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Neutral current cross-sections for neutrinos on protons

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Abstract. Defining the ratios $r_p = \sigma(\nu p \rightarrow \nu x)/\sigma(\nu p \rightarrow \mu^- x)$ and $\bar{r}_p = \sigma(\bar{\nu} p \rightarrow \bar{\nu} x)/\sigma(\bar{\nu} p \rightarrow \mu^+ x)$ we obtain the bounds $0{\cdot}28 < r_p < 0{\cdot}61$ and $0{\cdot}15 < \bar{r}_p < 0{\cdot}37$ using only the parton model and the data of CDHS group with iron target. We also give the complete set of parton-model relations which would allow the determination of all the neutral-current coupling constants from inclusive cross sections alone.

Keywords. Neutral current weak interaction; neutrinos; inclusive processes; parton model; chiral couplings; proton target.

1. Introduction

Soon after the experimental discovery of the neutral current, we had given a general phenomenological analysis with a four-parameter form of the neutral-current interaction (Rajasekaran and Sarma 1974). We had shown that two combinations of these four neutral-current coupling constants could be determined from the original inclusive data on heavy-nuclear target announcing the discovery of the neutral current.

By combining these two relations with two more relations for cross sections on semi-inclusive pion production processes, based on the quark-fragmentation-model, Sehgal (1977) has been able to determine all the four parameters, except for certain discrete ambiguities. These ambiguities have been resolved by Hung and Sakurai (1977) as well as by Abbott and Barnett (1978) by appealing to data on exclusive neutrino reactions, namely elastic scattering and exclusive one-pion production, respectively.

In the same paper referred to above, we had also given the equations which would determine all the four coupling constants (except for the discrete ambiguities) from inclusive data alone, provided cross-sections on proton target were measured. Now that such data are becoming available, one could complete that programme. This would provide an alternate determination of the coupling constants which would have the advantage of being dependent only on the original quark-parton model.

In the present paper we study how far this programme can be carried through at the present time. It is found that the present data are still not sufficiently accurate for this purpose, but remarkably enough, we are able to set rather stringent limits on the neutral current cross-sections on proton target using the parton model and the inclusive neutrino scattering data on isospin-averaged nucleon target.

For clarity of presentation, the paper is divided into short sections. In § 2, we introduce the notation and collect the earlier results. In § 3, we make the theoretical prediction for the neutral current cross-sections on proton target. Section 4 gives a more complete set of relations based on parton-model. Section 5 is devoted to a brief summary.

2. Notation and earlier results

The neutral-current interaction of the neutrinos can be written in the form

$$\mathscr{L}_{\text{int}} = -\frac{G}{\sqrt{2}}\bar{\nu}\,\gamma_\mu\,(1+\gamma_5)\,\nu\,N_\mu, \tag{1}$$

where
$$N_\mu = xV_\mu^3 + yA_\mu^3 + zV_\mu^0 + wA_\mu^0; \tag{2}$$

$$= \frac{x}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{y}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$$

$$+ \frac{z}{2}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \frac{w}{2}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d); \tag{3}$$

$$= \bar{u}\gamma_\mu\{u_L(1+\gamma_5) + u_R(1-\gamma_5)\}\,u$$

$$+ \bar{d}\gamma_\mu\{d_L(1+\gamma_5) + d_R(1-\gamma_5)\}\,d. \tag{4}$$

The set of four neutral-current coupling constants x, y, z and w or alternatively the set u_L, u_R, d_L and d_R is to be determined from experimental data.

The first step in the empirical determination of the neutral-current coupling constants is to consider the following inclusive neutrino reactions:

$\sigma_C(\nu\mathscr{N})$: $\nu + \mathscr{N} \to \mu^- +$ hadrons

$\sigma_N(\nu\mathscr{N})$: $\nu + \mathscr{N} \to \nu +$ hadrons

$\sigma_C(\nu p)$: $\nu + p \to \mu^- +$ hadrons

$\sigma_N(\nu p)$: $\nu + p \to \nu +$ hadrons

and the corresponding antineutrino reactions. We use subscripts N and C to denote the neutral and charged current reactions respectively whereas $\mathscr{N}$ and p stand for the isospin-averaged nucleon and proton respectively. We define the ratios of the cross-sections

$$r = \frac{\sigma_N(\nu\mathscr{N})}{\sigma_C(\nu\mathscr{N})};\ \bar{r} = \frac{\sigma_N(\bar{\nu}\mathscr{N})}{\sigma_C(\bar{\nu}\mathscr{N})};$$

$$r_p = \frac{\sigma_N(\nu p)}{\sigma_C(\nu p)};\ \bar{r}_p = \frac{\sigma_N(\bar{\nu} p)}{\sigma_C(\bar{\nu} p)}. \tag{5}$$

Using the simplest version of the parton model ignoring the sea and assuming the u and d partons to have the same distribution functions, it is straightforward to get the following equations for the neutral-current coupling constants in terms of the cross-section ratios:

$$u_L^2 + d_L^2 = \tfrac{1}{8}(9r - \bar{r}); \tag{6}$$

$$u_R^2 + d_R^2 = \tfrac{3}{8}(\bar{r} - r); \tag{7}$$

$$2u_L^2 + d_L^2 = \tfrac{1}{8}(9r_p - 2\bar{r}_p); \tag{8}$$

$$2u_R^2 + d_R^2 = \tfrac{3}{8}(2\bar{r}_p - r_p). \tag{9}$$

These are nothing but (14)–(17) of our earlier paper (Rajasekaran and Sarma 1974) rewritten in terms of the chiral coupling constants $u_L, \ldots d_R$. In fact if we use the latest Gargamelle data after correcting for the hadronic cut,

$$r = 0{\cdot}26 \pm 0{\cdot}04$$

(Blietschau *et al* 1977) (10)

$$\bar{r} = 0{\cdot}39 \pm 0{\cdot}06,$$

then (6) and (7) give

$$u_L^2 + d_L^2 = 0{\cdot}24 \pm 0{\cdot}04$$

$$u_R^2 + d_R^2 = 0{\cdot}05 \pm 0{\cdot}03 \tag{11}$$

which are essentially the same values as implied by (26) and (27) of the same paper.

The second step is to consider the semi-inclusive neutrino scattering with $\pi^{\pm}$ in the final state for which data are now available. Fitting these data to the quark-fragmentation model, Sehgal (1977) obtained two additional equations and using these in combination with (11), he got the following results*

$$u_L^2 = 0{\cdot}082 \pm 0{\cdot}035;$$

$$d_L^2 = 0{\cdot}158 \pm 0{\cdot}035;$$

$$u_R^2 = 0{\cdot}055 \pm 0{\cdot}025;$$

$$d_R^2 = 0{\cdot}001 \pm 0{\cdot}025. \tag{12}$$

*Sehgal has included the sea contribution in the formulae for the inclusive cross-sections, but not in those for the semi-inclusive cross-sections. However, the numbers are not affected much.

This complete determination of the magnitudes of the coupling constants is an important step forward.

The above, however, still leaves the signs of the coupling constants undetermined. The third step is to utilize the exclusive processes, namely elastic scattering of neutrinos and antineutrinos on protons as well as exclusive pion production processes. Using the data on these, Hung and Sakurai (1977), as well as Abbott and Barnett (1978) have been able to determine the signs of the coupling constants also (apart from an overall sign ambiguity).

3. Neutrino-proton cross-sections

As already mentioned our aim is to analyse the implications of inclusive cross-sections on protons. The following numbers have already been obtained from FNAL bubble chamber experiments on proton targets:

$$r_p = 0{\cdot}48 \pm 0{\cdot}17 \text{ (Harris } et\ al\text{ 1977)},$$

$$\bar{r}_p = 0{\cdot}42 \pm 0{\cdot}13 \text{ (Derrick } et\ al\text{ 1978)}. \tag{13}$$

Since these experiments were performed at higher neutrino energies, for the sake of consistency, one should consider the following higher-energy data for the isospin-averaged nucleon (corrected for cuts)

$$\left.\begin{aligned} r &= 0{\cdot}295 \pm 0{\cdot}01 \\ \bar{r} &= 0{\cdot}34 \pm 0{\cdot}03. \end{aligned}\right\} \text{(Holder } et\ al\text{ 1977)} \tag{14}$$

The first thing one may try is a straightforward substitution of the data (13) and (14) into our eq. (6)–(9). This gives the results:

$$u_L^2 = 0{\cdot}15 \pm 0{\cdot}19;$$

$$d_L^2 = 0{\cdot}14 \pm 0{\cdot}19;$$

$$u_R^2 = 0{\cdot}12 \pm 0{\cdot}12;$$

$$d_R^2 = -\,0{\cdot}10 \pm 0{\cdot}12. \tag{15}$$

We thus see that the present data on r_p and $\bar{r}_p$ are too poor to give any useful results on the coupling constants. The importance of, and need for more accurate experimental measurements of r_p and $\bar{r}_p$ are obvious, since these would allow an alternative determination of the coupling constants independent of the assumption of the quark-fragmentation model.

For the present, the better procedure appears to be to predict the values of r_p and $\bar{r}_p$. In fact, it turns out to be possible to obtain valuable information on r_p and $\bar{r}_p$

using the available data on r and $\bar{r}$ alone. To do this, we rewrite (6)–(9) in the following form:

$$r = u_L^2+d_L^2+\tfrac{1}{3}(u_R^2+d_R^2); \tag{16}$$

$$\bar{r} = u_L^2+d_L^2+3(u_R^2+d_R^2); \tag{17}$$

$$r_p = 2u_L^2+d_L^2+\tfrac{1}{3}(2u_R^2+d_R^2); \tag{18}$$

$$\bar{r}_p = \tfrac{1}{2}(2u_L^2+d_L^2)+\tfrac{3}{2}(2u_R^2+d_R^2). \tag{19}$$

Considering the expression for r_p first,

$$r_p = (u_L^2+d_L^2)+\tfrac{1}{3}(u_R^2+d_R^2)+(u_L^2+\tfrac{1}{3}u_R^2),$$

the last term $(u_L^2+\frac{1}{3}u_R^2)$ can be treated in two ways: By omitting it, we get

$$r_p \geqslant r.$$

By replacing $(u_L^2+\frac{1}{3}u_R^2)$ by $[u_L^2+d_L^2+\frac{1}{3}(u_R^2+d_R^2)]$ we get

$$r_p \leqslant 2r.$$

Thus we get the lower and upper bounds on r_p.

$$r \leqslant r_p \leqslant 2r. \tag{20}$$

A similar procedure for $\bar{r}_p$ yields*

$$\tfrac{1}{2}\bar{r} \leqslant \bar{r}_p \leqslant \bar{r}. \tag{21}$$

Inserting the CDHS experimental values of r and $\bar{r}$ given in (14) we get

$$0{\cdot}28 \leqslant r_p \leqslant 0{\cdot}61;$$

$$0{\cdot}15 \leqslant \bar{r}_p \leqslant 0{\cdot}37. \tag{22}$$

These limits are inclusive of one standard-deviation errors and correspond to a confidence level of 68%. Since the ranges of the limits are quite restrictive we shall regard the actual values of r_p and $\bar{r}_p$ to be the averages of these limits and the assigned errors to cover the ranges of the limits. With this procedure we have

$$r_p = \tfrac{1}{2}(3r \pm r) = 0{\cdot}44 \pm 0{\cdot}16;$$

$$\bar{r}_p = \tfrac{1}{4}(3\bar{r} \pm \bar{r}) = 0{\cdot}25 \pm 0{\cdot}11; \tag{23}$$

*It is easy to show that (20) and (21) are the best bounds that follow from eqs (16)–(19).

Table 1. Values of r_p and $\bar{r}_p$

	r_p	$\bar{r}_p$
From the inequalities (20) and (21)	$0{\cdot}44 \pm 0{\cdot}16$	$0{\cdot}25 \pm 0{\cdot}11$
From the analysis of Sehgal (1977)	$0{\cdot}36 \pm 0{\cdot}08$	$0{\cdot}33 \pm 0{\cdot}09$
From the analysis of Abbott and Barnett (1978)	$0{\cdot}40 \pm 0{\cdot}11$	$0{\cdot}29 \pm 0{\cdot}09$
Experiment:		
Harris *et al* (1977)	$0{\cdot}48 \pm 0{\cdot}17$	—
Derrick *et al* (1978)	—	$0{\cdot}42 \pm 0{\cdot}13$

where the experimental errors on r and $\bar{r}$ have also been folded in. In table 1 we have listed these values of r_p and $\bar{r}_p$ deduced from the inequalities (20) and (21) as well as those values implied by the coupling constants determined by the analysis of Sehgal (1977) and by the set given by Abbott and Barnett (1978). The latter values preferably are to be confronted with experiment at lower energies. The present experimental values given in (13) refer to FNAL energies, and are consistent with the predictions of all the analyses. It may be mentioned that the data are also consistent with the Weinberg-Salam model predictions $r_p=0{\cdot}42$, $\bar{r}_p=0{\cdot}29$ corresponding to $\sin^2\theta_w=0{\cdot}25$.

4. Inclusion of sea and other effects

For the sake of completeness, in this section we shall give the more complete formulae for all the neutrino-induced inclusive cross-sections based on the standard quark-parton model and also indicate how all the parameters can be determined.

The inclusive cross-sections on proton and neutron target for charged and neutral current processes are given by

$$\begin{aligned}
\sigma_C(\nu p) &= A(D+\tfrac{1}{3}\,\bar{U});\\
\sigma_C(\nu n) &= A(U+\tfrac{1}{3}\,\bar{D});\\
\sigma_C(\bar{\nu} p) &= A(\tfrac{1}{3}\,U+\bar{D});\\
\sigma_C(\bar{\nu} n) &= A(\tfrac{1}{3}\,D+\bar{U});
\end{aligned} \tag{24}$$

$$\begin{aligned}
\sigma_N(\nu p) &= A(C_1U+C_2D+C_3\bar{U}+C_4\bar{D});\\
\sigma_N(\nu n) &= A(C_2U+C_1D+C_4\bar{U}+C_3\bar{D});\\
\sigma_N(\bar{\nu} p) &= A(C_3U+C_4D+C_1\bar{U}+C_2\bar{D});\\
\sigma_N(\bar{\nu} n) &= A(C_4U+C_3D+C_2\bar{U}+C_1\bar{D});
\end{aligned} \tag{25}$$

where we have used the abbreviations

$$A \equiv \frac{2G^2ME}{\pi};$$

$$C_1 = u_L^2 + \tfrac{1}{3} u_R^2;$$

$$C_2 = d_L^2 + \tfrac{1}{3} d_R^2; \tag{26}$$

$$C_3 = \tfrac{1}{3} u_L^2 + u_R^2;$$

$$C_4 = \tfrac{1}{3} d_L^2 + d_R^2;$$

and U, D, $\bar{U}$ and $\bar{D}$ represent the contributions from the corresponding quark-parton. For instance,

$$U = \int_0^1 dx\, x\, u(x), \tag{27}$$

with similar equations for D, $\bar{U}$ and $\bar{D}$; $u(x)$, $d(x)$, $\bar{u}(x)$ and $\bar{d}(x)$ denote the corresponding quark-density functions with fractional momentum x inside the proton.

Measurement of the four charged-current cross-sections lead to a determination* of the four parton parameters U, D, $\bar{U}$ and $\bar{D}$ through (24). Then, measurement of the four neutral-current cross-sections lead to a determination of the four neutral-current coupling constants. u_L^2, u_R^2, d_L^2 and d_R^2 through eqs (25) and (26).

However, it is more convenient to consider the ratios of cross-sections. Define the following charged-current ratios:

$$R_c = \frac{\sigma_c(\bar{\nu}\mathscr{N})}{\sigma_c(\nu\mathscr{N})};\quad R_{cp} = \frac{\sigma_c(\bar{\nu}p)}{\sigma_c(\nu p)};\quad \rho_p = \frac{\sigma_c(\nu p)}{\sigma_c(\nu\mathscr{N})}. \tag{28}$$

Then, we have**

$$u_L^2 + d_L^2 = \frac{r - R_c^2\bar{r}}{1 - R_c^2};$$

$$u_R^2 + d_R^2 = \frac{R_c(\bar{r} - r)}{1 - R_c^2}; \tag{29}$$

$$u_L^2 + u_R^2 = \frac{1}{4}\,\frac{(1+R_c)(r_p+R_{cp}\bar{r}_p)\rho_p+\{1+R_c-2\rho_p(1+R_{cp})\}(r+R_c\bar{r})}{(1+R_c)\{1+R_c-\rho_p(1+R_{cp})\}};$$

$$u_L^2 - u_R^2 = \frac{2(1-R_c)(r_p-R_{cp}\bar{r}_p)\rho_p-\{1-R_c+\rho_p(1-R_{cp})\}(r-R_c\bar{r})}{2(1-R_c)\{1-R_c-\rho_p(1-R_{cp})\}}.$$

*The deep-inelastic electron or muon scattering data on protons and deuterons also can be used for this purpose.

**We assume $R_c \neq 1$. The present experimental value at SPS energies is $R_c = 0{\cdot}48 \pm 0{\cdot}03$.

Thus, the four neutral-current coupling constants can be determined from the four eqs in (29), provided the charged-current ratios R_c, R_{cp} and ρ_p as well as the neutral-current ratios r, $\bar{r}$, $r_{\bar{p}}$ and $\bar{r}_p$ are known.

The effect of including the $q\bar{q}$ sea in the proton can be seen by evaluating the first two of the general relations (29) using the inclusive data (14) for two cases: $R_c=\frac{1}{3}$ (i.e. sea is absent) and $R_c=0{\cdot}48$ which is the value given by CDHS collaboration;

$$\begin{array}{lll} & \underline{R_c = \frac{1}{3}} & \underline{R_c = 0{\cdot}48} \\ u_L^2 + d_L^2 = & 0{\cdot}289 \pm 0{\cdot}012 & 0{\cdot}281 \pm 0{\cdot}016 \\ u_R^2 + d_R^2 = & 0{\cdot}017 \pm 0{\cdot}012 & 0{\cdot}028 \pm 0{\cdot}020 \end{array} \tag{30}$$

The effects of including the sea thus result in changes in numerical values which lie within the errors. From the values listed above for $R_c=0{\cdot}48$ we note the obvious upper limits on the coupling constants,

$$\begin{array}{ll} u_L^2 \leqslant 0{\cdot}3, & d_L^2 \leqslant 0{\cdot}3 \\ u_R^2 \leqslant 0{\cdot}05, & d_R^2 \leqslant 0{\cdot}05. \end{array} \tag{31}$$

Our eqs (6–9) or (16–19) follow from (29) if we assume $U=2D$ and $\bar{U}=\bar{D}=0$ which imply $R_c=\frac{1}{3}$, $R_{cp}=\frac{2}{3}$, $\rho_p=\frac{2}{3}$. The accuracy of experimental data at the present stage do not seem to warrant a more detailed analysis. However, once all the cross-sections are measured with sufficient accuracy, these approximations can be dispensed with and the full version of (29) can be exploited.

5. Conclusions

Considerable progress has been achieved recently in the empirical determination of the neutral-current coupling constants. Our aim is to augment the generality of this analysis by including empirical data on the inclusive neutrino cross-sections on protons.

Although, in principle, the new measurements on the inclusive neutrino cross-sections on proton target could have led to an alternate determination of all the neutral-current coupling constants which would be independent of the assumptions of the quark-fragmentation model, the present data are not accurate enough to accomplish this. Complete formulae have been given which would enable the experimenters to do this once they achieve the requisite accuracy in their data.

For the present, we have reversed the procedure and tried to predict the neutral-current cross-sections on protons. Using only our knowledge of the neutral-current cross-sections on isospin-averaged nucleon, we are able to set upper as well as lower bounds on the ratios r_p and $\bar{r}_p$, which are quite restrictive.

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