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The resolution of a paradox which arises in the elementary discharge theory of an ion source

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Abstract. It is shown that the inclusion of a small amount of primary ionisation makes the solution to the discharge equilibrium problem single valued.

Keywords. Ion source; elementary discharge theory.

Although ion sources are of considerable interest for thermonuclear research, substantial improvements are needed in both efficiency and charge species selectivity (Martin and Green 1976). Theoretical modelling of these discharges (Green and Goble 1974) begins with the application of the elementary equations of particle and energy balance and has application to laboratory plasmas (Bollinger *et al* 1972) and thermonuclear experiments as well (for instance, the Levitron device, Riviere and Jones unpublished). The archetypal equation of this sort is the balance equation for ion production and ion loss in a monatomic gas (Martin and Green 1976):

$$n_i/n_n = t_{\text{loss}}/t_{\text{ionise}} = t_{\text{loss}}\, n_{e'} \langle \sigma\, v_{e'} \rangle_{\text{ionise}} \tag{1}$$

with $n_e = n_i$

where n_i, n_e, and n_n are the ion, electron, and neutral particle densities, t_{loss} and t_{ionise} are the ion containment time and generation time, and $\langle \sigma\, v_{e'} \rangle_{\text{ionise}}$ is the electron-gas ionisation rate coefficient. $n_{e'}$ is either the primary electron density, or simply reduces to the electron plasma density, depending upon the appropriate source of gas ionisation. If primary ionisation dominates, $v_{e'}$ is the known velocity of primaries. If primaries can be neglected then $v_{e'}$ is related to the suitable average involving thermalised plasma electrons. n_e is the total electron density in the source.

For present day Penning ion Gauge sources at low magnetic field (and various other designs as well) ion loss can be treated (Green and Goble 1974) according to the 'Langmuir free fall' model (Tonks and Langmuir 1929 and Langmuir 1961) giving:

$$t_{\text{loss}} = V/(A\alpha c_s), \tag{2}$$

where V is the source chamber volume, A is the area of the source walls (electrodes), c_s is the ion acoustic speed, and α is a coefficient of order unity. Simply explained, to maintain plasma charge neutrality electron loss AJ_e/e with:

$$J_e = e\, n_e\, (2KT_e/m_e)^{1/2} \exp\,(eU/KT_e), \tag{3}$$

must just balance ion loss

$$A\, J_{i_{sat}}/e = A\, J_e/e, \tag{4}$$

where $$J_{i_{sat}} = \alpha e n_e c_s. \tag{5}$$

KT_e is the electron plasma temperature, e is the electronic charge, and U is the potential that the wall develops, with respect to the plasma, in order to assure equal ion and electron loss.

It is worth pointing out that the exact value of α in (5) depends on the spatial profile of the potential in the body of the plasma (Dunn and Self 1964). The potential, in turn, is obtained by scaling the Poisson equation in terms of the Debye length and (8) (Jones 1977a; Caruso and Cavaliere 1962).

Dunn and Self (1964) have carefully evaluated α by numerical means and we have used their value of $\alpha = 1/3$. In certain inhomogeneous magnetic field configurations mirroring may reduce this value even further. Collisions will also act to reduce α.

Combining (3), (4) and (5) and neglecting secondary emission

$$-\, eU/KT_e = \ln\, [\alpha^{-1}\, (m_i/m_e)^{1/2}]. \tag{6}$$

Briefly, the faster electrons (transiently) escape, charge the walls negatively, and establish a potential difference between the walls and plasma which (in steady state) accelerates ions out (the so called 'free fall' in accordance with (5)) and electrostatically traps all but the fastest of the mobile plasma electrons (i.e. those with kinetic energy E_e greater than U). The fast primaries can also escape. Neither are they properly treated by a sheath model which assumes a Maxwellian distribution of electrons. Our interest here, however, will be in the limit of very low primary electron density.

Substituting for c_s we can combine (1) and (2) giving:

$$\frac{n_i}{n_n} = \frac{V\, n_{e'}\, \langle \sigma\, v_{e'} \rangle_{\text{ionise}}}{A\, \alpha\, (2KT_e/m_i)^{1/2}} \tag{7}$$

Cross-section data needed to evaluate the discharge eq. (7) have been given by Lotz (1967), Freeman and Jones (1974) and Martin (1976) for various gases. From these we have plotted, figures 1 and 2, the rate coefficients for hydrogen where ionisation is due to either primary electrons (E_e) or thermalised plasma electrons (T_e). The appropriate Maxwellian distribution of velocities is assumed in computing the rate coefficient in the latter case.

In some ion sources ionization is due largely to primary electrons (Sältz *et al* 1961) while in others (Harrison and Thompson 1959 and Bollinger *et al* 1972) (larger sources) the discharge relaxes quickly and thermal electron ionisation predominates

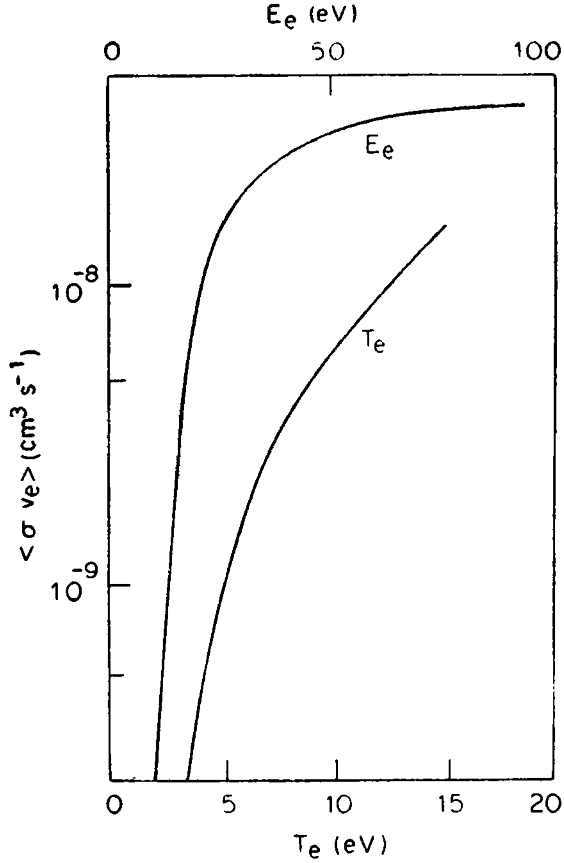

Figure 1. Electron ionisation rate coefficients for atomic hydrogen (scale log-linear).

In the former case $v_{e'}$ is given whereas in the latter case it depends upon T_e, which must be solved for as an integral part of the self-consistent problem.

In order to obtain a result more readily comparable with experiment we take cgs units, $n_{e'} \to n_e = n_i$ (i.e. neglect primaries), relate n_n to p, the neutral gas pressure in torr at 20°C, and substitute T_e in electron volts for KT_e. Equation (7) then becomes:

$$Vp/A = 8 \times 10^{-23} (T_e/m_i)^{1/2} \langle \sigma v_e \rangle_{\text{ionise}}^{-1} \tag{8}$$

which we plot in figure 3 for a wide range of parameters.

A paradox can arise in the thermal electron ionisation problem because the curve $\langle \sigma v_e \rangle_{\text{ionise}}$ falls off again at large values of v_e (or equivalently T_e). Two values of temperature T_e can produce the same value of $T_e^{1/2} \langle \sigma v_e \rangle^{-1}_{\text{ionise}}$. Two possible equilibrium solutions, for the same discharge conditions, Vp/A and discharge power W, are indicated as A and B in figure 3. Which discharge will be produced? One with high T_e and low n_i/n_n, state 'B', or the one with low T_e and high n_i/n_n, state 'A'? No such paradox arises in the primary ionisation problem since $v_{e'}$ is then fixed and T_e does not enter into the rate coefficient but only into the loss rate.

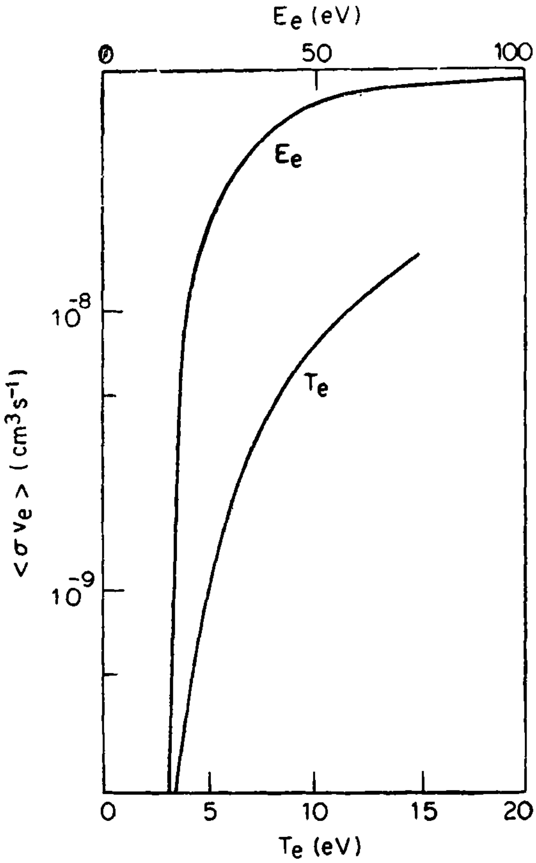

Figure 2. Electron ionisation rate coefficients for molecular hydrogen (scale log-linear).

The same paradox, due to the characteristic shape of the ionisation rate coefficient, will also arise if eq. 2 is replaced by some other plasma loss mechanism (such as instabilities, Jones 1977b). The attainment of the *B* state would, of course, be quite interesting because of its high plasma temperatures. Yet no such state is seen in the Levitron or in other devices where (8) is, otherwise, perfectly valid.

To obtain a resolution to this paradox we do not have to evoke enhanced losses (recombination, radiation, etc.) in the high temperature branch, nor is the total energy of the two branches different (in this formalism at least).

It is easy to obtain a second fundamental discharge equation (that of energy balance) to compliment (8). The energy balance is obtained by equating the power deposited in the plasma *W* with the power carried out by electrons:

$$W_e = A\, J_e\, (3/2\; T_e)/e = A\, J_{i_{\rm sat}}\, (3/2\; T_e)/e, \tag{9}$$

plus that accounted for by ion loss:

$$W_i = A\, J_{i_{\rm sat}}\, (E_I + |eU|)/e, \tag{10}$$

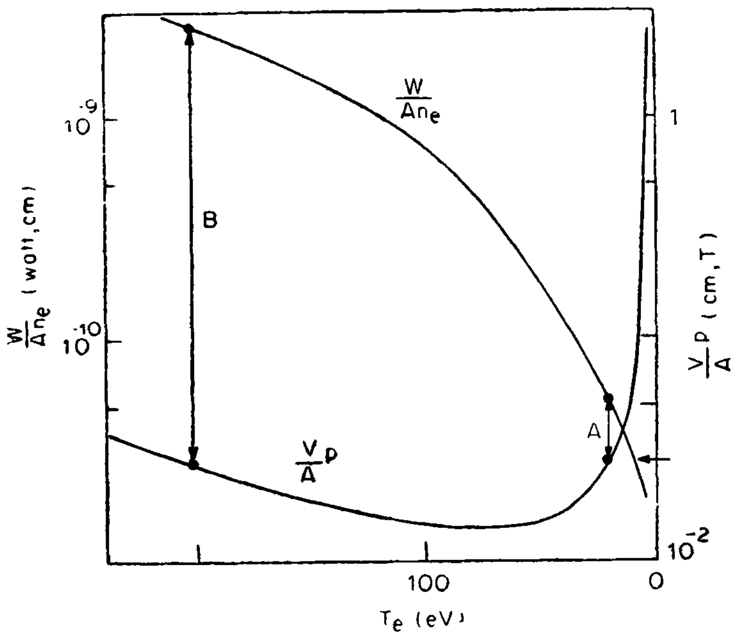

Figure 3. Energy and particle balance relations (scale log-linear).

where E_I is the ionisation energy of the gas. In cgs units, but with power in watts and T_e, E_I and eU in eV

$$W/An_e = 2 \times 10^{-25} (T_e/m_i)^{1/2} (E_I + |eU| + 3/2\ T_e). \tag{11}$$

Equation (11) for hydrogen is plotted in figure 3, where U is obtained from (6).

Figure 3 can be used to estimate the discharge parameters for various steady state plasmas and gives values in good agreement with realistic laboratory experiments (Bollinger *et al* 1972). For a plasma of given size (V and A) and fill gas pressure (p), the plasma density (n_e) is linearly proportional to delivered power W. While W is not the available power, and radiation loss must be accounted for in some (though by no means all) experiments, such factors can usually be estimated. [In RF discharge, for instance, reflected power can be measured in addition to the incident values. Radiation losses, etc., are estimated in the various literature (for instance Basile and Lagrange 1964)]. Plasma temperature can be increased by changing the gas pressure.

We see from figure 3 that, for a fixed power W, energy balance, as well as particle balance, is satisfied for both discharge branches A and B. Yet experimentally we have only observed the A branch of the solution. Why is this?

For all but the lowest values of Vp/A, $n_e(B)$ is an order of magnitude or more smaller than $n_e(A)$. While it is possible to neglect primary ionisation as compared with $n_e(A)$ and obtain branch A as a valid solution, it is not possible to neglect the contribution of primaries as compared to $n_e(B)$. Even a very small amount of primary induced ionisation (small as compared to $n_e(A)$) will render the B state inaccessible. For typical discharges, like those of Bollinger *et al* (1972) and the present author

(Jones 1978a), we find (Liu 1974 and Jones 1977c) that primaries contribute an ionisation that is at least 10% (and usually more) of $n_e(A)$. This point is brought home by considering very low pressure discharges (Jones 1978b). Whereas figure 3 permits no discharge whatever to occur below a certain value of Vp/A, experimentally we find that discharges can be sustained at low pressure if primary electron ionisation is taken into account.

This qualitative explanation can be seen analytically if we include a (previously neglected) primary ionisation term in the particle balance equation:

$$n_i/t_{\text{loss}} \approx n_e n_n \langle \sigma v_e \rangle_{\text{ionise}} + n_p n_n \langle \sigma v_p \rangle, \tag{12}$$

or

$$A/Vn_n = \frac{1}{\alpha(T_e/m_i)^{1/2}} \left(\langle \sigma v_e \rangle_{\text{ionise}} + \frac{n_p}{n_i} \langle \sigma v_p \rangle \right), \tag{13}$$

where the first term on the right hand side of (13) is the plasma electron ionisation (as before) and the second term is the (new) primary ionisation term. Although

$$\frac{n_p \langle \sigma\ v_p \rangle}{n_e \langle \sigma\ v_e \rangle_{\text{ionise}}}$$

is now non-zero the ratio of primary to plasma density, n_p/n_e is still $\ll 1$. This allows us to retain (Jones 1978b) a Maxwellian electron dominated loss rate (2), and an unmodified plasma electron ionization term.

If we consider a fixed discharge temperature T_e we see that the inclusion of the primary term in (13) results in a decrease in the quantity Vn_n/A. That is, the low pressure cut-off is withdrawn.

For higher values of n_p/n_i (but still $\ll 1$) primary ionisation will actually dominate completely (but, again, with n_p/n_i small ambipolar loss still scales as (2), see Jones 1978b) and (13) is approximately:

$$A/Vn_n = \frac{1}{\alpha(T_e/m_i)^{1/2}} \frac{n_p}{n_i} \langle \sigma v_p \rangle, \tag{14}$$

and the discharge equilibrium solution has become single valued. n_p is not arbitrary. It is set by diode filament emission equations, etc. (Jones 1978b). If n_p is externally determined (say by injection of primaries from some independent outside source), along with A, V and n_n then (14) is an equation in n_i and T_e only which can be solved simultaneously with the power balance equation to obtain $n_i(W)$ and $T_e(W)$.

Equation (13) goes over to (8) in the limit of $n_p/n_i \to 0$. While $n_i(A)$ is relatively large and (8), branch A, is a valid equilibrium, $n_i(B)$ is small, n_p/n_i is not small enough and (8), branch B, is an invalid limit of (13).

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