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Analysis of signal-to-noise enhancement of Box-Car averagers

K NEELAKANTAN and S DATTAGUPTA
Reactor Research Centre, Kalpakkam 603 102

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Abstract. The signal-to-noise improvement ratio (SNIR) of a Box-Car averager is calculated for various noise sources such as random white noise, exponentially correlated noise, etc. For a time constant (RC), and a sampling time ϵ, the quoted value of SNIR as $(2RC/\epsilon)^{1/2}$ is shown to be strictly correct only when the noise is white and deviation from this law is expected for other non-white noise sources. The validity of some of the calculated expressions is established by direct measurement of noise output.

Keywords. Signal averaging; Box-Car averager; signal-to-noise improvement ratio.

1. Introduction

The technique of signal averaging has been widely used to recover signals entirely buried in noise. Two criteria must be satisfied before signal averaging can be employed: (i) the signal must be repetitive, and (ii) a reference trigger must accompany the signal source (PARC Manual 1976). The basis of signal averaging lies in the assumption that the noise, being random, is uncorrelated to the trigger and should be averaged out in an incoherent fashion, while the signal should build up coherently (Heiftje 1972; Klein and Barton 1963).

It has been suggested that when signal averaging is used, the signal-to-noise improvement ratio is proportional to the square root of the number of samples, in the case of *linear* signal averaging (Jardetzky *et al* 1965). Ernst (1965) has made a detailed analysis to show that this $(n)^{1/2}$—law holds exactly in the case of white noise but there are deviations from this law in the case of non-white noise. The Box-Car averager, on the other hand, uses a *non-linear* averaging process and there does not exist in the literature, to our knowledge, a treatment which parallels Ernst's for linear averaging. It has been mentioned in one or two places in the literature that for a Box-Car with a time constant (RC) and a sampling time ϵ, the signal-to-noise improvement ratio (SNIR) is proportional to $(2RC/\epsilon)^{1/2}$ (PARC Manual 1976). The purpose of this paper is to show that this result, which in some sense is the analogue of the $(n)^{1/2}$ law for linear averaging, is again strictly correct only for a white noise. We present also expressions for SNIR for other noise sources.

In section 2, we discuss very briefly the principle of a Box-Car averager. Section 3 contains the mathematical analysis, some unedifying details of which are relegated to the appendix. In section 4, the measured values of the noise output for the cases of white noise and exponentially correlated noise sources are presented and are compared with the calculated values. Finally, some concluding remarks are made in section 5.

2. Box-car averager

Numerous Box-Car averager circuits have been described in the literature (Clark and Kerlin 1967, Reichert and Townsend 1964, Collins and Katchinoski 1973, etc.). A simplified block diagram is shown in figure 1. The noisy signal is amplified and fed to an analogue gate. This analogue gate blocks the signal voltage when the gate is closed. On receiving a sampling pulse (derived from the trigger) the gate opens and allows the signal to be integrated. The integrated value of the sampled signals forms the output. The width of the sampling pulse and its position in time relative to the trigger can be varied. Figure 2 shows the various timings and figure 3 shows the capacitor voltages during this process.

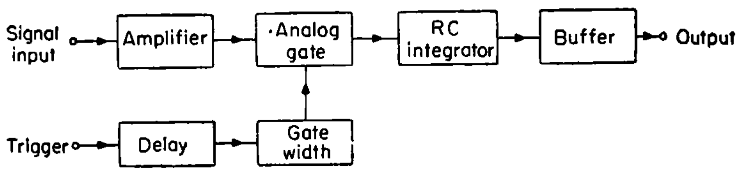

Figure 1. Block diagram of Box-Car averager

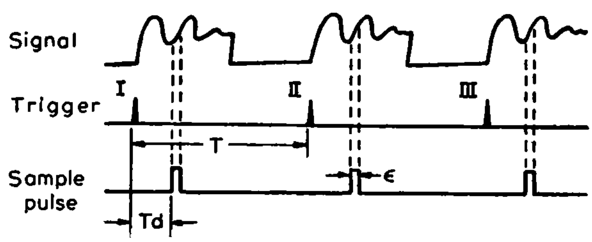

Figure 2. Timing diagram of Box-Car averager

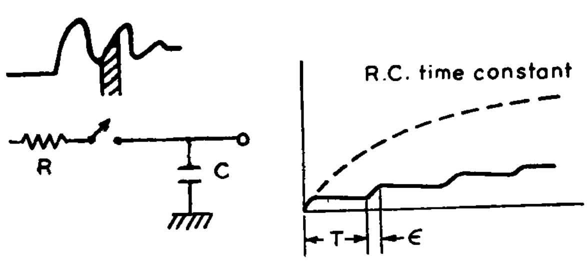

Figure 3. Capacitor voltage as a function of time

3. Analysis

The mathematical analysis is based on the following definitions (Ernst 1965).

$$\mathrm{S/N} = \frac{\text{Peak value of the signal voltage}}{\text{rms value of the noise voltage}},$$

$$\text{SNIR (signal-to-noise improvement ratio)} = \frac{(\mathrm{S/N})_{\text{output}}}{(\mathrm{S/N})_{\text{input}}}.$$

The noise is assumed to be stationary, ergodic and to have zero mean value.

For the purpose of the present analysis, we assume the signal to be periodic. Although the Box-Car averager only requires the signal to be repetitive, the assumption of periodicity simplifies the calculations without seriously affecting the results. At every repetition, the noisy signal is sampled for a time ϵ and the sampled voltages are passed through an RC filter. We assume that the sampling time is so short that the signal voltage is essentially constant within that interval. The capacitor is uncharged at the beginning of the first sampling while at the end of it and at the beginning of the subsequent samplings, the capacitor is charged. The charge across the capacitor at any time is governed by the equation

$$R(dq/dt) + (q/C) = V(t), \tag{1}$$

whose solution for an arbitrary initial condition reads

$$q(t_2) = \exp[-\lambda(t_2 - t_1)] \left[q(t_1) + (1/R) \int_{t_1}^{t_2} dt' \exp[\lambda(t' - t_1)] \, (V + V_N(t')) \right],$$
$$t_2 > t_1, \ \lambda = (\mathrm{RC})^{-1}, \tag{2}$$

where V is the constant value of the signal voltage over the sampling time ϵ and $V(_N t)$, the noise voltage.

Thus at the end of the first sampling, the voltage across the capacitor is

$$V_1 = \lambda \exp(-\lambda\epsilon) \int_0^\epsilon dt' \exp(\lambda t') \, (V + V_N(t')). \tag{3}$$

At the end of the second sampling, the voltage is

$$V_2 = \lambda \exp(-2\lambda\epsilon) \int_0^\epsilon dt' \exp(\lambda t') (V + V_N(t')) + \lambda \exp[-\lambda(T+\epsilon)]$$
$$\times \int_T^{T+\epsilon} dt' \exp(\lambda t') \, (V + V_N(t')), \tag{4}$$

using (2) and (3). Repeating this n times, the voltage across the capacitor after n samplings can be written as

$$V_n = \lambda \sum_{k=1}^{n} \exp[-\lambda(k\epsilon + (n-k)T] \int_{(n-k)T}^{(n-k)T+\epsilon} dt' \exp(\lambda t') \, (V + V_N(t')). \tag{5}$$

The output signal and noise components may be separated in (5) as

$$V_n = V_{SO} + V_{NO}, \tag{6}$$

where $V_{SO} = V(1 - \exp(-n\lambda\epsilon))$, (7)

and

$$V_{NO} = \lambda \sum_{k=1}^{n} \exp[-\lambda(k\epsilon + (n-k)T)] \int_{(n-k)T}^{(n-k)T+\epsilon} dt' \exp(\lambda t')\, V_N(t'). \tag{8}$$

It is clear from (7) that the signal component of the output voltage builds up to a peak value V after a sufficiently large number of samplings have been performed ($n\lambda\epsilon \gg 1$).

Assuming the noise to be completely uncorrelated to the signal part, the mean square output noise voltage after n samplings is given by

$$\langle V_{NO}^2 \rangle = \lambda^2 \sum_{k,l=1}^{n} \exp(-\lambda(k+l)\epsilon) \int_0^\epsilon d\tau \int_0^\epsilon d\tau' \exp(\lambda(\tau+\tau')) \times F(\tau - \tau' - (k-l)T), \tag{9}$$

where $F(\tau)$ is the noise correlation function defined by

$$F(\tau) = \langle V_N(\tau)\, V_N(0) \rangle, \tag{10}$$

where $\langle \ldots . \rangle$ denotes an ensemble average. In writing (9) we have assumed that the noise is stationary:

$$\langle V_N(t)\, V_N(t') \rangle = \langle V_N(t-t')\, V_N(0) \rangle = \langle V_N(t'-t)\, V_N(0) \rangle. \tag{11}$$

It may be pointed out that in (9), the terms in the double summation for which $k=l$, correspond to noise correlations within a sampling interval or 'intra-gate correlations' while the terms for which $k \neq l$, correspond to 'inter-gate correlations'.

For the purpose of later applications, (9) can be simplified to the form (see appendix I).

$$\langle V_{NO}^2 \rangle = \frac{2\lambda \exp(-\lambda\epsilon)}{1 - \exp(-2\lambda\epsilon)} \int_0^\epsilon d\tau \sinh(\lambda(\epsilon - \tau)) \Big\{ (1 - \exp(-2n\lambda\epsilon))\, F(\tau) + \sum_{k=1}^{n-1} \exp(-k\lambda\epsilon)\, [1 - \exp(-2(n-k)\lambda\epsilon)] \times [F(kT+\tau) + F(kT-\tau)] \Big\}. \tag{12}$$

As mentioned earlier, in order to extract the signal completely, the number of samplings must be such that

$$n\lambda\epsilon \gg 1. \tag{13}$$

This yields

$$(\mathrm{S/N})_{\mathrm{output}} = V/\langle V^2_{\mathrm{NO}}\rangle^{1/2}, \tag{14}$$

and

$$\mathrm{SNIR} = (\langle V^2_{\mathrm{N}}\rangle/\langle V^2_{\mathrm{NO}}\rangle)^{1/2}. \tag{15}$$

3.1. *White noise*

White noise is characterised by a constant spectral power density independent of the frequency. Its correlation function may be represented as

$$F(\tau) = 2\pi R' P \delta(\tau), \tag{16}$$

where R' is an arbitrary resistance across which the power is measured and P is the power spectral density. Examples of white noise are random noise of thermal origin like the Johnson noise or space-charge-limited shot noise.

Substituting (16) in (12), it is clear that in the white noise case, only the ' intra-gate correlations ' survive (since $T > \epsilon$ usually), and we obtain

$$\langle V^2_{\mathrm{NO}}\rangle = 2\pi\lambda P R' \,[1 - \exp(-2n\lambda\epsilon)]. \tag{17}$$

On the other hand, for a sampling interval ϵ, the mean square input noise voltage is

$$\langle V^2_{\mathrm{N}}\rangle = \frac{2\pi P R'}{\epsilon/2}, \tag{18}$$

which implies that in the white noise case,

$$\langle V^2_{\mathrm{NO}}\rangle = (\epsilon\lambda/2)\,\langle V^2_{\mathrm{N}}\rangle\,(1 - \exp(-2n\lambda\epsilon)). \tag{19}$$

Imposing the condition (13) on (19), we finally have

$$(\mathrm{SNIR})_{\text{white noise}} = (2\mathrm{RC}/\epsilon)^{1/2}. \tag{20}$$

This result may be viewed as the analogue of the $(n)^{1/2}$ law derived earlier by Ernst (1965) in the case of *linear* averaging. For the output voltage to reach *to within* 0·7 % of its final value, we must have $n\epsilon \geqslant 5$ RC (cf. condition (13)) so that in a sense $(2\mathrm{RC}/\epsilon)^{1/2}$ is related to $(n)^{1/2}$.

3.2. *Exponentially correlated noise*

When pure white noise is passed through a low-pass RC filter, the resulting noise has an exponential correlation function:

$$F(\tau) = \langle V^2_{\mathrm{N}}\rangle \exp(-\alpha\,|\,\tau\,|), \tag{21}$$

where α is the inverse of the RC-time constant of the filter. This is easily shown from (1) which has a structure similar to the ordinary Langevin equation for Brownian motion (Wax 1954). Substituting (21) in (12) and after some algebra we obtain

$$\langle V_{\rm NO}^2\rangle = \frac{\lambda\langle V_{\rm N}^2\rangle}{(\lambda^2-\alpha^2)}\left\{\left(\frac{1-\exp(-2n\lambda\epsilon)}{1-\exp(-2\lambda\epsilon)}\right)[(\lambda-\alpha)+\exp(-2\lambda\epsilon)(\lambda+\alpha)\right.$$

$$-2\lambda\exp(-\lambda+\alpha)\epsilon] + \frac{2\lambda}{\sinh(\lambda\epsilon)}[\cosh(\lambda\epsilon)-\cosh(\alpha\epsilon)]$$

$$\left.\times(S_1-S_2)\right\}, \tag{22}$$

where

$$S_1 = [1-\exp(-n(\alpha T+\lambda\epsilon))]\,[\exp(\alpha T+\lambda\epsilon)-1]^{-1}, \tag{23}$$

$$S_2 = \exp(-2n\lambda\epsilon)\,[1-\exp(n(\alpha T-\lambda\epsilon))]\,[\exp(\alpha T-\lambda\epsilon)-1]^{-1} \tag{24}$$

We note that since a delta function may be regarded in the limit as

$$\delta(\tau) = \mathop{\rm Lt}_{\alpha\to\infty} \alpha/2\exp(-\alpha|\tau|), \tag{25}$$

we are able to retrieve the white noise result, viz., (17) from (22) by first taking α to be very large and then setting α equal to $2/\epsilon$, where ϵ is the sampling time. The latter condition amounts to the fact that even in a pure white noise case, what we have in practice is a band-width ($=2/\epsilon$)—limited white noise.

Returning to (22) which is too complicated for the purpose of analysis, we look at a simpler result which follows when the exponential noise correlation has a time constant that matches with the RC-time constant of the Box-Car, i.e.,

$$\alpha = \lambda. \tag{26}$$

If we also have

$$T \gg \epsilon, \tag{27}$$

then

$$\langle V_{\rm NO}^2\rangle = \tfrac{1}{2}\langle V_{\rm N}^2\rangle(1-\exp(-2n\lambda\epsilon))$$

$$\times\left[\left(1-\frac{2\lambda\epsilon\exp(-2\lambda\epsilon)}{1-\exp(-2\lambda\epsilon)}\right)+\frac{2\lambda\epsilon}{(\exp(\lambda T-1))}\right]. \tag{28}$$

Assuming further that $\lambda\epsilon \ll 1$, still retaining $n\lambda\epsilon \gg 1$, however (cf. (13)), we obtain

$$\langle V_{\rm NO}^2\rangle \approx \lambda\epsilon\langle V_{\rm N}^2\rangle(\tfrac{1}{2}+\exp(-\alpha T)). \tag{29}$$

In this case

$$\text{SNIR} \approx (2\text{RC}/\epsilon)^{1/2}\,(1+2\exp(-\alpha T))^{-1/2}. \tag{30}$$

Equations (28)–(30) suggest that generally, the dependence of SNIR on the parameters of the Box-Car is more complicated than what a simple $(2\text{RC}/\epsilon)^{1/2}$–relation would predict. In this connection, it may be noted that a SNIR proportional to $(\text{RC}/\epsilon)^{\gamma}$ where the exponent $\gamma<1$, has been quoted in the literature for a Box-Car averager (Collins and Katchinoski 1973). The present analysis implies that such an apparent violation of 'the $(\text{RC}/\epsilon)^{1/2}$–law' may be attributed to the non-white character of the noise.

3.3. *Other noise sources*

An important class of noise autocorrelation one encounters in practice is the so-called exponential-cosine type:

$$F(\tau) = \langle V_N^2 \rangle \exp(-\alpha\,|\,\tau\,|)\cos(\beta\tau). \tag{31}$$

Since the above expression can also be written as

$$F(\tau) = \langle V_N^2 \rangle \,\text{Re}\,[\exp(-\tau(\alpha - i\beta))]; \quad \tau>0, \tag{32}$$

the analysis in this case is a straightforward extension of the one worked out in 3.2

The behaviour of noise with quite arbitrary correlations may also, in some cases, be encompassed within (32). As an example of this, we consider next the case of $1/f$—noise which may be generated, for example, in semiconductors. The power spectrum of such a noise source can be expressed as

$$P(\omega) = P\,|\,\omega\,|^{-\nu}, \quad \nu > 0. \tag{33}$$

We consider the case for which the exponent ν lies between 0 and 1. The power spectrum in the time space can then be written as (Abramowitz and Stegun 1965)

$$P(\tau) = \pi\, P\tau^{(\nu-1)}\,[\cos(\pi\nu/2)\,\Gamma(\nu)]^{-1}, \tau>0, \tag{34}$$

where $\Gamma(\nu)$ is the gamma function of argument ν. Equation (34) can be further expressed as (Abramowitz and Stegun 1965)

$$P(\tau) = 2\,P\sin(\pi\nu/2)\int_0^{\infty} d\alpha\;\alpha^{-\nu}\exp(-\alpha\,|\,\tau\,|). \tag{35}$$

Therefore, the power spectrum in this case is simply proportional to a superposition of the power spectra of the exponential correlation type, weighted by a factor $\alpha^{-\nu}$. An expression of SNIR can be obtained by combining (22) and (35), and again a simple $(2\text{RC}/\epsilon)^{1/2}$–type relation does not hold, in general. This same conclusion is expected to be also valid in the case of noise spectra which are more complicated than that of $1/f$ noise of the type discussed above. We do not pursue such an analysis, however.

4. Experimental verification

In order to verify the above calculations, a white noise source was constructed. This source uses the noise generated by the avalanche breakdown of the base-emitter junction of a high frequency *n-p-n* transistor and is expected to have frequency components up to 40 MHz (designed upper cut-off frequency of the amplifiers). Thus it produces essentially white noise for Box-Car integrator using $\epsilon \sim 1$ μs. Such circuits have been discussed in the literature (White 1975).

The noise was fed to a Box-Car integrator constructed at this research centre, and the instrument was triggered by means of a pulse generator. The output noise after exponential averaging by the Box-Car averager was then measured by a true rms voltmeter. It may be noted that feeding a pure noise rather than a signal buried in noise does not affect the validity of the comparison of the analysis with the measurement. This is because the signal and noise contributions to the output voltage of the Box-Car averager may be separated (cf. (6)).

First, the white noise case, where the SNIR is proportional to $(2RC/\epsilon)^{1/2}$ was verified. The RC-time constant of the instrument and the time T between triggers were

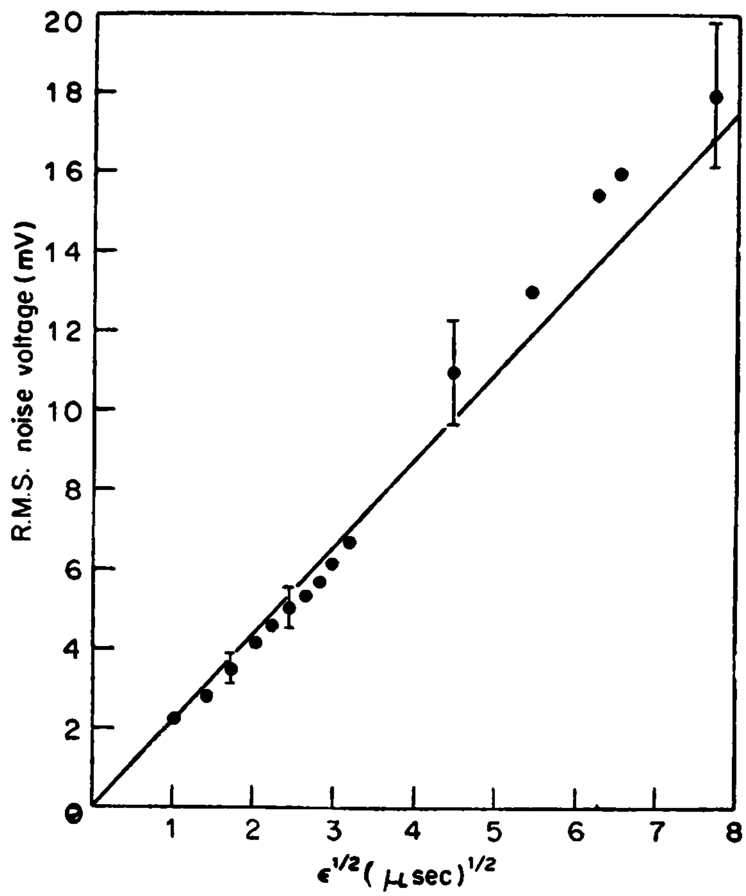

Figure 4. Plot of measured output noise voltage vs $\epsilon^{1/2}$ for a white noise source

fixed at definite values. The output noise voltage was then measured for various ϵ. A typical set of results is shown in figure 4 where the output noise voltage is plotted against $\epsilon^{1/2}$. The measured values are seen to be within 10% of the calculated points.

The output noise voltage as a function of the time T between triggers was measured for an exponentially correlated noise in the special case where $\alpha=\lambda$. To generate such a noise, the white noise output was filtered by a low pass filter with an RC time constant of 10 μs. This noise was fed to the Box-Car averager, with the latter's time constant also set at 10 μs. The results are shown in figure 5 (solid curve) where the measured output noise is plotted against the time T between triggers. Figure 5 also contains calculated values using (22) (dashed curves) for three different values of $1/\alpha$, 9, 10 and 11 μs with the value of $1/\lambda$ kept fixed at 10 μs. Of these, the best fit with the data seems to result in the case of $1/\alpha=9$ μs rather than 10 μs. This may be attributed to a lack of exact matching between α and λ during measurements, inherent noise of the Box-Car averager, etc. The comparison between the measured and calculated values is of course not very meaningful for very small values of T for which the used mathematical expressions lose their validity in view of the imposed condition (27). Finally, for the sake of comparison, the expected white noise situation in which

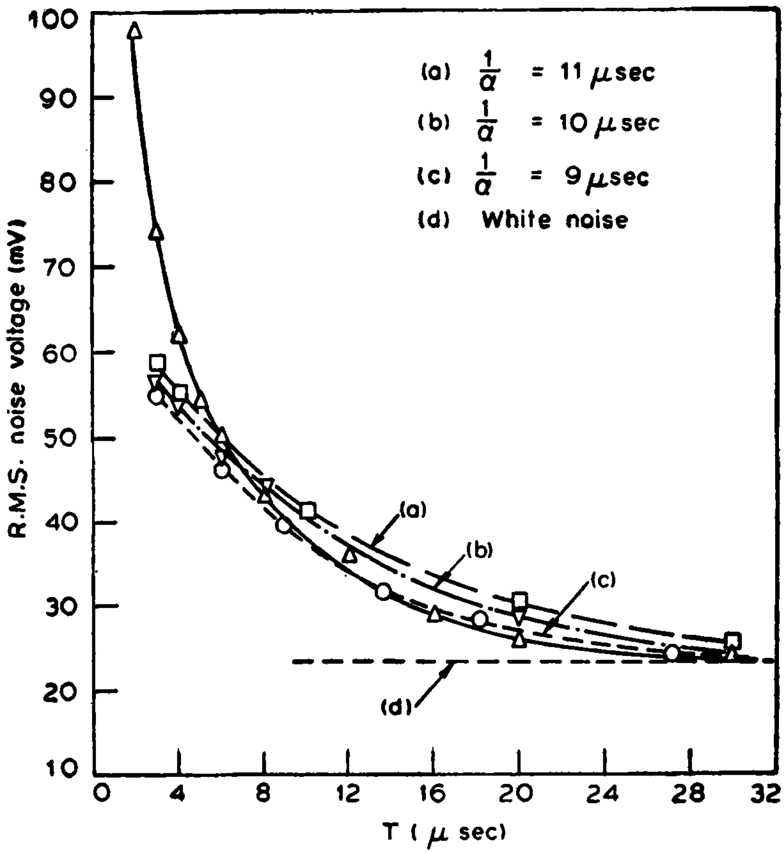

Figure 5. Plot of output noise voltage vs time between triggers (T) for exponentially correlated noise with $\alpha = (RC)^{-1}$. The solid curve represents the measured data while the dashed curves are based on calculations

the output noise voltage is independent of T, is also demonstrated in the lowermost curve of figure 5. It may be noticed that the behaviour in the exponentially correlated noise case deviates from the white noise behaviour when $T \lesssim 1/\alpha$ but tends towards the latter when $T > 3/\alpha$. This is due to the fact that contributions for inter-gate correlations become more important when $T \lesssim 1/\alpha$.

5. Conclusions

We have shown in this paper that the SNIR of a Box-Car averager depends very strongly on the nature of the noise input. In the case of pure white noise, the SNIR is proportional to $(2RC/\epsilon)^{1/2}$. However, for non-white noise, the effect of inter-gate correlations leads to complicated expressions. The additional noise contribution due to these inter-gate correlations depends on the time T between triggers.

The consideration of inter-gate correlations is very important from the point of view of time-efficiency of Box-Car averagers. It is required, on the one hand, to use a sufficiently large number of samples for extracting the signal completely ($n \gg RC/\epsilon$) which, for a time-efficient system, would require the time T between triggers to be small. The latter, on the other hand, would lead to enhanced inter-gate correlations and therefore an inferior signal-to-noise improvement for noise components which are not strictly white. Thus a balance has to be struck between the time efficiency and the SNIR as far as the choice for a value of the time between triggers is concerned.

Appendix I

To derive equation (12) from (9).

The double integral in (9), after a change of variable from τ' to $\tau - \tau' = t$, can be written as

$$\int_0^\epsilon d\tau \int_{\tau-\epsilon}^{\tau} dt\, F(t-(k-l)\,T) \exp[\lambda\,(2\tau - t)],$$

which, following an integration by parts, reduces to

$$\frac{1}{2\lambda}\Big[\exp(2\lambda\epsilon) \int_0^\epsilon d\tau \exp(-\lambda\tau)\, F(\tau-(k-l)T) - \int_0^\epsilon d\tau \exp(\lambda\tau)$$

$$\times F((k-l)T+\tau) - \int_0^\epsilon d\tau \exp(\lambda\tau)\, F(\tau-(k-l)T)$$

$$+ \int_0^\epsilon d\tau \exp(\lambda\tau) \exp(\lambda\epsilon)\, F(\tau-\epsilon-(k-l)T)\Big],$$

where we have used (11).

A further change in the integration variable and use of (11) in the last term within the parenthesis simplifies the above expression to

$$\frac{\exp(\lambda\epsilon)}{\lambda} \int_0^\epsilon d\tau \sinh(\lambda(\epsilon-\tau))\, [F((k-l)\,T+\tau) + F((k-l)\,T-\tau).$$

Substituting in (9) we obtain

$$\langle V_{NO}^2 \rangle = \lambda \exp(\lambda\epsilon) \int_0^\epsilon d\tau \sinh(\lambda(\epsilon-\tau)) \Big\{ \sum_{k=1}^n \exp(-2k\lambda\epsilon)$$

$$\times [F(\tau)+F(-\tau)] + \sum_{k=1}^n {\sum_{l=1}^n}' \exp[-\lambda(k+l)\epsilon]$$

$$\times [F((k-l)T+\tau)+F((k-l)T-\tau)] \Big\},$$

where $\lambda = (\mathrm{RC})^{-1}$ and the prime in the summation over l implies that the $l = k$ term is to be excluded.
Using (11),

$$\langle V_{NO}^2 \rangle = \lambda \exp(\lambda\epsilon) \int_0^\epsilon d\tau \sinh(\lambda(\epsilon-\tau)) \left\{ \frac{2[1-\exp(-2n\lambda\epsilon)]}{\exp(2\lambda\epsilon)-1} F(\tau) \right.$$

$$\left. + [S(\tau)+S(-\tau)] \right\}, \tag{A1}$$

where $S(\tau)$ is of the form

$$S(\tau) = \sum_{k=1}^n {\sum_{l=1}^n}' \exp[-\lambda\epsilon(k+l)] F((k-l)T+\tau). \tag{A2}$$

To evaluate $S(\tau)$ it is helpful first to examine its structure for a few finite terms, such as

$$S(\tau) = [F(T+\tau)+F(T-\tau)] \exp(-3\lambda\epsilon), \text{ for } n = 2, \tag{A3}$$

$$S(\tau) = [F(T+\tau)+F(T-\tau)][\exp(-3\lambda\epsilon)+\exp(-5\lambda\epsilon)]$$

$$+ [F(2T+\tau)+F(2T-\tau)] \exp(-4\lambda\epsilon), \quad \text{for } n = 3, \tag{A4}$$

$$S(\tau) = [F(T+\tau)+F(T-\tau)][\exp(-3\lambda\epsilon)] + \exp(-5\lambda\epsilon) + \exp(-7\lambda\epsilon)]$$

$$+ [F(2T+\tau)+F(2T-\tau)][\exp(-4\lambda\epsilon)+\exp(-6\lambda\epsilon)]$$

$$+ [F(3T-\tau)+F(3T+\tau)] \exp(-5\lambda\epsilon), \quad \text{for } n = 4, \tag{A5}$$

and so on. In writing (A3-5) we have made use of (11).
An inspection of (A3-5) enables us to write (A2) as

$$S(\tau) = \sum_{k=1}^{n-1} [F(kT+\tau)+F(kT-\tau) [\exp[-\lambda\epsilon(k+2)]$$

$$\times [1+\exp(-2\lambda\epsilon)+\ldots+\exp[-2(n-k-1)\lambda\epsilon]]$$

$$= \sum_{k=1}^{n-1} [F(kT+\tau)+F(kT-\tau)] \exp(-k\lambda\epsilon) \frac{[1-\exp(-2(n-k)\lambda\epsilon)]}{[\exp(2\lambda\epsilon)-1]} \tag{A6}$$

It is clear from (A6) that $S(\tau)=S(-\tau)$. Substituting (A6) into (A1) therefore yields (12) of the text.

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