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Shape of spectrum with ensemble of 2-body random Hamiltonians

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Abstract. Monte-Carlo calculations in *d-s* shell space have been done using two-body random interactions, to obtain ensemble-averaged *m*-particle scalar moments up to fourth order. A shift of the spectrum shape from semi-circle to Gaussian with respect to the increase in number of particles can be clearly seen in terms of the ensemble-averaged fourth moment.

Keywords. Monte-Carlo calculations; random Hamiltonians; moments.

It is well known that the ensemble-averaged eigenvalue density function for an ensemble of $N \times N$ symmetric random matrices, where the matrix elements are distributed randomly around a mean value zero, is a semi-circle for large values of N. Each matrix element here is randomly chosen, so that there is no apparent correlation between any two matrix elements, and hence the dominant part of the interaction turns out to be of body-rank k, equal to the number of paricles m under consideration. On the contrary, in nuclear physics and also for many problems in solid state physics, the interactions are predominantly 1-body and at most 2-body in nature. This particular feature is lost when each matrix element of $N \times N$ symmetric matrix ensemble is chosen independently.

Wong and French (1972) have conclusively shown that the shape of the eigenvalue density function changes from a semi-circle to a Gaussian as the body rank of the interaction decreases from $k = m$ to small values of k, such that $k \ll m < N$, where N is the number of single-particle states. Their demonstration consists of (i) construction of an exact m-body Hamiltonian matrix starting from the basic k-body random interaction, (ii) diagonalization of this matrix to obtain all the eigenvalues and then (iii) evaluation of the ensemble-averaged eigenvalue density function. Obviously, due to the computational difficulties with large matrices, this method cannot be applied to very large matrices.

An alternative approach to this is not to consider the exact eigenvalues at all, but to go on to evaluate the low-order moments of the eigenvalue distribution in many particle spaces and then infer about the shape of the ensemble-averaged eigenvalue distribution. Propagation techniques, for evaluation of low-order moments in many particle spaces, which do not require the construction of an exact Hamiltonian and consequently its diagonalization, have been developed by French and coworkers (French and Ratcliff 1971; Chang *et al* 1971; Potbhare 1975, Li *et al* unpublished). The input information needed for this corresponds to the traces of H^p (small value of p) over a net of few particle and/or few hole spaces; this information can then be

propagated to evaluate the low-order moments in many particle spaces. Computer codes which exploit these methods are available for evaluation of low-order (up to fourth) moments of the Hamiltonian (usually (1+2)—body in nature) over various spaces. The entire *d-s* shell calculation for scalar moments of the Hamiltonian takes $\simeq$ 120 sec on IBM 360/44 machine.

In principle, one needs all the moments to obtain the exact eigenvalue distribution. In practice however, only few lower moments can be calculated; hence there is no way to obtain a unique eigenvalue distribution. But, one can get a feeling about how the eigenvalue distribution should look like. For example, with the information corresponding to first four moments, centroid and width tell us about the physical location and the spread of the spectrum, while the spectrum shape information is given by skewness and excess.* If the values of the skewness and the excess are close to zero, then the eigenvalue distribution can be said to be close to a Gaussian. If, however, the excess is in the neighbourhood of -1, with the skewness around zero, the eigenvalue distribution nears a semi-circle.

In this paper, we have evaluated ensemble-averaged quantities (centroid, width, skewness and excess) for scalar distributions in *d-s* shell space. Scalar distribution treats all values of J and T on equal footing; that is, all the information about J and T is averaged out. In *d-s* shell, as usual, iso-scalar rotationally invariant Hamiltonian is described by 63 two-body matrix elements (TBME) and 3 single-particle energies. 96 sets of 63 TBME were generated randomly using Gaussian random variate. The 6048 numbers generated this way had following statistical properties: mean=—0·027, variance=1·009, skewness=0·0 and excess=—0·106. The three single-particle energies have been taken to be zero so as to study the effects of random nature of the TBME only. Table 1 gives the relevant necessary ensemble-averaged scalar quantities for various values of m, the number of particles. The dimensionality of the space is given by $N!/(N-m)!m!$ where N, the single-particle states in *d-s* shell equals 24.

One immediately observes that the values of the ensemble-averaged skewness is very close to zero throughout the *d-s* shell. The value of the ensemble-averaged excess, however, increases from —0·65 for $m=3$ to —0·20 for $m=16$ and has again decreased

Table 1. Ensemble-averaged scalar quantities in *d-s* shell.

m	σ Width	$\sqrt{\text{var } \sigma}$	γ_1 Skewness	$\sqrt{\text{var } \gamma_1}$	γ_2 Excess	*$\sqrt{\text{var } \gamma_2}$
2	1·7758	0·2166	0·0025	0·3593	—0·3540	0·5930
3	2·9754	0·3681	—0·0070	0·2242	—0·6519	0·2190
4	4·0642	0·5163	—0·0068	0·1803	—0·5769	0·1404
8	7·5290	1·1590	—0·0019	0·1510	—0·3152	0·1084
12	9·4757	1·9688	—0·0056	0·1780	—0·2210	0·1184
16	9·7577	2·7839	0·0093	0·2317	—0·2023	0·1559
20	7·9369	3·2037	0·0035	0·3361	—0·2727	0·2695

*Var=Variance

*The skewness (γ_1) and the excess (γ_2) are related to the central moments: $\gamma_1=M_3/(M_2)^{3/2}$ and $\gamma_2+3=M_4/(M_2)^2$ where M_p is the p-th central moment.

to $-0{\cdot}27$ by the time $m=20$ is reached. The value of the ensemble-averaged excess for $m=2$ is exceptional. One would expect it to be closer to -1 (at least smaller than -0.65 for $m=3$), but it turns out to be $-0{\cdot}35$. At present, we have no explanation for this. Otherwise, from 3-particle onwards the ensemble-avearged eigenvalue distribution can be seen in terms of the ensemble-averaged excess to go from a semi-circle to a Gaussian as the particle number increases. (According to an empirical rule given by French 1974, a distribution with $|\gamma_1|$, $|\gamma_2| < 0{\cdot}3$ can be hardly distinguished from a Gaussian by an untrained eye.)

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