Pramāna, Vol. 12, No. 1, January 1979, pp. 77–88, © printed in India

Connection between elastic scattering and inclusive k_T^2 distribution

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MS received 14 September 1978

Abstract. A connection between the elastic scattering and the inclusive one-particle k_T^2 distribution is pointed out in the context of the s channel unitarity. One of the implications of this connection is that the slope of the k_T^2 distribution at $k_T^2 = 0$ is about a factor two larger than the slope of the elastic scattering at $t = 0$.

Keywords. Elastic and inelastic hadron collisions; s-channel unitarity; inclusive k_T^2 distribution; elastic t distribution.

1. Introduction

Several attempts have been made to understand the observed features of high energy elastic scattering of hadrons starting from s channel unitarity (Zachariasen 1971). One specifies a model for the multiparticle production amplitudes or parametrises the so-called inelastic overlap function and uses unitarity to relate it to the absorptive part of the elastic amplitude (Van Hove 1963, 1964; Henzi and Valin 1974; Miettinen 1974). The purpose of this paper is to exhibit the connection between the elastic scattering and the one-particle inclusive k_T distribution on the basis of the s channel unitarity. In §2 we present a geometrical description of the unitarity, which brings forth the intimate connection between the two distributions qualitatively, while the corresponding expressions are given in §3. In §4 we note the implications for the slopes of the t and k_T^2 distributions for two simple examples of profile functions. In §5, we comment on the slope of the inclusive k_T^2 distribution using the available experimental data. In §6 we present our derivation of the connection between the k_T^2 and t distributions. Starting from an N-particle amplitude which is factorisable in the k_T and k_L variables we derive in §7 a relation involving the slope parameters and the elastic and total cross sections. We conclude with some remarks in §8.

2. Geometrical argument based on unitarity

Experimental data suggest that the particles emitted in the scattering and in the production processes exhibit similar damping in the values of transverse momenta (Narayan and Sarma 1963; Krisch 1963 and Orear 1964). This suggests that the inclusive one-particle k_T^2 distribution may have a close connection to the t distribution in the elastic scattering. The existence of such a connection can be inferred through

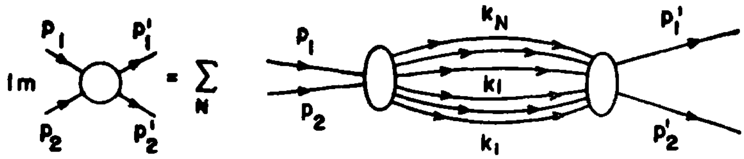

Fig. 1a

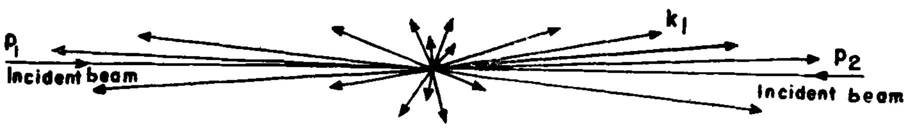

Fig. 1b

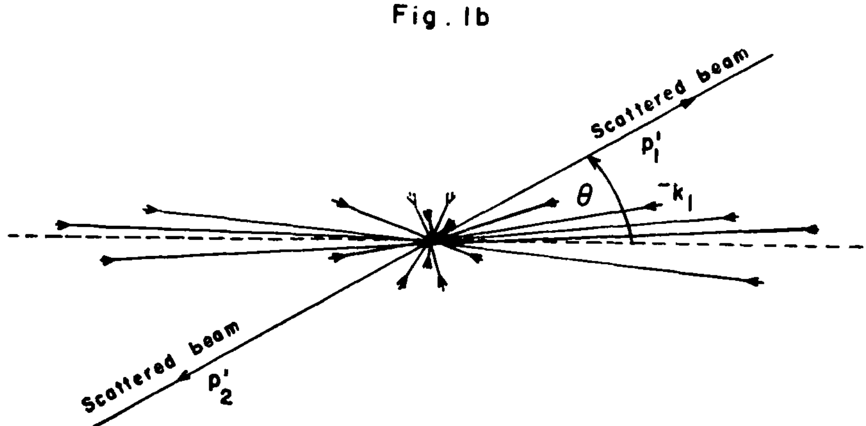

Fig. 1c

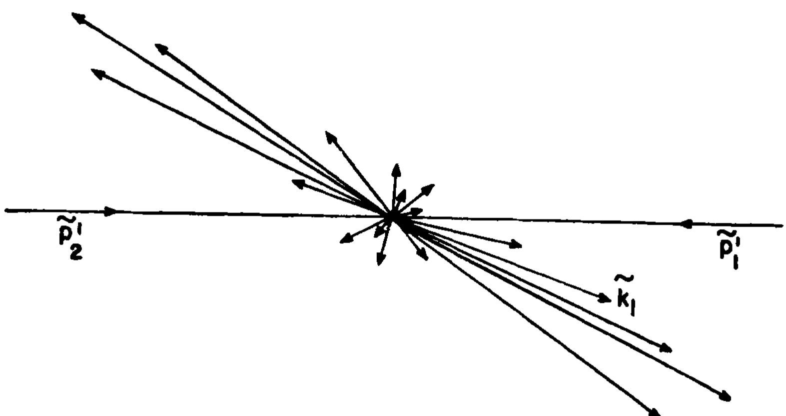

Fig. 1d

Figure 1. A schematic representation of the unitarity relation and the implications of the damping of transverse momenta in multiparticle amplitudes (see text).

the constraint imposed by unitarity (Narayan 1968). To visualise the nature of this constraint, suppose that the values of the transverse momenta k_T are damped in a production process, while there is no such constraint on the k_T values in a scattering process. We examine the implications of this supposition for the unitarity, which is shown schematically in figure 1a.

The right hand side of the unitarity relation in figure 1a involves two physical processes; one in which two particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ interact to produce a variable number of N particles with momenta $\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_N$, all with limited transverse momenta with respect to the incident beam as shown in figure 1b, and the other, in which the produced particles interact to emit the final scattered particles with momenta $\mathbf{p}'_1$ and $\mathbf{p}'_2$, as shown in figure 1c. We now reverse the directions of all particles in figure 1c and rotate all momenta about the origin through an angle θ (θ being the angle of the elastic scattering in the CM system), so as to bring the direction of the scattered beam to coincide with the direction of the incident beam. The resulting event with momenta $\tilde{k}_i$ $(i=1, 2, \ldots, N)$, shown in figure 1d, represents a physically realisable production process. When the scattering angle θ is large, corresponding to large k_T values of the scattered particles, the event of figure 1d is in conflict with the typical configuration depicted in figure 1b, which reflects our assumption about the damping of transverse momenta in a production process. In order that the event in figure 1d may be compatible with the event in figure 1b, we require that the particles scattered in an elastic event have also limited transverse momenta.

The converse of the above argument, however, is not valid. Starting with the assumption that the elastic t-distribution is peaked about $t=0$, one sees that unitarity alone would not forbid the occurrence of an average inelastic collision involving relatively large k_T values.

3. Relation between k_T^2 and t distributions

We shall assume for simplicity that there is only one type of particle and that it has no spin or internal degrees of freedom. A simple generalisation of our analysis can be made to incorporate internal degrees of freedom. The elastic scattering amplitude $f(s, t)$ satisfies the s-channel unitarity relation,

$$\operatorname{Im} f(s, -\mathbf{q}^2) = \frac{1}{4\pi k} \int d^2q' f^*(s, -\mathbf{q}'^2) f(s, -(\mathbf{q}-\mathbf{q}')^2) + F(s, t), \tag{1}$$

where s and t are the usual Mandelstam variables, k is CM momentum and $t=-\mathbf{q}^2$. This relation takes a simple form in the impact parameter space (Blankenbecler and Goldberger 1962; Amaldi *et al* 1976),

$$2 \operatorname{Im} \eta(s, b) = |\eta|^2 + G(s, b), \tag{2}$$

where $$\eta(s, b) = \frac{1}{2\pi k} \int d^2 \exp(i\mathbf{b}\cdot\mathbf{q}) f(s, b - \mathbf{q}^2) \tag{3}$$

and $$G(s, b) = \frac{1}{\pi k} \int d^2q \exp (i\mathbf{b}\cdot\mathbf{q})\, F(s, -\mathbf{q}^2)$$

$$= \frac{2}{k} \int_0^\infty q dq\, J_0 (bq)\, F(s, -\mathbf{q}^2). \tag{4}$$

The function $\eta(s, b)$ is referred to as the profile function and $G(s, b)$ is the Van Hove overlap function. Assuming that the elastic amplitude is purely imaginary and goes to zero in the limit $G \to 0$ (shadow scattering) the differential cross-section $d\sigma_{el}/dt$ is given by

$$d\sigma_{el}/dt = \pi \left| \int_0^\infty bdb\, J_0 (b \sqrt{-t})\, [1-\sqrt{1-G}] \right|^2. \tag{5}$$

By inverting the relation in (4) for $F(s, t)$ and putting $t=0$, one gets

$$F(s, o) = (k/2) \int_0^\infty bdb\, G(s, b). \tag{6}$$

From the definition of $F(s, t)$, one can also show (Tiktopoulos and Treiman 1972) that

$$F(s, o) = (k/2) \int_0^\infty k_T\, dk_T\, \frac{d^2 \sigma_{\text{inc}} (k_T)}{d^2k_T} \tag{7}$$

Here the inclusive k_T -distribution is defined by

$$\frac{d^2\sigma_{\text{inc}}(k_T)}{d^2k_T} = \frac{1}{2} \int_{-1}^{+1} dx\, k_0 \frac{d^3\sigma_{\text{inc}}(\mathbf{k})}{d^3k} \tag{8}$$

where $$k_0 \frac{d^3\sigma_{\text{inc}}(\mathbf{k})}{d^3k}$$

is the usual one particle inclusive cross-section and $x = 2k_L/\sqrt{s}$ is the Feynman variable. Equations (5), (6) and (7) clearly exhibit a connection between

$$\frac{d^2\sigma_{\text{inc}}(k_T)}{d^2k_T} \text{ and } \frac{d\sigma_{el}}{dt}.$$

A simple and suggestive ansatz to satisfy the equality of the integrals on the right hand sides of (6) and (7) is

$$\sqrt{G(s,b)} = \frac{1}{\lambda} \int_0^\infty k_T\, dk_T\, J_0\left[\frac{bk_T}{\lambda}\right] \left[\frac{d^2\sigma_{\text{inc}}(k_T)}{d^2 k_T}\right]^{1/2}, \tag{9a}$$

or the inverse relation

$$\left[\frac{d^2\sigma_{\text{inc}}(k_T)}{d^2 k_T}\right]^{1/2} = \frac{1}{\lambda} \int_0^\infty b\,db\, J_0\left[\frac{bk_T}{\lambda}\right] \sqrt{G(s,b)} \tag{9b}$$

where λ is a constant. A derivation of the relation (9b) is provided in §6 and 7 in terms of N particle amplitudes, wherein it turns out that the λ is the root mean square value of x. A relation of the type in (9) seems to be implied also in the work of Savrin *et al* (1976).

4. Simple examples

We do not attempt any detailed discussion of (5) and (9) because of absence of data on x-integrated k_T^2 distributions at high energies. We want to point out, however, their implication for the slopes of the elastic scattering and the inclusive k_T^2 distributions at $t=0$ and $k_T^2 = 0$, respectively, by considering some examples of the profile function $\eta(s, b)$.

For the case of scattering from an opaque disc of radius R,

$$\eta(b, s) = iC \text{ for } b < R,$$
$$= 0 \text{ for } b > R,$$

we obtain (for $\lambda=1$)

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{\pi C^2 R^2}{q^2} [J_1(Rq)]^2,$$

$$\frac{d\sigma_{\text{inc}}}{d(k_T^2)} = \pi(2C - C^2) \frac{R^2}{k_T^2} [J_1(Rk_T)]^2,$$

implying that the normalised k_T^2 distribution of the inclusive cross-section is identical to the normalised t-distribution of the elastic scattering and has a slope given by $R^2/4$. For $\lambda < 1$, the k_T^2 distribution has a larger slope than the t-distribution.

Another example of physical interest is the Gaussian profile

$$\eta = iC \exp(-b^2/R^2)$$

for which we obtain

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{\pi C^2 R^4}{4} \exp(R^2 t/2)$$

$$\frac{d\sigma_{inc}}{d(k_T^2)} = 2\pi C R^4 \Big[\exp(-R^2k_T^2/2) - \frac{C}{12}\exp(-R^2k_T^2/6) - \frac{C^2}{160}\exp(-R^2k_T^2/10-)\ldots\Big]^2$$

implying that the slope of the k_T^2 distribution is steeper than the slope of the elastic scattering by a factor 2. Except for the case of the opaque disc, the slope of the inclusive k_T^2 distribution seems to be generally larger than the slope of the elastic scattering, a feature which, as shown in § 7, follows directly by considering a simple form of the N particle amplitudes.

5. Comparison with experiment

There is no experimental data for the x-integrated k_T^2 distribution at small k_T to fix the slope at $k_T^2=0$. But there are accurate data at ISR energies for the inclusive k_T^2 distribution for pions at $x=0$ in the range $0{\cdot}0016 < k_T^2 < 0{\cdot}16$ (GeV/c)^2 (Guettler 1976). The distribution can be well fitted to a form

$$\exp[-\beta(k_T^2+m_\pi{}^2)^{1/2}] \text{ with } \beta=7{\cdot}1 \text{ (GeV/c)}^{-1}.$$

This gives a slope for the inclusive k_T^2 distribution equal to $\beta/2m_\pi=25$ (GeV)$^{-2}$. Assuming an approximate factorisation in the inclusive x and k_T^2 distributions, the x-integrated k_T^2 distribution would be the same as the k_T^2 distribution at $x=0$. The ISR experiments thus suggest that the slope of the k_T^2 distribution is about two times the slope of the t distribution which has a value $=12$(GeV)$^{-2}$, consistent with the expectations based on (5) and (9).

6. Derivation of the results

We give a brief derivation of the result in (9b). The function $F(s, t)$ in (1) is, by definition, given by

$$F(s,t) = \frac{1}{16\pi\sqrt{s}}\sum_{N=3}^{\infty}\frac{1}{N!}\int\prod_i(dk_i)(2\pi)^4\delta^4(k_1+k_2+\ldots+k_N-P_1-P_2)$$

$$T^*_N(k_1k_2\ldots k_N;\, P_1P_2)T_N(k_1k_2\ldots k_N;\, P'_1P'_2), \tag{10}$$

where

$$(dk_i) \equiv \frac{d^3k_i}{(2\pi)^3 2k_{io}}$$

and T_N is the N-particle amplitude. We rewrite (10) following a procedure adopted by Tiktopoulos and Treiman (1972). After performing integrations with respect to $\mathbf{k}^3$

and k_{30} by making use of the δ-functions in the integrand in (10), $F(s, t)$ can be rewritten as:

$$F(s, t) = \frac{k}{4\pi} \int_0^\infty k_{1T}\, dk_{1T} \int_0^{2\pi} d\phi \int_0^1 dx$$

$$\sum_{N, V} \rho_N (V) \tau^*_N(k_{1T}, k_{1L}, \phi_1; V) \tau_N (k'_{1T}, k'_{1L}, \phi_1; V), \tag{11}$$

where $\quad k_{1T}(k'_{1T}) = k_1 \sin\theta_1(\sin\theta'_1);\ k_{1L}(k'_{1L}) = k_1 \cos\theta_1(\cos\theta'_1). \quad (12)$

θ_1, ϕ_1 (θ'_1, ϕ'_1) are the polar and azimuthal angles of the incident (scattered) beam with respect to the co-ordinate system in which the z-axis is taken along $\mathbf{k}_1$; V stands for the variables θ_3, ϕ_3, $\mathbf{k}_4, \ldots \mathbf{k}_N$ and $\sum_{N, V} \rho_N (V)$ stands for the integration operator

$$\sum_{N, V} \rho_N (V) = \frac{1}{2(2\pi)^3 4k\sqrt{s}} \sum_N \frac{(2\pi)^4}{(N-1)!} \int d\Omega_3 (dk_4) \ldots (dk_N),$$

and $$\tau_N = \left[\frac{k_3}{k_{20}} \left[\frac{\partial f}{\partial k_{30}}\right]^{-1}\right]^{1/2} \cdot\ T_N, \tag{13}$$

$$f = k_{10} + k_{20} + \ldots + k_{N0}.$$

As we are ignoring spins in the problem, the N-particle amplitude T_N depends on the variables k_1, θ_1, ϕ_1 only through the variables k_{1T} and k_{1L} and does not depend on ϕ_1, and by virtue of (13), τ_N is also independent of ϕ_1. Further the x-integrated inclusive k_T distribution can be written as,

$$\frac{d^2 \sigma_{\rm inc} (k_T)}{d^2 k_T} = \int_0^1 dx \sum_{N, V} \rho_N (V) \,|\, \tau_N (k_T, x; V) |^2. \tag{14}$$

As seen from (11), the dependences of $F(s, t)$ on t is through the variables k'_{1T} and k'_{1L} present in the factor $\tau_N(k'_{1L}, k'_{1T}; V)$. Most of the dependence on t, however, comes through the variable k'_{1T} and only a very weak dependence from k'_{1L}. To see this, we make a Taylor expansion of $\tau_N (k'_{1L}, k'_{1T}; V)$ about k_{1L} as

$$\tau_N (k'_{1L}, k'_{1T}; V) = \tau_N (k_{1L}, k'_{1T}; V) + \triangle k_{1L}\, a_L + \ldots, \tag{15}$$

where $$a_L \equiv \frac{\partial}{\partial k_{1L}} \tau_N (k_{1L}, k'_{1T}; V)$$

and from the relations

$$\cos\theta'_1 = \cos\theta \cos\theta_1 + \sin\theta \sin\theta_1 \cos\phi_1,$$

$$\cos\theta = 1 + (2t/4k^2),$$

$$\sin\theta = 2(-t/4k^2)^{1/2}\,[1+(t/4k^2)]^{1/2} \simeq 2(-t/4k^2)^{1/2} \text{ for } |t| \ll 4k^2,$$

one has approximately

$$\triangle k_{1L} = 2(-t/4k^2)^{1/2}\,(k_{1T}\cos\phi_1 - (k_{1L}/2k)(-t)^{1/2}), \tag{16a}$$

$$k'^2_{1T} = k^2_{1T} + (2k_{1L}\,k_{1T}/k)(-t)^{1/2}\cos\phi_1 + (k^2_{1L}/k^2)(-t). \tag{16b}$$

From (16a) one finds that $\triangle k_{1L}$ decreases as $1/\sqrt{s}$ for a fixed t. If the one-particle inclusive distribution exhibits scaling in the Feynman variable $x_1 = 2k_{1L}/\sqrt{s}$, we expect a_L to decrease as $1/\sqrt{s}$. So the second term on the RHS of (15) decreases as $1/s$ and the dependence of $\tau_N(k'_{1T}, k'_{1L}; V)$ on t through k'_{1T} cannot be ignored even at high energies. The reasons for this are two-fold; as the values of k_{1L} and k may be comparable, the difference $k'^2_{1T} - k^2_{1T}$ given by (16b) remains always finite, and there is no Feynman scaling in the variable k_{1T}.

The above analysis shows that $\tau_N(k'_{1T}, k'_{1L}; V)$ in (11) can be replaced by $\tau_N(k'_{1T}, k_{1L}; V)$. We now introduce the transformed N-particle amplitude $\bar{\tau}_N(b, k_{1L}; V)$ defined by

$$\tau_N(k_{1T}, k_{1L}; V) = \int_0^\infty b\,db\,J_0(bk_{1T})\,\bar{\tau}_N(b, k_{1L}, V). \tag{17}$$

Using (17) in (11), we get

$$F(s,t) = (k/4\pi)\int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 \int_0^\infty k_{1T} dk_{1T} \int_0^1 dx_1\, J_0(b_1 k_{1T})$$

$$\int_0^{2\pi} d\phi_1\, J_0(b_2 k'_{1T}) \sum_{N,V} \rho_N(V)\, \bar{\tau}^*_N(b_1, k_{1L}; V)\, \bar{\tau}_N(b_2, k_{1L}; V). \tag{18}$$

Using (16b) in (18), the integration with respect to ϕ_1 can be performed (Watson 1966) to give

$$\int_0^{2\pi} d\phi_1\, J_0(b_2\, k'_{1T}) = 2\pi J_0(b_2 k_{1T})\, J_0(b_2 x_1\sqrt{-t}). \tag{19}$$

Inserting (19) in (18) and performing k_{1T} integration, one finally gets

$$F(s,t) = (k/2)\int_0^\infty b\,db \int_0^1 dx_1\, J_0(bx_1\sqrt{-t})\, U(b, b; x_1). \tag{20}$$

Further $$\frac{d^2\sigma_{\rm inc}}{d^2k_T} = \int_0^\infty b_1\, db_1\, J_0(b_1 k_T) \int_0^\infty b_2\, db_2\, J_0(b_2 k_T) \int_0^1 dx_1\, U(b_1, b_2; x_1) \tag{21}$$

where $$U(b_1, b_2; x_1) = \sum_{N,V} \rho_N(V)\, \bar{\tau}_N^*(b_1, k_{1L}; V)\, \bar{\tau}_N(b_2, k_{1L}; V). \tag{22}$$

Using the mean value theorem, we rewrite (20) as

$$F(s, t) = (k/2) \int_0^\infty b db\, J_0(b\lambda\sqrt{-t})\, R(s; b, b), \tag{23}$$

where $$R(s; b_1, b_2) = \int_0^1 dx\, U(b_1, b_2; x), \tag{24}$$

and λ is some mean value of x which may in general depend on $b\sqrt{-t}$. It turns out, however, that this dependence is weak. For example, the integral of $x^n J_0(yx)$ ($y=b\sqrt{-t}$, $n=1, 2,\ldots$) in the range 0 to 1, can be evaluated in a closed form which may be compared with the value obtained by the mean value theorem. We have found that λ lies between 0·6 and 0·9 for $n \leqslant 10$ and $0 \leqslant y \leqslant 4$, with an average around 0·8. Now the functional dependence of $U(b, b; x)$ in x is essentially the inclusive x distribution, which can be regarded as a polynomial in x whose coefficients can be expected to be damped functions in b. In such a case, the example considered above is relevant and we infer that the value of λ in (23) would have a weak dependence on the value of the multiplicative factor $b\sqrt{-t}$ in $J_0(\lambda b\sqrt{-t})$. We thus regard λ as a fixed parameter to be determined by experiment.

Differentiating with respect to t both (20) and (23) and taking the limit $t\to 0$, one finds

$$\lambda^2 = \frac{\int_0^1 x^2\, U(b, b; x)\, dx}{\int_0^1 U(b, b, x) dx}$$

which shows that λ is the root mean square $\sqrt{\langle x^2\rangle}$ of the inclusive x distribution. Inserting (24) in (4) and performing the integration, one finds

$$G(s, b) = (1/\lambda^2)\, R(s; b/\lambda, b/\lambda). \tag{25}$$

Next we assume that the transverse momenta of the particles in N-particle amplitude are limited in magnitude and are uncorrelated except for the constraint of energy momentum conservation. This assumption implies that the N-particle amplitude $\tau_N(k_{1T}; x_1; V)$ factorises as $h(k_{1T})\beta_N(x_1; V)$, where we take $h(k_{1T})$ to be real. In the impact parameter space the factorisation of the N-particle amplitude implies that the function $R(s;\, b_1, b_2)$ factorises

$$R(s;\, b_1, b_2) = g(s, b_1)\, g(s, b_2), \tag{26}$$

and (25) becomes

$$G(s, b) = \frac{1}{\lambda^2}\, [g(s, b/\lambda)]^2. \tag{27}$$

Substituting (24) and (26) in (21) and using (27) we get (9) which together with (5) gives the desired connection between

$$\frac{d^2\sigma_{\rm inc}(k_T)}{d^2k_T} \text{ and } d\sigma/dt.$$

The generalisation to include internal degrees of freedom involves only a simple extension. When there are more than one type of particles (9a) gets replaced by

$$\sqrt{G(s, b)} = \sum\nolimits_c (1/\lambda_c) \int_0^\infty k_T dk_T\, J_0\, (b\, (k_T/\lambda_c))\, (d\sigma^c_{\rm inc}/d^2k_T)^{\frac{1}{2}} \tag{28}$$

where Σ_c denotes the summation over the different types c of particles. If the differences in the values of λ_c's are small, one may replace λ_c in (28) by an average value $\bar{\lambda}$ and (28) takes the form:

$$\sqrt{G(s, b)} = 1/\bar{\lambda} \int_0^\infty k_T dk_T\, J_0\, (b\, (k_T/\bar{\lambda})\, \sum\nolimits_c (d\sigma^c_{\rm inc}/d^2k_T)^{\frac{1}{2}} \tag{29}$$

whose inverse is

$$\sum\nolimits_c (d\sigma^c_{\rm inc}/d^2k_T)^{1/2} = 1/\bar{\lambda} \int_0^\infty b db\, J_0\, (b\, (k_T/\bar{\lambda}))\, \sqrt{G(s, b)}. \tag{30}$$

7. Relation between slopes for factorisable N-particle amplitudes

The factorization of $\tau_N(k_{1T}, x_1;\, V)$ has been important in our derivation of (9). The implications of the factorisation can perhaps be seen more directly by starting with the following simple form;

$$\tau_N(k_{1T}, x_1;\, V) = \exp\, [-(a/2)k^2_{1T}]\, \beta_N(x_1;\, V). \tag{31}$$

Using (31) in (11), one can show that

$$F(s, t) = \frac{k}{8a} \int_0^1 dx_1 \sum\nolimits_{N,\, V} \rho_N(V)\, |\, \beta_N\, (x_1, V)\, |^2 \exp -(ax_1^2\, t/4)$$

and hence $$\left[\frac{\partial F(s,t)}{\partial t}\right]_{t=0} = \frac{a}{4}\langle x^2\rangle F(s,o)$$

$$= \frac{a}{4}\langle x^2\rangle \frac{k}{4\pi}(\sigma_T - \sigma_{el}). \tag{32}$$

Assuming that the scattering amplitude has the form

$$f(s,t) = i(k\sigma_T/4\pi)\exp[(B/2)t] \tag{33}$$

one obtains

$$\left[\frac{\partial}{\partial t}\operatorname{Im} f(s,t)\right]_{t=0} = \frac{Bk\sigma_T}{8\pi} \tag{34}$$

$$\left[\frac{\partial}{\partial t}\int d^2q' f^*(s,-\mathbf{q}'^2) f(s,-(\mathbf{q}-\mathbf{q}')^2)\right]_{t=0} = \frac{\pi}{4}\left(\frac{k\sigma_T}{4\pi}\right)^2. \tag{35}$$

Using (32), (34) and (35) in (1), we obtain

$$a = \frac{2B}{\langle x^2\rangle}\ \frac{\sigma_T}{\sigma_T - \sigma_{el}}\left[1 - \frac{\sigma_T}{32\pi B}\right]. \tag{36}$$

Once again, we see in (36) that the slope of the inclusive k_T^2 distribution is about a factor two larger than that of the elastic scattering. This result is (36) would also be valid in the uncorrelated Jet Model and it is interesting to compare our result with that obtained by de Groot (1972).

8. Concluding remarks

We have shown that a damping in the values of k_T^2 in production processes implies, as a consequence of s channel unitarity, a similar damping in the values of $|t|$ in the elastic scattering. One of the implications of our results is that the slope of the k_T^2 distribution at $k_T^2=0$ is about a factor two larger than the slope of the elastic scattering at $t=0$. There is indirect experimental evidence in support of the above conclusion. One, however, needs to test the relation among the slopes (equation 36) using data on x-integrated inclusive cross-sections.

The relations in (5) and (9a) can be used to construct the differential elastic scattering cross section, using the information about k_T^2 distributions in production processes. The dip in pp scattering at $-t \simeq 1{\cdot}4$ (GeV/c)^2 has been a long standing mystery. The dip has been reproduced by Henzi and Valin (1974) by choosing the profile function to be the sum of two functions of different forms. But the choice of the two functions has been arbitrary and no physical significance can be attached to the functions. We suggest the possibility that the dip could arise due to the presence of two or more components in the k_T^2 distribution. The problem is to get a suitable parametrisation of the experimental data on k_T^2 distribution as a sum of two or more functions. The choice of the functions is dictated by the requirement that they reproduce the dip on the basis of (5) and (9a). The approach is similar to that of Henzi and Valin (1974) but the distribution functions and hence also the corresponding profile functions would have physical significance.

Acknowledgement

We would like to thank Professor V Singh for his helpful comments.

References

Amaldi U, Jacob M and Matthiae G 1976 *Ann. Rev. Nucl. Sci.* **26** 385
Blankenbecler R and Goldberger M L 1962 *Phys. Rev.* **126** 766
de Groot E H 1972 *Nucl. Phys.* **B48** 295
Guettler K *et al* 1976 *Phys. Lett.* **B64** 111
Henzi R and Valin P 1974 *Phys. Lett.* **B48** 119
Krisch A D 1963 *Phys. Rev. Lett.* **11** 217
Miettinen 1974 CERN Preprint, TH. 1864—CERN
Narayan D S 1968 *Phys. Rev.* **176** 2154
Narayan D S and Sarma K V L 1963 *Phys. Lett.* **5** 365
Orear J 1964 *Phys. Rev. Lett.* **12** 112
Savrin V I, Semenov S V and Tyurin N E 1976 *Sov. J. Nucl. Phys.* **23** 447
Tiktopoulos G and Treiman S B 1972 *Phys. Rev.* **D6** 2045
Van Hove L 1963 *Nuovo Cimento* **28** 798
Van Hove L 1964 *Rev. Mod. Phys.* **36** 655
Watson G N 1966 *A treatise on the theory of Bessel functions* (Cambridge: University Press) ed 2. p. 367
Zachariasen F 1971 *Phys. Rep.* **C2** 1