m_g^2$. For $Q^2 < 2m_g^2$ the colour octet part, and hence the total, of $F_2^{\mu p}$ will be an increasing function of Q^2 till it acquires its full scaling weight at $Q^2 \simeq 2m_g^2$. For such a model the region $Q^2 < 2m_g^2$ is a threshold region. This is analogous to the behaviour of the colour singlet part of F_2 which, although it scales for $Q^2 \simeq 1$ GeV^2, acquires its full scaling value only for $Q^2 \gtrsim 2$ m$_\rho^2$. Thus the predicted behaviour (18) of $F_2^{\mu p}$ at small x in excitable colour models will set in only for $Q^2 \gtrsim 5$ GeV^2 (if m_g=1–2 GeV). In the present muon scattering experiments at small x, Q^2 is constrained to be 2–5 GeV^2 because the beam energy is small. In the light of the above discussion, $F_2^{\mu p}$ in this Q^2 range will be increasing with Q^2 which is not inconsistent with what is observed. On the other hand, in the large x region of the muon scattering experiments $Q^2 \gtrsim 5$ GeV^2, and $F_2^{\mu p}$ is a decreasing function of Q^2 consistent with the predictions of the broken colour symmetry model. However, it is possible that the decrease in the structure function with increasing Q^2 at large x could only be partly due to colour gluon effects and in a more refined analysis one should also take into account logarithmic scale breaking effects expected on the basis of an asymptotically free theory.* This would require more accurate data and, given this, would also require the exact location of colour threshold and the colour gluon mass. What we wish to emphasise here is that for $Q^2/m_g^2 \gtrsim 2$ the gluons having a sealike distribution (16) give a sizeable contributions to F_2 at small x (F_2^{NS} is the colour nonsinglet part of F_2)

*The spontaneously broken colour gauge theory, where the symmetry is broken by elementary scalar fields, will be 'temporarily' asymptotically free in the present energy regime as long as the effective quartic couplings of the scalar fields at present energies are sufficiently small ($\lesssim e$). Otherwise asymptotic freedom applies if the spontaneous symmetry breaking is dynamical.

e.g.,

$$\frac{F_2^{\rm NS}(0{\cdot}01)}{F_2(0{\cdot}01)} \simeq 0{\cdot}40,$$
$$\frac{F_2^{\rm NS}(0{\cdot}1)}{F_2(0{\cdot}1)} \simeq 0{\cdot}25 \tag{19}$$

for $Q^2/m_g^2 = 10$. Thus a large increase ($\sim 50\%$) in structure functions at small $x(\simeq 0{\cdot}01)$ with $Q^2 \gtrsim 5$ GeV^2 is likely to signal colour gluon contribution if the gluons have a sea-like momentum distribution. Such an effect at small x cannot be seen in the present day muon scattering experiments where, because the energy is small, Q^2 is constrained to be 2–5 GeV^2. However, it can be seen in the Drell-Yan process at ISR energies where the sea (small $\sqrt{\tau}$) region can be probed for large Q^2. Note that such gluon effects will not be clearly visible in the integrated structure function F_2 (Pati and Salam 1976; Rajasekaran and Roy 1976) nor in the integrated Drell-Yan cross-section because the gluon effects die out rapidly for large x. Of course if the colour is not excited in the muon scattering experiments (sea distribution given by (17) then the colour effects can only be seen in the ISR experiments.

4. Conclusions

We have studied the predictions of the spontaneously broken colour gauge theory for the dimuon production in proton-proton collisions under two rather different assumptions. (i) The threshold for producing coloured hadron states has already been crossed in the muon scattering experiments at Fermilab, or (ii) the threshold has not been crossed at Fermilab but lies somewhere in the ISR region ($\sqrt{s} \simeq 50$). At the highest ISR energy ($\sqrt{s}=62$) we have compared these predictions for $M^3 d^2\sigma/dMdy|_{y=0}$ with what is expected from the quark parton model (with three colours), and shown that they can differ by as much as a factor of 3 – 4. While the present data (Cobb *et al* 1978) is unable to distinguish between the three alternatives, namely (i) no colour excitation at ISR, (ii) Colour excitation at ISR but not at Fermilab and (iii) colour excitation at ISR and Fermilab but not at SLAC, we hope that more accurate data at ISR ($\sqrt{s}=62$ GeV) in the region $0{\cdot}03 \lesssim \sqrt{\tau} \lesssim 0{\cdot}2$ will help in settling the issue. We have not taken into account the conventional scale breaking effects which will only add to the present effect and will not change our conclusions.

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