Pramāna, Vol. 11, No. 6, December 1978, pp. 755–760, © printed in India.

Effect of Gaunt factor correction in soft x-ray plasma diagnostics

S V DESHMUKH
Physical Research Laboratory, Ahmedabad 380 009

MS received 25 March 1978; revised 9 August 1978

Abstract. The effect of Gaunt factor correction on temperature estimation over a range 100 to 1500 eV had been studied. Greene's analytical expression for the quantum mechanical Gaunt factor averaged over Maxwellian distribution is used. Transmission ratios are calculated with and without Gaunt factor for various combinations of beryllium foils, taking into account x-ray emission due to free-free transitions. A significant difference ($\gtrsim 15\%$) is observed between the temperatures estimated from classical and quantum mechanical curves, above 600 eV. Selection of foil combinations useful for estimating higher temperatures is also discussed.

Keywords. X-ray diagnostics; Gaunt factor; electron temperature; transmission ratio; free-free transitions; beryllium foils; photon energy.

1. Introduction

Soft x-ray diagnostic technique is now commonly employed for measuring electron temperature (kT_e) in plasma heating experiments. When $kT_e > 3 \cdot \eta_H Z^2$, where η_H is the ionisation potential of hydrogen and Z is the atomic number of the atoms composing the plasma, x-ray emission from the plasma is mainly due to free-free electron transitions. Electron temperature is determined by measuring the intensities of these x-rays transmitted through absorber foils of two different thicknesses. This technique is discussed by Jahoda *et al* (1960), Elton and Anderson (1967) and Stratton (1972). They have used the classical expression for free-free radiation intensity for calculating the transmission ratios. For kT_e above 100 eV and photon energies greater than 100 eV, it is essential to take into account the quantum mechanical corrections in the classical expression for the free-free radiation intensity. It is well-known that the quantum mechanical effects can be accounted for by multiplying the classical expression for intensity by an appropriate Gaunt factor. This has been discussed in detail by Healed and Wharton (1965), and Bekefi (1966). Robouch and Rager (1973) have calculated the Gaunt factor corrected transmission ratios for aluminium and gold foils for electron temperatures in the range of 1 to 25 keV.

Transmission ratios were calculated for beryllium foils for kT_e from 100 to 1500 eV for two reasons. Commercially available beryllium foils can be readily used in this temperature range. Secondly, beryllium does not have any absorption edge in the photon energy range of interest. Transmission ratios for various combinations of beryllium foils are calculated considering only the x-ray emission resulting from free-free transitions and the analytical expression for quantum mechanical Gaunt factor given by Greene (1959).

2. Transmission ratios

Assuming that plasma electrons follow Maxwellian distribution, the intensity of free-free radiation (bremsstrahlung) per Å is given (Jahoda *et al* 1960) as

$$I_\lambda \, d\lambda = 1{\cdot}9\times 10^{-28}\, N_e\, N_i\, Z^2\, \bar{g}\, (kT_e)^{-1/2}\, \lambda^{-2} \exp\left[-\frac{hc}{\lambda k T_e}\right] d\lambda \, \frac{\text{Watt}}{\text{cm}^3.\,\text{Å}}. \quad (1)$$

All notations in the above expression have their usual meaning. The total or integrated x-ray intensity from plasma can be obtained by integrating (1) over the appropriate wavelength range. The x-ray intensity transmitted through an absorber of thickness D is given by

$$I'_\lambda = I_\lambda.\exp(-K_\lambda \cdot D), \quad (2)$$

where K_λ is the mass absorption coefficient of the absorbing material at wavelength λ. The ratio of integrated intensities transmitted through absorbers of thicknesses $D1$ and $D2$ obtained by combining (1) and (2) is given as

$$\frac{\int \left[\bar{g}\, \lambda^{-2} \exp\left(-\frac{hc}{\lambda k T_e} - K_\lambda D1\right)\right] d\lambda}{\int \left[\bar{g}\, \lambda^{-2} \exp\left(-\frac{hc}{\lambda k T_e} - K_\lambda D2\right)\right] d\lambda}. \quad (3)$$

We have calculated this ratio as a function kT_e for different combinations of $D1$ and $D2$ for beryllium.

Various analytical expressions for the quantum machanical Gaunt factor are available in literature. Each of these expression is applicable over a particular range of photon energy and relative value of electron temperature. Brussard and van de Hulst (1962), and Hughes (1975) have discussed the different forms of Gaunt factor and the range of their applicability. To determine the proper expression of Gaunt factor, the ranges of electron temperature and photon energy E, under consideration must be clearly specified. As discussed by Stratton (1972), part of the x-ray continuum where $E > kT_e$, is useful to determine kT_e. This is the spectral region most affected by any change in kT_e. Secondly, when beryllium foils are used as absorbers, the low energy part of the continuum is cut-off or absorbed in the foils. We therefore consider kT_e from 100 to 1500 eV, and $E > kT_e$ and use the following analytical expression for the quantum mechanical Gaunt factor, averaged over Maxwellian distribution, given by Greene (1959).

$$\bar{g} = \frac{\sqrt{3}}{\pi} \exp\left[\frac{hc}{2\lambda\, kT_e}\right] K_0\left[\frac{hc}{2\lambda\, kT_e}\right], \quad (4)$$

where K_0 is the Bessel function of zero order. Brussard and van de Hulst (1962), and Hughes (1975), have shown that Greene's expression is of a general form because the other forms of Gaunt factor for $E \ll kT_e$ and $E \gg kT_e$ can be deduced as special cases of Greene's expression.

High purity beryllium foils of thickness ranging from 0·001 to 0·02 in. available from M/s Brush Wellman, USA are used in the present calculations. Mass absorption coefficients of beryllium given by Liebhafsky *et al* (1972) are used. These coefficients for the intermediate values of wavelengths were calculated using Lagrangian intrapolation. The transmission factor $\int [I_\lambda \exp(-K_\lambda D)] d\lambda$ was calculated for different foil thicknesses and electron temperatures between 100 to 1500 eV, with and without Gaunt factor correction. The ratio given by (3) was then calculated for different foil combinations using an IBM 360-model 45 computer. Integrations in (3) were carried over the wavelength range of 0·5 to 50 Å, with a step of 0·1 Å. The error in these calculations was less than 0·1 %.

3. Results and discussion

The results obtained without Gaunt factor correction ($\bar{g}=1$) as well as calculations useful for designing an experimental diagnostic system are reported elsewhere (Deshmukh 1977). Variations of the transmission factor with kT_e for four foils are shown in figure 1 and it is seen that transmission increases rapidly at lower temperatures. Transmission ratios as a function of electron temperature were calculated for 20 different foil combinations. These combinations were selected considering different criteria, such as foil thickness, difference in cut-off energy, and relative separation between their transmission curves. To determine kT_e from the transmission ratio curves, it is essential that for a given combination of foils, this ratio should increase continuously with kT_e. At the same time, the thickness of individual foil should be such that the transmitted x-ray intensity could be measured with sufficient accuracy.

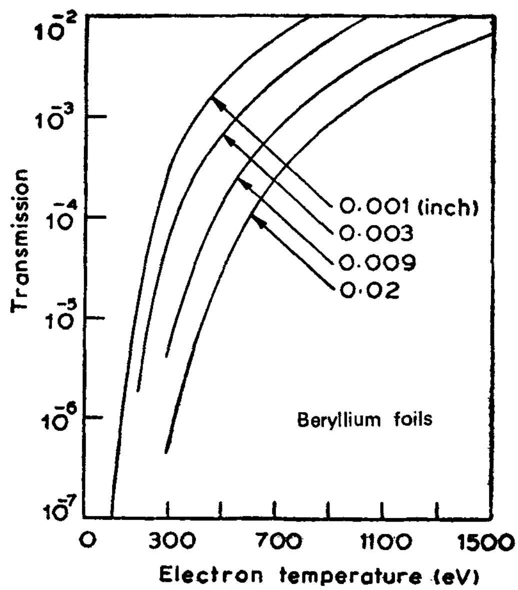

Figure 1. Variation of x-ray transmission through different beryllium foils with electron temperature.

Some foil combinations with thicknesses close to each other were selected. Variations of calculated transmission ratios with kT_e for few such combinations are shown in figure 2. It is seen that for most of the combinations, the ratio increases rapidly up to 500 to 600 eV and then slowly with further increase in kT_e. Combinations with close thickness can be used for evaluating temperatures up to 500 to 600 eV. Gaunt factor corrected ratios for some of the combinations are also shown in figure 2. At lower temperature range ($\lesssim 600$ eV), ratios calculated with and without Gaunt factor correction are close to each other, and the slope of the ratio curves are much larger compared to that at higher temperatures. This shows that at lower temperatures, error in the estimated temperature due to non-inclusion of Gaunt factor is negligible.

Of the three criteria mentioned earlier, it is found that only the thickness factor is useful to determine foil combinations suitable for estimating higher temperatures. It is also found that the combinations with thickness two to three times of each other are useful at higher temperature range as is evident from figures 2 and 3. Slopes of these curves are greater than others in the temperature range of 500 to 1500 eV. Foil combinations with thicknesses more than three times of each other were found to be unsuitable at higher temperatures. Figures 2 and 3 reveal that the Gaunt factor corrected ratios for combinations useful at higher temperatures are much smaller than those calculated by taking $\bar{g}=1$. For a given foil combination, this difference is found to increase with increasing kT_e. Also, the slopes of the curves useful in higher temperature range are smaller compared to that at lower temperatures. As a result, temperatures estimated from the Gaunt factor corrected curves are about 15% higher, depending upon the foil combination and intensity ratio.

The probable error in measuring the ratio of the x-ray intensities transmitted through some of the foil combinations was calculated for different electron

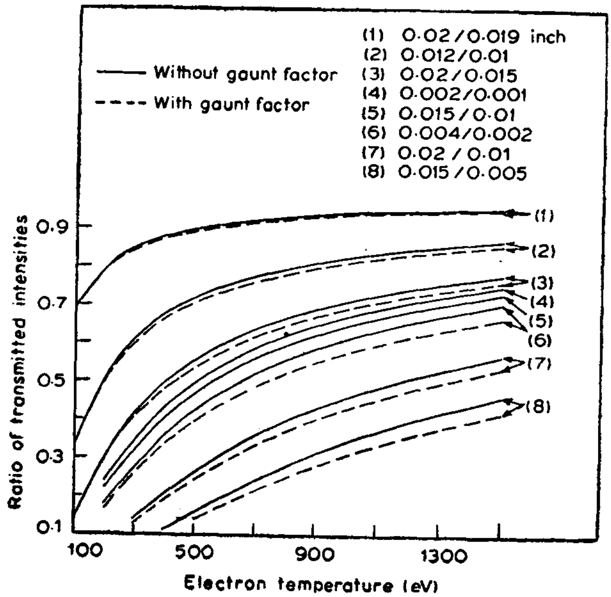

Figure 2. Variation of the ratio of x-ray intensities transmitted through beryllium foils with electron temperature.

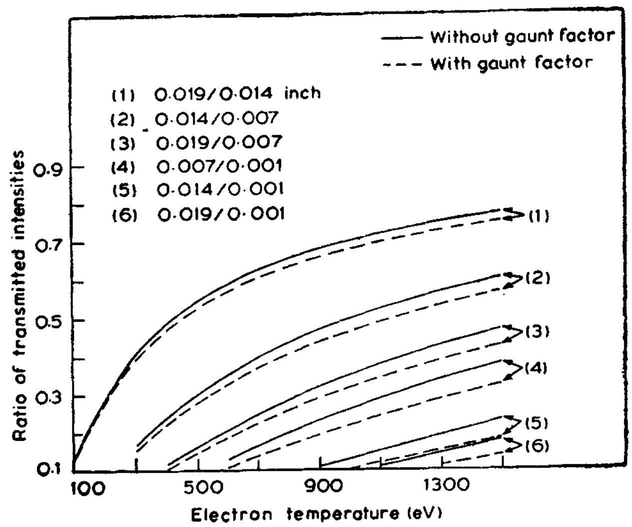

Figure 3. Variation of the ratio of x-ray intensities transmitted through beryllium foils with electron temperature.

temperatures and densities using x-ray photons from a hydrogen plasma and tramsmitted through these foils. The errors were negligible as compared to the errors due to non-inclusion of Gaunt factor. This again establishes the necessity of Gaunt factor correction at higher temperatures.

4. Conclusions

Soft x-ray transmission ratios for various combinations of beryllium foils are presented for electron temperatures between 100 to 1500 eV, with and without Gaunt factor correction. The ratios with Gaunt factor are always less than those calculated without the correction. This difference increases with increase in kT_e. Difference in the estimated temperatures below 500 eV from the corrected and classical curves is negligible. It is essential to use foil combinations with thicknesses two to three times of each other for estimating $kT_e > 500$ eV. Temperatures in higher range in ($\gtrsim 600$ eV), estimated from the corrected curves are about 15% higher than those obtained from classical curves. Error considerations show that it is essential to use the Gaunt factor corrected transmission ratios if the expected temperature in a low Z plasma is higher than 500 eV.

Acknowledgements

The author expresses his sincere thanks to Drs Y C Saxena and K H Bhatt for discussions and valuable suggestions. Thanks are also due to Mr V R Choksi for his help in computer programming.

References

Bekefi G 1966 *Radiation processes in plasmas* (New York: John Wiley) ch. 3

Brussard P J and van de Hulst M C 1962 *Rev. Mod. Phys.* **34** 507

Deshmukh S V 1977 PRL Tech. Note TN-77-03

Elton R C and Anderson A D 1967 *Naval Res. Lab (USA) Report No.* 6541

Greene J 1959 *Astrophys. J.* **130** 693

Heald M A and Wharton C B 1965 *Plasma diagnostics with microwaves* (New York: John Wiley) ch. 7

Hughes T P 1975 *Plasmas and laser light* (London: Adam Hilger) ch. 2

Jahoda F C, Little E M, Quinn W E, Sawyer G A and Stratton T F 1960 *Phys. Rev.* **119** 843

Liebhafsky H A, Pfeiffer H G, Wilson E H and Zemany P D 1972 *X-rays, electrons and analytical chemistry* (New York: Wiley Interscience) 525

Robouch B V and Rager J P 1973 *J. Appl. Phys.* **44** 1527

Stratton T F 1972 *Temperature—its measurement and control in science and industry* ed. Herzfeld C M Vol. 3 pt 1 (Reinhold Pub. Corp.) p. 663