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Interaction of a charged extended body with Magnetic field

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Abstract. It is shown that the conventional energy expressions need modification for the interaction of a charged extended body with a magnetic field. It is also shown that the correction term arises as a result of the rotation of the charged extended body about its centre of mass in a magnetic field. This can provide us with an additional source of energy that can be tapped with a suitable energy conversion device.

Keywords. Charged extended body; magnetic field; rotational energy.

1. Introduction

In this paper it is shown that the conventional energy expressions need modification for a charged body of extended dimensions interacting with a magnetic field. The physical meaning and the implications of the modified energy expression are discussed. In section 2, theoretical arguments have been presented to show the existence of the correction term which arises as a result of the spinning of the charged extended body about its centre of mass in the presence of a magnetic field. In section 3, a numerical estimate for this correction term has been discussed.

2. A modified energy—expression for a charged extended body in a magnetic field

According to the conventional theories, the energy of a charged particle in a magnetic field is given by the expression:

$$E = \frac{p^2}{2m} + \text{magnetic potential energy of the particle,} \tag{1}$$

where **p** stands for the linear momentum of the particle and m for its mass. Equation (1) is fine for a 'point-particle' but for a body with extended dimensions, it needs modification. To show this, let us consider, without any loss of generality, a simple case of a charged point-particle carrying a charge q entering a region of uniform static magnetic field (**B**) oriented at a right angle to the velocity vector of the charged particle. For this case, noticing that $p^2/2m = \frac{1}{2} I \omega_B^2$, (1) can be rewritten as

$$E = \tfrac{1}{2} I \omega_B^2 + \text{magnetic potential energy of the particle,} \tag{2}$$

where I is the moment of inertia of the charged point-particle about an axis parallel to **B** and passing through the centre of its circular orbit and $\vec{\omega}_B = q\mathbf{B}/mc$, the gyration or the precession angular velocity.

Equation (2) has been derived for the case of a charged point-particle. Next, let us consider the case of a charged extended body. If the magnetic field is regarded as the one that causes a purely rotational motion of the charged body, then (2) can be regarded as the fundamental equation valid not only for a charged point-particle but also for a charged extended body. This point would be further elaborated later in the text. However, for the present, it should suffice to say that, at least, one cannot present any arguments to disprove this description of the interaction of a body with magnetic field. Thus we proceed with the understanding that (2) describes not only the interaction of a charged point-particle with magnetic field but also that of an extended charged body with the same. But in the case of a charged extended body, using the parallel-axis-theorem for moment of inertia, the expression $\frac{1}{2} I \omega_B^2$ can be re-expressed as follows.

$$\tfrac{1}{2} I \omega_B^2 = \tfrac{1}{2} I_0 \omega_B^2 + \tfrac{1}{2} I_{\text{cm}} \omega_B^2, \tag{3}$$

where I_0 is the moment of inertia of the body, as if concentrated at its centre of mass about the original axis and I_{cm} is the moment of inertia about a parallel axis passing through its centre of mass. But since $\frac{1}{2} I_0 \omega_B^2 = p^2/2m$, (3) can be cast into the form:

$$\tfrac{1}{2} I \omega_B^2 = \frac{p^2}{2m} + \tfrac{1}{2} I_{\text{cm}} \omega_B^2. \tag{4}$$

Thus from (2) and (4), we get for the energy of a charged extended body interacting with magnetic field the expression:

$$E = \frac{p^2}{2m} + \tfrac{1}{2} I_{\text{cm}} \omega_B^2 + \text{magnetic potential energy of the body.} \tag{5}$$

For the case of a point-particle, this equation would clearly reduce to (1).

The physical meaning of the second term on the right hand side of (5) can be understood as follows. Figure 1 shows the motion of a charged extended body in magnetic field. From this figure, it is quite apparant that, in addition to its circular orbital motion, the body also spins about the axis passing through its centre of mass with the angular velocity $-\vec{\omega}_B$. Therefore, besides the kinetic energy $p^2/2m$ associated with the translational motion of the centre of mass of the body, there also exists $\frac{1}{2} I_{\text{cm}} \omega_B^2$ amount of kinetic energy associated with the spinning motion of the body about the axis passing through its centre of mass. Even when the particle is stationary, it is still spinning with the angular velocity $-\vec{\omega}_B$ and consequently it still possesses $\frac{1}{2} I_{\text{cm}} \omega_B^2$ amount of energy in the presence of the magnetic field. It should be noticed here that the orientation with which the body traverses with its centre of mass on the circular orbit would be such that the resultant force would always act at its centre of mass toward the centre of the circular orbit. But for any given orientation, the resultant force is determined by the distribution of the charge on the body. Therefore, it is quite apparent that the orientation with which the body orbits would be determined by the distribution of charge on it. A clear experimental evidence in support of the conclusion that in the presence of a static uniform magnetic field, a charged body spins about its centre of mass is provided by the observation that the intrinsic spin angular momentum vector of an electron precesses (figure 2) about the direction of the magnetic field.

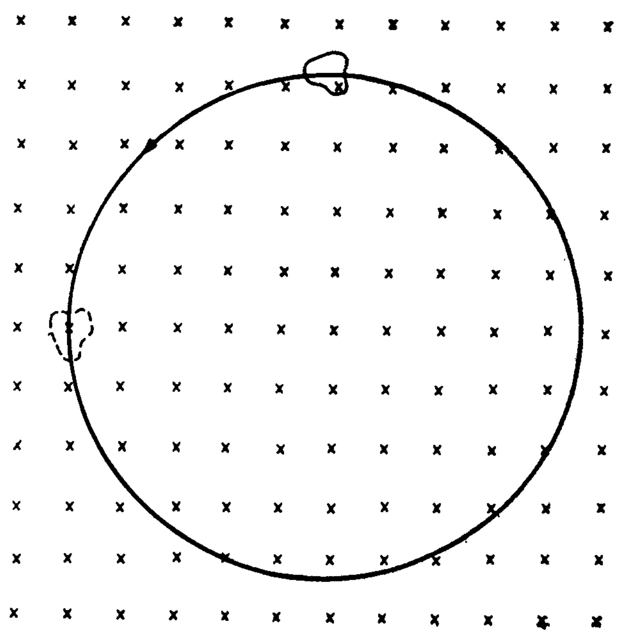

Figure 1. Motion of a charged extended body in a uniform static magnetic field. The magnetic field is directed into the page.

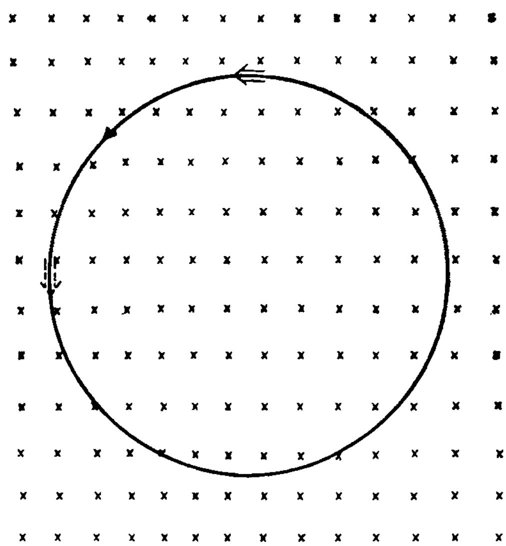

Figure 2. Motion of a Dirac spin-$\frac{1}{2}$ particle in a uniform static magnetic field. The arrow shows the direction of the spin of the particle. The magnetic field is directed into the page.

The second term in (5) can provide us with a mean to determine the moment of inertia of a given particle about an axis passing through its centre of mass. Let us consider a spin-$\frac{1}{2}$ particle entering the magnetic region with a definite helicity. Now if we assume the above mentioned classical model for the particle, then it is possible to determine the moment of inertia of the particle about an axis passing through its centre of mass and oriented at the right angle to the axis of its intrinsic spin. Within this classical model if we further assume the spherical shape for the particle, then since we know its spin angular momentum, we can now deduce a value for the angular velocity associated with its intrinsic spin. However, since one should expect only a very small value for I_{cm} for the fundamental particles, its experimental determination may not be easy.

3. A numerical estimate for the correction-term

We have seen above that in the presence of a magnetic field, an extended charged body rotates with an angular velocity $-\vec{\omega}_B$ about its centre of mass and that the energy associated with this rotation is given by the second term on the right hand side of (5). This observation can be put to an important practical use. It is possible to conceive of an energy conversion device that would convert this rotational kinetic energy into a useful power output by way of its conversion into translational kinetic energy. The power output of such a device would, of course, depend upon the moment of inertia of the charged body, the magnitude of the magnetic induction (**B**) and the efficiency of the energy conversion device. As an example, consider a typical MHD generator which employs a magnetic flux density of 3 webers /m^2. Assuming an electron to be a solid sphere with a classical radius of $2{\cdot}818\times10^{-15}$m, the rotational energy of the electron corresponding to the second term in (5) turns out to be $2{\cdot}51\times10^{-18}$ eV. Since a typical electron density in an MHD generator is of the order of 10^{21} m^{-3}, there exists about 2·51 keV/m^3 amount of energy density associated with only the rotational energy of the electrons. Therefore, we see that for this particular situation the contribution to the energy density from the rotational energy of the electrons is very small. But the above method for estimating the value for the moment of inertia of the electron is highly unreliable. This value for the moment of inertia of the electron could easily be off by several orders of magnitude from the actual value. Therefore, the above numerical estimate for the contribution to the energy density as a result of the spinning of the electron about its centre of mass is also highly unreliable. There would also be a contribution to the energy density from the rotational energy of the ions. But since an ion is much more massive than an electron, this contribution to the energy density would be even smaller.