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Scaling hypothesis, relativistic statistics and critical mass of dwarf stars

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Abstract. Assuming that field theoretic Green's functions scale, expressions for correlation function, number density, pressure are obtained for a relativistic statistical many-body system. The scaling parameter is related to anomalous dimension and is a measure of interaction. The effect of this interaction on the critical mass of dwarf stars is studied and is not found to be insignificant. A mass radius relation is deduced. The Chandrasekhar limit is reproduced in the limit of non-interaction.

Keywords. Green's function; correlation function; dwarf stars; critical mass.

1. Introduction

Recently Bowers *et al* (1973) have proposed a relativistic quantum many-body theory for the study of strongly interacting system. The authors have used the Green's functions approach for this system. A renewed interest to understand the properties of the matter at very high densities has been created. In the above work, an equation of state of relativistic interacting system was obtained and numerical estimates of the expected correction were given in some cases. Later, Bowers *et al* (1975) applied their results to construct a model for neutron stars. From this model they obtained a higher mass stability limit on the neutron star than that given by Oppenheimer and Volkoff (1939). To study statistical thermodynamics, the method discussed and reviewed by Zubarev (1960), i.e., the method of double time Green's functions has also been used profusely. Most of the useful thermodynamic properties of these systems can be written as correlation functions related to two point Green's functions.

Progress in solving interacting systems has also been made in another direction. Solutions of the renormalization group and Callan-Symanzik (Mueller and Trueman 1971) equations in the asymptotic limit suggest that the one-particle renormalized Green's functions for interacting system can be written in the momentum representation in the following form:

$$S_R(\mathbf{p}, w) = S_0(\mathbf{p}, w) \left[(w^2 - |\mathbf{p}|^2)/(m^2)\right]^{\alpha} \tag{1}$$

Where $S_R(\mathbf{p}, w)$ is the one-particle renormalized Green's function and $S_0(\mathbf{p}, w)$ is the free Green's function in the absence of interactions and α is a parameter which depends on the strength of the interaction and is a measure of anomalous dimension. Earlier, Gell-Mann and Low (1954) had obtained such a propagator for the electron in QED while studying the behaviour at small distances. An exact proof of asymptotic scaling where the theory is exactly soluble is still lacking. In most cases it is assumed and then shown to be consistent (Brout 1974). The purpose of the present paper is to investigate the thermodynamic behaviour of many-body systems where the Green's function scales and is given by (1).

We shall obtain expressions for correlation function, number density and pressure by standard methods. These results will be applied just as an illustration for the calculation of critical mass of a white dwarf star. The pressure in a white dwarf star (Huang 1963) is due to a highly degenerate electron gas. Under equilibrium condition this pressure is equal to the gravitational attraction that binds the star. The temperature in a white dwarf star is much lower than the Fermi temperature of the electron gas. The electron gas can be considered at absolute zero. In view of this point we have used the zero temperature thermodynamic Green's function in our calculations for this case. It is to be noted that the matter is very dense in such stars. The electrons are more like in a Wigner-Seitz solid. The energy of the electrons is also not that high so that there is obvious scaling. But it is of interest to know the possible effects on the properties of such stars if the dressed electron Green's function is a scaled one.

The Green's function of (1) has simple analytic properties for real α in the complex w-plane. It has the usual poles at $w=\pm E_p$ and also branch cuts extending from $-p$ to $+p$. In an earlier paper Deo and Kumar (1975) have solved the relativistic equation of motion of statistical Green's function in the Thirring model (Thirring 1958). Some thermodynamic properties of this self-interacting system of fermions were obtained. In Thirring model, the Green's function (Johnson 1961) exhibits scaling behaviour similar to that of (1) with $\alpha=g^2/4\pi^2\,(1-g^2/4\pi^2)$ where g is the four-fermion coupling constant. The Thirring model describes a system in one-space dimension only and is not realized in practice, even though the results obtained provide a basis for the calculations in a physical system. This work now extends the calculation to physical dimensions with an assumed Green's function analogous to the one in Thirring model.

In § 2, expressions for the retarded and advanced Green's functions and correlation function in terms of field operators are derived. Using these Green's functions and the related correlation function, number density and pressure equations are established in § 3. In § 4, the mass-radius equation of the white dwarf star is obtained and an upper limit on the mass of the star is found out. In § 5 we discuss our results.

2. Thermodynamic Green's functions and the correlation function

Vacuum expectation values of products of field operators with or without prescribed time ordering shall be called field theoretic Green's functions, i.e.

$$S(x-y)=\langle 0\,|\,T\,(\psi(x)\,\bar{\psi}(y))\,|\,0\rangle. \tag{2}$$

Callan-Symanzik equation (Mueller and Trueman 1971) in the asymptotic region for such functions can be written as

$$\left[m\frac{\partial}{\partial m}+\beta(g)\frac{\partial}{\partial g}-2\,\alpha(g)\right]S^{-1}\approx 0, \tag{3}$$

$S(p^2/m^2, g)$ is the momentum representation of $S(x-y)$. $\beta(g)$ in (3) is related to the derivative of g with respect to mass, i.e.,

$$\beta(g) = m\left(\frac{\partial}{\partial m} g\right), \tag{4}$$

and $\alpha(g)$ is the anomalous dimension factor.

For the coupling strength for which $\beta(g)$ vanishes

$$S(p^2/m^2, g) \approx (p^2/m^2)^{\alpha(g)}. \tag{5}$$

For purposes of direct calculation, the double time Green's functions method reviewed by Zubarev (1960) seems to be a better one. We define the following thermodynamic Green's and the correlation functions.

$$G_p(t-t') = -i\langle T(a(p, t)\, a^+(p, t'))\rangle, \tag{6}$$

$$F_p(t-t') = \langle a^+(p, t)\, a(p, t)\rangle, \tag{7}$$

where

$$\langle a^+(p, t')\, a(p, t)\rangle$$

$$= \mathrm{Tr}\{\exp[-\beta(\hat{H}-\mu\hat{N})]\}^{-1} \times \{\mathrm{Tr}\,[\exp(-\beta(\hat{H}-\mu\hat{N}))\, a^+(p, t')\, a(p, t)\} \tag{8}$$

$a^+(p, t')$ and $a(p, t)$ are the creation and annihilation operators in the Heisenberg representation and T denotes the time-ordered product. The operators $\hat{H}$ and $\hat{N}$ are the Hamiltonian and total number of particles operators. $\beta = 1/kT$ and μ denotes the chemical potential.

The unrenormalized operators $a^+(p, t)$ and $a(p, t)$ in Heisenberg representation for a Fermi system, for example can be written as

$$a(p, t) = 1/(2\pi)^{3/2}\, (m/E_p)^{1/2} \int d^3x\; U^+(p)\, \psi(x, t) \exp(i\mathbf{p}\cdot\mathbf{x}) \tag{9}$$

$$a^+(p, t) = 1/(2\pi)^{3/2}\, (m/E_p)^{1/2} \int d^3x\, \psi^+(x, t)\, U(p) \exp(-i\,\mathbf{p}\cdot\mathbf{x}), \tag{10}$$

where $E_p = (\mathbf{p}^2 + m^2)^{1/2}$ and a's are the interacting Heisenberg field operators for fermions. $U^+(p, s)$ and $U(p, s)$ are the usual Dirac spinors with spin s. We have omitted the spin index in the last equations. Field operator ψ's in (9) and (10) are the interacting fields.

The Green's function $G_p(t-t')$ can be written in terms of the interacting field operators as

$$G_p(t-t') = 1/(2\pi)^3\, (m/E_p) \int d^3x \int d^3x'$$

$$U^+(p)\, S_F(x-x', t-t')\, \gamma^0 U(p) \exp(i\,\mathbf{p}\cdot\mathbf{x}-\mathbf{x}') \tag{11}$$

where $S_F(x-x', t-t') = -i\langle T(\psi(x, t)\, \bar{\psi}(x', t'))\rangle. \quad (12)$

The hermitian conjugate operator $\psi^+ (x, t)$ is related to the adjoint operator $\bar{\psi}(x, t)$ by $\psi^+ = \bar{\psi}\, \gamma^0$. In writing (11) we have ignored the renormalization constant for simplicity. It can be easily incorporated at appropriate places.

We assume that the fourier transform of $S(x, t)$ given by S_R (p, w) scales and is written as

$$S_R(\mathbf{p}, w) = (\gamma \cdot p + m)/(p^2 - m^2)\,(p^2/m^2)^\alpha. \tag{13}$$

A word about the form chosen will be in order. Besides scaling and satisfying Callan-Symanzik equation, it also satisfies the on-shell requirement, i.e.,

$$(\gamma \cdot p - m)\, S_R(\mathbf{p} \cdot w)\,|_{\,\gamma \cdot p = m} = 1. \tag{14}$$

So the Green's functions written in this form may have a wider range of validity. The Green's function $G_p(t-t')$ in momentum representation is

$$G(\mathbf{p}, w) = 1/(w - E_p)\,[(w^2 - |\mathbf{p}|^2)/m^2]^\alpha \tag{15}$$

Retarded and advanced Green's functions can be contained from $G(\mathbf{p}, w)$ in the usual way. These functions are

$$G_r(\mathbf{p}, w) = \frac{1}{w - E_p + i\epsilon}\left[\frac{(w + i\epsilon)^2 - |\mathbf{p}|^2}{m^2}\right]^\alpha, \tag{16}$$

$$G_a(\mathbf{p}, w) = \frac{1}{w - E_p - i\epsilon}\left[\frac{(w - i\epsilon)^2 - |\mathbf{p}|^2}{m^2}\right]^\alpha. \tag{17}$$

Identical expressions can be obtained for the Bose systems also.

After getting these Green's functions the correlation function can be found easily. $F_p(t-t')$ has the following spectral representation

$$F_p(t-t') = \int_{-\infty}^{\infty} J(\mathbf{p}, w) \exp[-iw(t-t')]\, dw \tag{18}$$

Where the spectral function $J(\mathbf{p}, w)$ is related to the Green's functions given in (16) and (17) as

$$J(\mathbf{p}, w) = (1/2\pi i)\,[G_a(\mathbf{p}, w) - G_r(\mathbf{p}, w)]\,(1/[\exp \beta(w-\mu) + 1]) \tag{19}$$

After substituting relation (19) together with expressions (16) and (17) into (18) we arrive at the following spectral representation of $F_p(t-t')$

$$F_p(t-t') = (1/2\pi i)\,(1/(m^2)\alpha \int_{-\infty}^{\infty} [(1/\exp \beta(w-\mu)]_{+1} \exp[-iw(t-t')]$$

$$\times \left[\frac{\{(w-i\epsilon)^2 - |\mathbf{p}|^2\}^\alpha}{w - E_p - i\epsilon} - \frac{\{(w+i\epsilon)^2 - |\mathbf{p}|^2\}^\alpha}{w - E_p + i\epsilon}\right] dw. \tag{20}$$

This is the relation incorporating asymptotic scaling and on-shell constraint and will be used in the next section for the calculation of some thermodynamic quantities.

3. Expressions for number density, pressure and equation of state

In the limit $t'=t$ the correlation function (20) reduces to density of particles with momentum p, i.e.,

$$F_p(0) = \langle a^+(p, t)\, a(p, t)\rangle \equiv \langle n_p\rangle. \tag{21}$$

The number density $[n(\beta, \mu) = N/V]$ can be obtained by integrating (21) over all values of momenta (Kadonoff and Baym 1962), i.e.,

$$n(\beta, \mu) \equiv N/V = (2s+1)/(2\pi)^3 \int_0^\infty \langle n_p\rangle\, d^3p. \tag{22}$$

In the above N is the total number of particles in a volume V. $(2s+1)$ is the spin degeneracy factor. Now substituting the value of $\langle n_p\rangle$ from (20) into (22) we get

$$n(\beta, \mu) = -\frac{2i}{(2\pi)^4}\frac{1}{(m^2)^\alpha}\int_{-\infty}^{\infty}\frac{dw}{\exp[\beta(w-\mu)]+1}\int_0^\infty d^3p$$

$$\times\left[\frac{\{(w-i\epsilon)^2-|\mathbf{p}|^2\}^\alpha}{w-E_p-i\epsilon}-\frac{\{(w+i\epsilon)^2-|\mathbf{p}|^2\}^\alpha}{w-E_p+i\epsilon}\right]. \tag{23}$$

The pressure can be obtained from the relation

$$P(\beta, \mu) = \int_0^\mu n(\beta, \mu')\, d\mu'. \tag{24}$$

In the limit $\alpha\to 0$ (23) and (24) reduces to the number density and pressure of a free relativistic Fermi gas.

Our next task is to perform p and w integrations occurring in these expressions.

Let us consider the following integral coming in (23)

$$I(w) = \int_0^\infty\left[\frac{\{(w-i\epsilon)^2-|\mathbf{p}|^2\}^\alpha}{w-E_p-i\epsilon}-\frac{\{(w+i\epsilon)^2-|\mathbf{p}|^2\}^\alpha}{w-E_p+i\epsilon}\right]d^3p. \tag{25}$$

In this expression $d^3p=4\pi p^2dp$ and $E_p=(|\mathbf{p}|^2+m^2)^{1/2}$.

For the purpose of this paper we shall evaluate this expression for asymptotic values of E_p where scaling hypothesis is presumably correct. We expand the factors $(w-E_p\pm i\epsilon)^{-1}$ in (25) by Taylor series expansion around $m^2=0$ and retain only the lowest order term in m^2, i.e.,

$$(w-E_p\pm i\epsilon)^{-1} \approx (w-\mathbf{p}\pm i\epsilon)^{-1} + (m^2/2\mathbf{p})\,(w-p\pm i\epsilon)^{-2}$$

Substituting this expansion into (25) and after some calculations we obtain the result of the integrations, for $|a| < \frac{1}{2}$, as

$$I(w) = 8\, i\pi^2 C_1(a)\, w^{2+2a} - 4\pi i\, m^2\, C_2(a)\, w^{2a}, \tag{26}$$

where $$C_1(a) = \left[\frac{1}{a} - \frac{1}{a+\frac{1}{2}}\right] \Gamma(1+a)/(\pi)^{1/2}\, \Gamma(\tfrac{1}{2}-a)\, (\sin \pi a)/\pi,$$

and $$C_2(a) = \left[\frac{1}{a} + \frac{1}{1-a}\right] \Gamma(1+a)/(\pi)^{1/2}\, \Gamma(\tfrac{1}{2}-a)\, (\sin \pi a)/\pi.$$

Now substituting the expression (26) in the equation of number density (23) we arrive at

$$n(\beta, \mu) = \frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^a} \int_{-\infty}^{\infty} \frac{w^{2+2a}}{\exp\,[\beta(w-\mu)] + 1}\, dw - \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)^a} \int_{-\infty}^{\infty} \frac{w^{2a}}{\exp\,[\beta(w-\mu)] + 1}\, dw. \tag{27}$$

The expression for pressure is the following

$$P(\beta, \mu) = \int_0^{\mu} du' \left[\frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^a} \int_{-\infty}^{\infty} \frac{w^{2+2a}}{\exp\,[\beta(w-\mu)] + 1}\, dw - \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)^a} \int_{-\infty}^{\infty} \frac{w^{2a}}{\exp\,[\beta(w-\mu)] + 1}\, dw\right]. \tag{28}$$

The grand partition function is related to the pressure P and volume V by the relation

$$\log Z = \beta P V. \tag{29}$$

Taking suitable derivatives with respect to the variables like temperature and chemical potential one can easily get expressions for energy, specific heat, entropy, free energy, etc. Thus we are able to deduce the thermodynamic properties of a system in the asymptotic regions.

At this stage, for the application to dwarf stars to be discussed in the succeeding section, we specialise to Fermi system and that again to regions where $\mu/KT \gg 1$. In this limit the w-integrations in (27) and (28) can be performed approximately (Landau and Lifshitz 1959). The results in the lowest order of temperature are

$$n(\beta, \mu) = \frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^a} \left[\frac{\mu^{3+2a}}{(3+2a)} + \frac{\pi^2}{3} (KT)^2\, (1+a)\, \mu^{1+2a}\right] - \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)^a} \left[\frac{\mu^{1+2a}}{(1+2a)} + \frac{\pi^2}{3} (KT)^2\, a\, \mu^{2a-1}\right], \tag{30}$$

$$P(\beta, \mu) = \frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^a} \left[\frac{\mu^{4+2a}}{(4+2a)(3+2a)} + \frac{\pi^2}{6} (KT)^2 \mu^{2+2a} \right]$$

$$- \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)^a} \left[\frac{\mu^{2+2a}}{(2+2a)(1+2a)} + \frac{\pi^2}{6} (KT)^2 \mu^{2a} \right]. \quad (31)$$

The equation of state can be obtained by substituting the value of μ from (30) into (31). As a ready check, letting $a=0$, we get back

$$n(\beta, \mu) = (1/\pi^2) [(\mu^3/3) + (\pi^2/3) (KT)^2 \mu] - (m^2\mu/2\pi^2), \quad (32)$$

$$P(\beta, \mu) = (1/\pi^2) [(\mu^4/12) + (\pi^2/6) (\mu kT)^2] - (m^2/2\pi^2)$$

$$\times [(\mu^2/2) + (\pi^2/6) (kT)^2]; \quad (33)$$

which are the expressions for number density and pressure of a free, degenerate, extreme relativistic electron gas given by Landau and Lifshitz (1959).

4. Critical mass of a star

In this section we shall apply the results obtained in the previous section to the case of a white dwarf star as an illustration. These stars are composed mainly of helium nuclei, which are responsible for gravitational binding and electron gas. There are nuclear interactions between helium nuclei and also electromagnetic interaction between charged particles. Since the Fermi energy for electrons is of the order of 20 MeV, pion production by electrons is not energetically feasible. However, the pressure built up is mainly due to the electron gas and will surely be affected by the electrons interacting among themselves. We assume that the net effect of this and of other coulombic, weak interactions is to give a scaled Green's function in the asymptotic limit. This assumption is supported by the work of Gell-Mann and Low (1954) for QED at small distances. As the temperature of a white dwarf star ($\approx 10^7$ °K) is much lower than Fermi temperature for the electrons ($\approx 10^{11}$ °K), we can take the limit $T \to 0$ in equations (30) and (31). In this limit we get

$$N/V = n(\beta, \mu) \approx \frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^d} \frac{\mu^{3+2a}}{(3+2a)} - \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)a} \frac{\mu^{1+2a}}{(1+2a)}, \quad (34)$$

$$P(\beta, \mu) \approx \frac{1}{\pi^2} \frac{C_1(a)}{(m^2)^a} \frac{\mu^{4+2a}}{(4+2a)(3+2a)} - \frac{m^2}{2\pi^2} \frac{C_2(a)}{(m^2)a} \frac{\mu^{2+2a}}{(2+2a)(1+2a)}. \quad (35)$$

Now we shall eliminate μ from (35) with the help of (34). In the first approximation, the value of the chemical potential will be obtained from the first term only of (34), i.e.,

$$\mu = [(3+2a)\, \pi^2\, (m^2)^a\, N/VC_1(a)]^{1/3+2a} \quad (36)$$

The density of particles (N/V) is related to the mass and radius of the white dwarf star (Huang 1963) by

$$N/V = 3M/(8\pi m_p R^3) \tag{37}$$

where M is the total mass of the white dwarf star and R is its radius. m_p is the mass of a proton. Further two other dimensionless quantities can be defined as

$$\bar{M} = (9\pi/8)\,(M/m_p) \text{ and } \bar{R} \doteq mR. \tag{38}$$

We have used the notations $\hbar = c = 1$. m is the mass of the electron. In terms of $\bar{M}$ and $\bar{R}$, relation (37) becomes

$$N/V = (\bar{M}/3\pi^2)\,(m^3)/(\bar{R})^3. \tag{39}$$

Now we substitute this value of N/V into (36). Thus we obtain

$$\mu = \left[\frac{(3+2a)}{3}\frac{(m^2)^a}{C_1(a)}\, m^3\, \frac{\bar{M}}{(\bar{R})^3}\right]^{1/(3+2a)}. \tag{40}$$

The value of μ so obtained is substituted back in the expression (35). The resultant expression for pressure built up for the dwarf star is

$$P = \frac{1}{\pi^2}\frac{C_1(a)}{(m^2)^a}\frac{1}{(4+2a)\,(3+2a)}\left[\frac{(3+2a)}{3(C_1(a)}\, m^{3+2a}\, \frac{\bar{M}}{(\bar{R})^3}\right]^{(4+2a)/(3+2a)}$$

$$-\frac{m^2}{2\pi^2}\frac{C_2(a)}{(m^2)^a}\frac{1}{(1+2a)\,(2+2a)}\left[\frac{(3+2a)}{3C_1(a)}\, m^{3+2a}\, \frac{\bar{M}}{(\bar{R})^3}\right]^{(2+2a)/(3+2a)}. \tag{41}$$

Now from the equilibrium condition (Huang 1963), i.e., for stability against gravitational collapse we get

$$\frac{1}{\pi^2}\frac{C_1(a)}{(m^2)^a}\frac{1}{(4+2a)\,(3+2a)}\left[\frac{(3+2a)}{3C_1(a)}\, m^{3+2a}\, \frac{\bar{M}}{(\bar{R})^3}\right]^{(4+2a)/(3+2a)}$$

$$-\frac{m^2}{2\pi^2}\frac{C_2(a)}{(m^2)^a}\frac{1}{(1+2a)\,(2+2a)}\left[\frac{(3+2a)}{3C_1(a)}\, m^{3+2a}\, \frac{\bar{M}}{(\bar{R})^3}\right]^{(2+2a)/(3+2a)}$$

$$= \delta/4\pi\; G(8mp/9\pi)^2\, m^4\, (\bar{M})^2/(\bar{R})^4. \tag{42}$$

The r.h.s. of (42) represents the contraction pressure due to gravitational attraction, G is the gravitational constant and δ is a pure number of the order of unity. We take $\delta = 1$. We can solve this equation for critical mass approximately.

Examining (42) carefully we see that the powers of m cancels from both sides. Introducing constants A, B, and K by the following definitions,

$$A = \frac{1}{\pi^2} \frac{C_1(\alpha)}{(4+2\alpha)(3+2\alpha)} \left[\frac{(3+2\alpha)}{3C_1(\alpha)}\right]^{(4+2\alpha)/(3+2\alpha)}, \tag{43}$$

$$B = \frac{1}{2\pi^2} \frac{C_2(\alpha)}{(2+2\alpha)(1+2\alpha)} \left[\frac{(3+2\alpha)}{3C_1(\alpha)}\right]^{(2+2\alpha)/(3+2\alpha)}, \tag{44}$$

$$K = \frac{1}{4\pi} G \left(\frac{8m_p}{9\pi}\right)^2, \tag{45}$$

equation (40) becomes

$$A\left(\frac{\bar{M}}{(\bar{R})^3}\right)^{(4+2\alpha)/(3+2\alpha)} - B\left(\frac{\bar{M}}{(\bar{R})^3}\right)^{(2+2\alpha)/(3+2\alpha)} = K\frac{(\bar{M})^2}{(\bar{R})^4}. \tag{46}$$

This is the equation that gives the required relation between mass and radius of the star. The existence of the solution of this equation will place a limit on the mass of the white dwarf star. We assume that α is small since it is more likely to be of the order of electromagnetic interaction strength and write a further approximate equation

$$B(\bar{M})^{(2+2\alpha)/(3+2\alpha)} (\bar{R})^2 + K(\bar{M})^2 - A(\bar{M})^{(4+2\alpha)/(3+2\alpha)} = 0. \tag{47}$$

This equation has a solution for $\bar{R}$, namely

$$\bar{R} = \left(\frac{A}{B}\right)^{1/2} (\bar{M})^{(1)/(3+2\alpha)} \left[1 - \frac{(\bar{M})^{(2+2\alpha)/(3+2\alpha)}}{A/K}\right]^{1/2}. \tag{48}$$

The upper limit on the mass of the star can be obtained from the positivity of the quantity under the square root, namely

$$\frac{(\bar{M})^{(2+2\alpha)/(3+2\alpha)}}{A/k} = 1, \tag{49}$$

$$\bar{M} = \left(\frac{A}{k}\right)^{(3+2\alpha)/(2+2\alpha)}. \tag{50}$$

From (38) and (50) we obtain the maximum mass limit on the white dwarf star, i.e.,

$$M = \frac{8m_p}{9\pi}\left(\frac{A}{k}\right)^{(3+2\alpha)/(2+2\alpha)}$$

$$= \frac{3}{2} \frac{m_p}{(4+2\alpha)} \frac{\hbar c}{G(m_p)^2} \left[\frac{9\pi}{16}\left(\frac{3+2\alpha}{4+2\alpha}\right) \frac{1}{C_1(\alpha)} \left(\frac{\hbar c}{Gm_p^2}\right)\right]^{(1)/(2+2\alpha)}. \tag{51}$$

This is our final expression for the critical mass of a white dwarf star in the presence of interactions. In this expression appropriate factors of $\hbar$ and c are introduced

Table 1. Variation of critical mass with the scaling parameter a

Scaling parameter a	Mass of the star in units of sun's mass
0·1	0·02
0·01	0·73
0·001	1·08
0·00058	1·10
−0·1	156·82
−0·01	1·77
−0·001	1·18
−0·00058	1·11

to have proper dimension of mass on the right hand side. It is easy to see that in the limit $a \to 0$, we get back the Chandrasekhar limit. Values of the critical mass for different values of a are given in table 1.

5. Conclusions

In this paper zero temperature thermodynamic Green's functions are obtained from the corresponding field theoretic Green's functions. Using these functions we have studied the thermodynamic properties of a Fermi system. Further the pressure equation obtained by us is applied to a white dwarf star as an illustration. We have computed the critical mass for various values of the coupling constants. It is noticed that the corrections due to interactions are quite significant. This type of study can be made for neutron stars and similar other systems.

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