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Negative muon capture in ^{3}He

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Abstract. We compute the partial capture rate of negative muons in ^{3}He by following the analysis of Peterson to include the relativistic corrections and the exchange effects, for various values of the g_p/g_A ratio. We also calculate the total capture rate. The ground state of ^{3}He is assumed to be spherical. The radial dependence of the ground state wave function is taken to be (a) one parameter Irving function, (b) a modified three-parameter Irving function and (c) a function having 'soft-core', whose parameters have been fixed in a variational calculation of the binding energy of the triton using a non-local momentum-dependent potential involving p^2 terms. The calculated values of the capture rates are compared with the experimental data to find a value for the g_p/g_A ratio.

Keywords. Muon capture in ^{3}He; total and partial capture rates; softcore function; momentum-dependent potential.

1. Introduction

A number of theoretical calculations (Primakoff 1959; Goulard *et al* 1964; Oakes 1964; Yano 1964; Peterson 1968; Peachey 1969; Phillips *et al* 1975) on the muon capture rate in ^{3}He have been reported in the literature. The aim of these calculations are either to determine the ratio of the induced pseudoscalar and axial vector muon dressed coupling constants (g_p/g_A), or to calculate the capture rate assuming a suitable value for the above ratio. Most of these calculations are based on the theory given by Primakoff (1959). The results of both Peterson (1968) and Peachey (1969) clearly show that the effects of the mixed symmetric (S') state and the D-state probabilities (and also the parameters used for these states) in the partial capture rate calculations for the reaction,

$$\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu, \tag{1}$$

are quite insensitive. A similar result has been reported for the total muon capture rate in ^{3}He by Phillips *et al* (1975). Further, Peachey (1969) finds that the partial capture rate is sensitive to the size parameter of S-state. If this was fixed by fitting the Coulomb energy, a rather different size parameter resulted by fitting the r.m.s. radius of the trinucleon. In order to eliminate this uncertainty in the size parameter determination we have taken in this investigation, the values of the parameters which are obtained in a variational calculation of the binding energy of the triton. Of course, a variational calculation is not free from defects. However, for a size parmeter fixed in the variational calculation by minimising the energy, we can simultaneously obtain a good fit for both the r.m.s. radius and the Coulomb energy of the trinucleon.

In this paper we report the results for the partial and the total muon capture rates obtained with (a) single parameter Irving function (b) a modified three-parameter Irving function and (c) a soft-core function whose parameters have been determined in variational calculations of the binding energy of the triton using the momentum-dependent potential (MDP) of Srivastava (1965) (which is a representative of the soft-core potentials). We compare our calculated values of the capture rate with the available experimental data in order to extract a value for the g_p/g_A ratio. In view of the findings of Peachey (1969) and Phillips *et al* (1975) we consider the *S*-state only. For computing the partial capture rate we have used the expression given by Peterson (1968), who includes both the relativistic corrections and the exchange effects. We have followed the analysis of Goulard *et al* (1964) to calculate the total muon capture rate in the closure approximation, which includes the relativistic corrections.

2. Ground state wave function and size parameter determination

The ground state wave function for the three-nucleon system can be written as (when the *S*-state alone is present)

$$\psi_S = \Phi_S(\phi^m \bar{\eta}^t - \bar{\phi}^m \eta^t), \tag{2}$$

where ϕ^m, $\bar{\phi}^m$ and η^t, $\bar{\eta}^t$ are respectively, the spin and iso-spin doublet functions. We choose the radial function Φ_S to be of the following forms:

$$\Phi_S = N_S \left\{ \exp\left[-\alpha \left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right] + A \exp\left[-\lambda \left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right]\right\} \tag{3}$$

$$\Phi_S = N_{S'} \exp\left[-\beta \left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right] \Big/ \left(\sum_{i<j} r_{ij}^2\right)^n \prod_{i<j} r_{ij} \tag{4}$$

in which α, λ, A, β and n are the variational parameters. Equation (3) represents a modified three-parameter Irving function and when $A=0$ it becomes the one-parameter Irving function. The soft-core function is given by (4) and is found to be flexible in the charge form factor and pion-photo-production calculations (Yadav *et al* 1972; Lazard and Maric 1973). Using the function given by (3), Jain and Srivastava (1969) have calculated the binding energy of the triton variationally, employing the MDP of Srivastava (1965). Raghavan (1975) has used the same two-body potential with the soft-core function and evaluated the binding energy. His calculated values of the binding energy, the r.m.s. radius of the triton and the Coulomb energy of ^{3}He are shown in table 1 along with those obtained by Jain and Srivastava (1969).

From the table we find that the one parameter Irving wave function gives almost the same value for the binding energy of ^{3}H as the soft-core function. However, the r.m.s. radius and the Coulomb energy obtained with the soft-core function are better

Table 1. Properties of the tri-nucleon

Wave function	Binding energy (^3He) (MeV)	r.m.s. radius (fm) (^3He)	Coulomb energy (^3He) (MeV)	Dip in F_{ch} (^3He) (fm^{-2})	Sec. Max in F_{ch} (^3He) (fm^{-2})	Best values of the variational parameters			
						β or α(fm^{-1})	λ(fm^{-1})	A	n
One parameter Irving+	6·84	1·96	0·659	Nil	Nil	0·55	—	0	—
Three-parameter Irving+	8·21	1·68	0·724	Nil	Nil	0·70	1·23	—1·2	—
Soft-core*	6·87	1·56	0·701	12·5	18	1·01	—	—	$\frac{3}{4}$
Experiment	8·48	1·64	0·764	11·6	18·2	—	—	—	—

+Jain and Srivastava (1969)

*Raghavan (1975)

than those obtained with the one parameter Irving function. A comparison of the results obtained for the modified three-parameter Irving function with those obtained using the soft-core function shows that the former describes the three-nucleon ground state properties well. But this is to be expected, since Jain and Srivastava (1969) have shown that the binding energy has converged for the three parameter function. On the other hand the soft-core function can be made to include more free parameters, for example,

$$\Phi_S = \frac{N_{S'}\left\{\exp\left[-\beta\left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right] + B\exp\left[-\delta\left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right]\right\}\prod_{i<j} r_{ij}}{\left(\sum_{i<j} r_{ij}^2\right)^n} \tag{5}$$

in which B and δ are additional parameters. By using this function one can expect the binding energy to increase and possibly show convergence. Because of the enormous numerical work involved, for the present, we have not used the above radial function for calculating the binding energy.

It has been found (Raghavan 1975) that the soft-core function is able to reproduce a diffraction minimum and a secondary maximum in the ^{3}He charge form factor calculations in accordance with the experimental observation (McCarthy *et al* 1970). But the Irving functions could not reproduce these observables. [It is to be noted that the form factor data favour a core in the wave function (McCarthy *et al* 1970)]. Considering the form factor data and other properties of the three-nucleon system obtained with the soft-core function we believe that the MDP and the soft-core function may constitute a reasonably good model for the trinucleon.

3. Partial capture rate calculations

The partial capture rate for the reaction (1) is given by (Peterson 1968):

$$\Lambda = (R/2\pi^2)\ [2m'_\mu/137]^3 k_\nu^2\ G^2/(1+k_\nu/m_{\mathrm{He}}) \times \tfrac{1}{2}\sum_{\text{spins}}\left\{A\left|\int 1\right|^2 + B\left|\int \sigma\right|^2\right\} \tag{6}$$

in which $A = G_V^2 - 2G_V F_V k_\nu/3M$

$$B = G_A^2 + \tfrac{1}{3}G_p(G_p - 2G_A) + 2F_A k_\nu (G_p - G_A)/9M \tag{7}$$

In (6) R is the reduction factor which accounts for the averaging process and for finite size of the nucleus, m'_μ is the reduced mass of the muon and k_ν is the photon energy. $|\int 1|^2$ and $|\int \sigma|^2$ are the form factor integrals and G, G_V, G_A and G_p are the coupling constants which are explicitly given in Peterson's (1968) paper. This calculation also includes the effect of spatial extension of the nuclear charge distribution on the atomic wave function of the muon. It has been shown by Peterson (1968) that the exchange effects are very important in the partial capture rate calculations, and so we have included this correction by means of a phenomenological treatment.

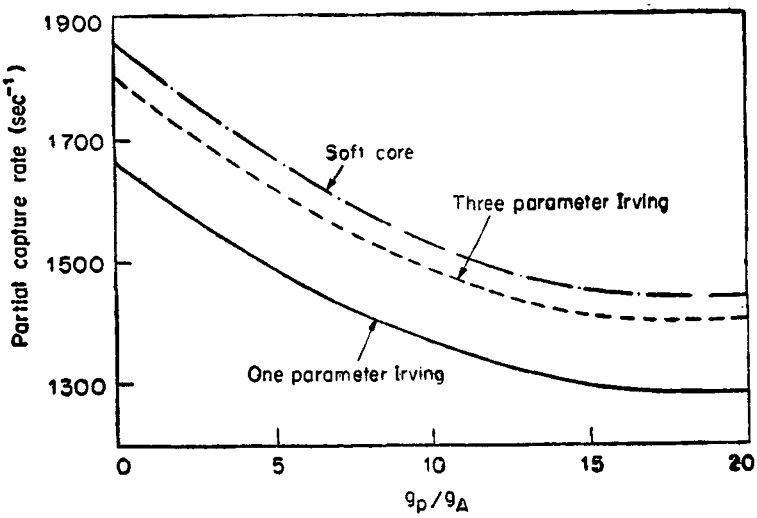

Figure 1. Partial muon capture rate in ^{3}He vs the g_p/g_A ratio.

As mentioned earlier we restrict ourselves to the *S*-state of the tri-nucleon. Using (6) and (7) we have evaluated the capture rates for the Irving and soft-core functions for various values of the g_p/g_A ratio and our results are shown in figure 1.

4. Total capture rate calculations

In the closure approximation the total muon capture rate in ^{3}He, when the relativistic corrections are included, is given by (Goulard *et al* 1964):

$$\Lambda^{(\mu)}(^3\text{He}) = Z^4_{\text{eff}}\,(\langle\eta\rangle_a)^2(272\ \text{sec}^{-1})R$$

$$\times \left\{1-\tfrac{1}{2}(\alpha_a^{(+)} + \alpha_a^{(-)})\,(1-x_a)\right\}, \tag{8}$$

in which

$$\alpha_a^{(+)} + \alpha_a^{(-)} = \int\!\!\int\!\!\int j_0\,(\langle\nu\rangle_a\,|\mathbf{r}-\mathbf{r}'|\,)\,|\phi_a(\mathbf{r},\mathbf{r}',\mathbf{r}'')|^2$$

$$\times\ \delta(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')d\mathbf{r}\ d\mathbf{r}'\ d\mathbf{r}'' \tag{9}$$

In the above equation $\phi_a(\mathbf{r}, \mathbf{r}', \mathbf{r}'')$ is the position-space wave function of ^{3}He. The term $\alpha_a^{(+)}+\alpha_a^{(-)}$ is called the exclusion principle inhibition factor and this alone requires the explicit consideration of the ground state wave function of ^{3}He. The quantities Z^4_{eff}, $(\langle\eta\rangle_a)^2$ and R have been evaluated following the analysis of Goulard *et al* (1964). We have evaluated the integral in (9) with our wave functions and have calculated $\Lambda^{(\mu)}$ (^{3}He) for various values of the g_p/g_A ratio. The calculated values are shown in figure 2.

5. Discussion

From the figures we note that the differences in the capture rates obtained with the modified three-parameter Irving wave function and the soft-core function are small. But the one-parameter Irving function predicts capture rates which are far away from those obtained with the other two functions. This is due to the fact that the capture rate, to a good approximation, depends on the mean square radius (Oakes 1964) and the radius obtained for the one-parameter function is very large. So, for the rest of the discussion we omit the results obtained with the one parameter Irving function.

The experimental values (Auerbach *et al* 1965; Clay *et al* 1965) of the partial capture rate for the reaction (1) are in the range 1460 to 1530 sec^{-1}, with a weighted average of 1470 sec^{-1} (Peterson 1968). For this value of the capture rate, we find from figure 1 that the g_p/g_A ratios are 10·7 and 13 respectively, for the modified three-parameter Irving function and for the soft-core function.

The total muon capture rate in ^3He has been measured by two experimental groups (Zimaidoroga *et al* 1963, Auerbach *et al* 1965). Zimaidoroga *et al* (1963) report that this rate is 2170^{+170}_{-430} sec^{-1}, while Auerbach *et al* find this value to be 2140 ± 180 sec^{-1}. The upper limits for these experiments are very close but the lower limits differ significantly. From figure 2 we note that for both the lower limits of the experimental results the g_p/g_A ratio falls well beyond 20. Hence, neglecting the negative deviation of the experimental values, we find from figure 2 that the g_p/g_A ratios are in between 10 and 18 for the modified three parameter Irving function and from 8·5 to 15 for the soft-core function. A more reliable experimental value of

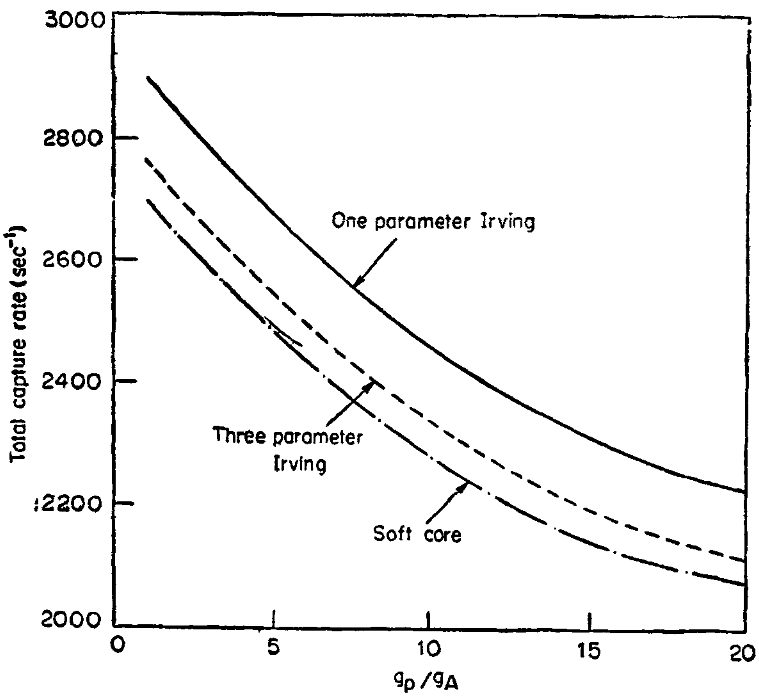

Figure 2. Total muon capture rate in ^3He vs the g_p/g_A ratio.

the total capture rate would have enabled us in finding a near exact value of the g_p/g_A ratio.

Considering the results obtained with the soft-core function—as this type of function is favoured by the electro-magnetic form factor data (McCarthy *et al* 1970)—we note that the range of values for the g_p/g_A ratio obtained in our calculations agree reasonably well with those of Peachey (1969), who estimates $g_p/g_A=9\pm4$, and with those of Peterson (1968) who finds the above ratio to be in between 6 and 32 with the most probable value 11. Also Clay *et al* (1965) find the g_p/g_A ratio in the range 1 to 17, with the probable value 11·6. Our estimated values differ from that of Goldberger and Treiman (1968) who find $g_p/g_A=7$ on the basis of PCAC hypothesis. However, the radiative muon capture experiments which are sensitive to the value of g_p, give $g_p/g_A = 13{\cdot}3 \pm 2{\cdot}7$ (Conversi *et al* 1964) and $16{\cdot}5 \pm 3{\cdot}4$ (Fearing 1966). It is heartening to find that our results agree reasonably well with these experimental results.

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