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A bag model study of D mesons

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Abstract. An investigation of the newly discovered charmed mesons D^0 and D^+, particularly their non-leptonic decay modes, is carried out in the framework of the MIT bag model. The amplitude for a number of two-body final state decays are explicitly evaluated and compared with other available estimates.

Keywords. MIT bag model; charm; charmed mesons; non-leptonic decay.

1. Introduction

The MIT bag model (Chodos *et al* 1974a) in its cavity approximation has been quite successful in reproducing several of the static properties of light hadrons such as their mass, magnetic moment, axial-vector coupling constant and the charge radius of the proton (De Grand *et al* 1975). The model has also been applied successfully to the study of weak non-leptonic decays of baryons and mesons (Donoghue *et al* 1975; Katz and Tatur 1976) and the radiative decays of some of the vector mesons (Katz and Tatur 1977), although the bag decay amplitudes for electromagnetic and weak leptonic decays of mesons obtained by Hays and Ulehla (1976) are not in good agreement with experiment. In the light of the relative simplicity of the ideas on which the bag model is based, its successes should be regarded as remarkable. However, no serious attempt has been made so far to understand the newly discovered hadrons in terms of this model in an exhaustive manner. Evidence for the existence of a fourth quark with the quantum number ' charm ' (Glashow *et al* 1970) is growing stronger with the discovery of the new narrow resonances. Even before the discovery of charmed hadrons, an elaborate SU(4) classification of charmed mesonic and baryonic states and their possible decay modes was worked out by Gaillard *et al* (1975) by extending the familiar notions of the colour triplet quark model to the four quark scheme of Glashow *et al* (1970). The recently observed narrow resonances named D^0, D^+ around 1·87 GeV at SPEAR by Goldhaber *et al* (1976), and by Peruzzi *et al* (1976) furnish an important test for the validity of the phenomenological bag model in the charm regime. In this paper we are primarily concerned with the non-leptonic decays, particularly the two-body decay channels of these charmed pseudoscalar mesons. Previous studies (for example Cabibbo *et al* 1975; Donoghue and Holstein 1975) predict a vanishing of the amplitude for the mode $D^+ \to \bar{K}^0\pi^+$, but recent experimental data (Galtieri 1977) indicate an amplitude comparable to that of the mode $D^0 \to K^-\pi^+$. The present bag model calculations gives a result which is at least in qualitative agreement with the experimental observation.

In section 2 of the present work we discuss the bag parameters of the charmed meson, and calculate its bag radius and mass. In section 3 we compute the decay amplitudes in a current–current picture, making use of PCAC and soft meson techniques. Section 4 contains a brief discussion of the results and our conclusions.

2. The charmed bag

The bag model is essentially a colour quark model with the quark and gluon fields confined to a finite region of space. In the cavity approximation the region is a fixed sphere of radius R. The field equations and the bag boundary conditions determine the quark wave functions (Chodos *et al* 1974b; De Grand *et al* 1975). For the lowest frequency mode we have

$$\psi(\mathbf{r}, t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i\left(\frac{\omega + m}{\omega}\right)^{1/2} j_0\left(\frac{x\,r}{R}\right) u \\ -\left(\frac{\omega - m}{\omega}\right)^{1/2} j_1\left(\frac{x\,r}{R}\right) \sigma \cdot \hat{\mathbf{r}}\, u \end{pmatrix} \exp(-i\omega t/R) \tag{1}$$

for quarks, and

$$\Phi(\mathbf{r}, t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} -i\left(\frac{\omega - m}{\omega}\right)^{1/2} j_1\left(\frac{x\,r}{R}\right) \sigma \cdot \hat{\mathbf{r}}\, u \\ \left(\frac{\omega + m}{\omega}\right)^{1/2} j_0\left(\frac{x\,r}{R}\right) u \end{pmatrix} \exp(i\omega t/R) \tag{2}$$

for anti-quarks. Here j_0 and j_1 are spherical Bessel functions of order 0 and 1, and u is a 2-component Pauli spinor; ω is the lowest quark (anti-quark) mode frequency which is a function of the quark (anti-quark) mass m and the bag radius R;

$$\omega(m, R) = \frac{1}{R}[x^2 + (m\,R)^2]^{1/2}, \tag{3}$$

where $x = x(m\,R)$ is the smallest positive root of the transcendental equation

$$\tan x = \frac{x}{1 - mR - \sqrt{x^2 + (mR)^2}}. \tag{4}$$

The normalization constant N in (1) and (2) is given by

$$N = \frac{1}{Rj_0(x)} \left\{\frac{\omega(\omega - m)}{2\omega(R\omega - 1) + m}\right\}^{1/2} \tag{5}$$

We choose all the bag parameters except the charmed quark mass from De Grand *et al* (1975). In their paper results are presented for two different non-strange quark masses $m_n = 0$ and $m_n = 108$ MeV. The results are, however, not very sensitive to the choice of the value of m_n. Hence for simplicity we set $m_n = 0$. The strange quark is, however, given a mass with a view to breaking the SU(3) degeneracy. Our choice of the strange quark mass M_s is 300 MeV. The bag pressure parameter B which determines the stability of the bag gives best fit to the hadronic masses for the choice $B^{1/4} = 145$ MeV. Following Gaillard *et al* (1975) and Donoghue and Golowich (1976) we fix the charmed quark mass $m_c = 1{\cdot}5$ GeV. The bag radius R is determined by minimising the hadron mass, a procedure demanded by the quadratic bag boundary condition.

The mass of a hadron of bag radius R is given by the energy equation

$$M(R) = E_v + E_o + E_q + E_m + E_e. \tag{6}$$

Here $E_v = 4/3\pi R^3 B$, known as the volume energy, gives part of the zero-point energy; $E_o = -Z_o/R$ is the remainder of the zero-point energy (Z_o is a positive parameter which is of order unity and whose phenomenological value is around 2); $E_q = \Sigma_j N_j\, \omega\,(m_j, R)$ represents the rest and kinetic energies of the quarks, with N_j the number of quarks or antiquarks of flavour j and ω the frequency defined by (3). E_m and E_e in (6) are the 'magnetic' and 'electric' quark-gluon interaction energies. The magnetic spin-spin interaction energy is

$$E_m = \sum_{i>j} \lambda\, a_{ij}\, M_{ij}, \tag{7}$$

with $\lambda = 2$ for mesons and 1 for baryons. The coefficients a_{ij} are expressed as

$$a_{ij} = \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \tag{8}$$

and

$$M_{ij} = \frac{8\alpha_c}{36R}\, \frac{[4\alpha_i + 2\lambda_i - 3]}{[2\alpha_i\,(\alpha_i - 1) + \lambda_i]}\, \frac{[4\alpha_j + 2\lambda_j - 3]}{[2\alpha_j(\alpha_j - 1) + \lambda_j]}\, I\,(m_i R, m_j R) \tag{9}$$

where $\alpha_i = R\omega\,(m_i, R)$, $\lambda_i = m_i R$ and $I\,(m_i R,\ m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$ (De Grand *et al* 1975), i and j being quark flavour indices. To first order in quark-gluon interaction the coupling constant $\alpha_c = g^2/4\pi \sim 0{\cdot}55$. We have determined $I\,(m_i R,\ m_j R)$ with the help of a computer for all relevant sets of values of $m_i R$ and $m_j R$. These are presented in table 1.

Table 1. $I\,(m_i R,\ m_j R)$ values for different sets of $m_i R,\ m_j R$.

$m_i R$	$m_j R$	$I\,(m_i R, m_j R)$
0	0	1·44
1	0	1·49
5	0	1·59
5	1	1·60

The gluon ' electric ' interaction energy E_e for the D meson is given by

$$E_e = -\frac{8\alpha_c}{3R}[2f(x_n, x_c) - f(x_n, x_n) - f(x_c, x_c)], \tag{10}$$

where

$$f(x_i, x_j) = R\int_0^R dr\, \frac{1}{r^2}\, \rho_i(r)\, \rho_j(r), \tag{11}$$

$\rho_i(r)$ being the fraction of the quark charge density within a radius r and is a function of m_i, ω_i, x_i and r. Computing the $f(x_i, x_j)$ numerically E_e is estimated. It yields a significant contribution to the hadron mass, namely 165 MeV which is nearly twice the volume energy. It is to be noted that this part of the hadron mass is usually neglected in calculations for ordinary hadrons for which this turns out to be negligibly small (<5MeV) as a result of nearly equal quark masses. For charmed mesons the quark masses are substantially different: $m_n=0$, $m_c=1\cdot5$ GeV, and it was conjectured by De Grand *et al* (1975) that the electric contribution in such cases might be appreciable. Our computation confirms their speculation.

Following the standard procedure (De Grand *et al* 1975; Johnson 1975), the radius of the charmed bag (bag radius of the D meson) is estimated in the zero quark mass limit by minimising M (equation 6) with respect to R. In this limit $I\,(m_i, R, m_j\, R) = 1\cdot44$ (table 1) and $x = 2\cdot04$. With these parameter values and with $Z_0=1\cdot84$ and $B^{1/4}=145$ MeV, we get a bag radius $3\cdot3$ GeV^{-1} for the D meson. Our calculation does not distinguish between the charged and the neutral states. Under these assumptions, the bag radius of the strange K meson also must have the same value. Here we note that the bag radius seems to be almost independent of the mass of the hadron. This is a consequence of putting the quark mass equal to zero. As the results are not very sensitive to the bag radius this need not be regarded as a serious anomaly.

With the values $R=3\cdot3$ GeV^{-1}, $m_n=0$, $m_c=1\cdot5$ GeV, we have $m_nR=0$, $m_cR=5$ and $I\,(5, 0)=1\cdot59$ (table 1). Based on these data the charmed D meson mass is computed to be $1\cdot805$ GeV as against the reported experimental values: $1\cdot865\pm 0\cdot015$ for D^0 (Goldhaber *et al* 1976) and $1\cdot876\pm0\cdot015$ GeV for D^+ (Peruzzi *et al* 1976).

3. Weak nonleptonic decays

3.1. *Interaction Hamiltonian*

In the GIM scheme (Glashow *et al* 1970) the weak hadron current is written

$$J_\mu{}^h = \bar{q}\, C_h\, \gamma_\mu\, (1+\gamma_5)\, q, \tag{12}$$

where q is the 4 component quark column vector ($cuds$), and the matrix

$$C_h = \begin{pmatrix} 0 & 0 & -\sin\theta & \cos\theta \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

θ being the Cabibbo angle. Consequently

$$J_\mu{}^h = -\sin\theta\,\bar{c}\,\gamma_\mu(1+\gamma_5)\,d + \cos\theta\,\bar{u}\,\gamma_\mu(1+\gamma_5)\,d$$
$$+\cos\theta\,\bar{c}\,\gamma_\mu(1+\gamma_5)\,s + \sin\theta\,\bar{u}\,\gamma_\mu(1+\gamma_5)\,s, \tag{13}$$

where u and d denote ordinary up and down quarks, s the strange quark and c the charmed quark. We take the weak non-leptonic Hamiltonian in the current-current form

$$\mathcal{H}_w = -\frac{G}{\sqrt{2}} J_\mu{}^h (J_\mu{}^h)^+. \tag{14}$$

We consider here parity-violating, charm-changing and strangeness-changing decays for which the following selection rules hold.

$$\triangle s = \triangle c = \pm 1, \text{ so that } \triangle s/\triangle c = +1,$$
$$-\triangle s = \triangle c = 1, \text{ so that } \triangle s/\triangle c = -1. \tag{15}$$

These when applied to (14) yield the following interactions

$$\mathcal{H}_w^{\triangle s/\triangle c=1} = -\frac{G}{\sqrt{2}}\cos^2\theta\,\{\bar{d}\,\gamma_\mu(1+\gamma_5)\,u\,\bar{c}\,\gamma_\mu(1+\gamma_5)\,s$$
$$+\bar{s}\,\gamma_\mu(1+\gamma_5)\,c\,\bar{u}\,\gamma_\mu(1+\gamma_5)\,d\} \tag{16}$$

$$\mathcal{H}_w^{\triangle s/\triangle c=-1} = \frac{G}{\sqrt{2}}\sin^2\theta\,\{\bar{d}\,\gamma_\mu(1+\gamma_5)\,c\,\bar{u}\,\gamma_\mu(1+\gamma_5)\,s$$
$$+\bar{s}\,\gamma_\mu(1+\gamma_5)\,u\,\bar{c}\,\gamma_\mu(1+\gamma_5)\,d\}. \tag{17}$$

3.2. *Decay amplitudes*

In order to determine the two-body decay amplitudes soft meson techniques are invoked. This involves the transformation of matrix elements of the form $\langle B\pi \mid \mathcal{H}_w \mid A\rangle$ into the form $\langle B \mid \mathcal{H}_w \mid A\rangle$. This may be a drastic approximation since we let the meson momentum vanish in the soft meson limit, and the latter may imply omission of terms of appreciable magnitude. However, it should be remembered that a bag model calculation of non-leptonic decay amplitudes cannot but depend on such techniques as we are still not in a position to handle more than two bags at a time in this sort of computation.

Using the PCAC hypothesis

$$\partial_\mu A_{k\mu}(x) = f_\pi m_\pi^2 \pi_k, \tag{18}$$

we can write in the soft pion limit (De Alfaro *et al* 1973)

$$\langle B\pi_a \mid \mathcal{H}_w^{\text{pv}}(0) \mid A \rangle = -\frac{1}{f_\pi} \langle B \mid [F_a^5, \mathcal{H}_w^{\text{pv}}(0)] \mid A \rangle$$

$$= -\frac{1}{f_\pi} \langle B \mid [F_a, \mathcal{H}_w^{\text{pc}}(0)] \mid A \rangle, \tag{19}$$

in which pv stands for ' parity violating ' and pc for ' parity conserving.' Thus

$$\langle B\pi^+ \mid \mathcal{H}_w^{\text{pv}}(0) \mid A \rangle = -\frac{1}{\sqrt{2} f_\pi} \langle B \mid [F_-^5, \mathcal{H}_w^{\text{pv}}(0)] \mid A \rangle$$

$$= -\frac{1}{\sqrt{2} f_\pi} \langle B \mid [F_-, \mathcal{H}_w^{\text{pc}}(0)] \mid A \rangle. \tag{20}$$

For the specific decays studied in the present investigation the following transformations are obtained.

$$\langle K^- \pi^+ \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^0 \rangle = -\frac{1}{\sqrt{2} f_\pi} \langle \bar{K}^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle,$$

$$\langle \bar{K}^0 \pi^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^0 \rangle = -\frac{1}{f_\pi} \langle \bar{K}^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle,$$

$$\langle K^+ \pi^- \mid \overset{\Delta s/\Delta c = -1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^0 \rangle = -\frac{1}{\sqrt{2} f_\pi} \{ \langle K^0 \mid \overset{\Delta s/\Delta c = -1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle$$

$$- \langle K^+ \mid \overset{\Delta s/\Delta c = -1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^+ \rangle \}, \tag{21}$$

$$\langle \bar{K}^0 \eta^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^0 \rangle = \frac{\sqrt{3}}{2 f_K} \langle \bar{K}^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle,$$

$$\langle K^0 \eta^0 \mid \overset{\Delta s/\Delta c = -1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^0 \rangle = -\frac{\sqrt{3}}{2 f_K} \langle K^0 \mid \overset{\Delta s/\Delta c = -1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle.$$

where in the last two cases the soft kaon limit has been taken.
Again,

$$\langle \bar{K}^0 \pi^+ \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pv}}(0)} \mid D^+ \rangle = \frac{1}{\sqrt{2} f_\pi} \langle \bar{K}^0 \mid \overset{\Delta s/\Delta c = 1}{\mathcal{H}_w^{\text{pc}}(0)} \mid D^0 \rangle,$$

$$\langle K^0 \pi^+ | \mathcal{H}_w^{\text{pv}}(0) | D^+ \rangle^{\Delta s/\Delta c = -1} = -\frac{1}{\sqrt{2} f_\pi} \{ \langle K^+ | \mathcal{H}_w^{\text{pc}}(0) | D^+ \rangle^{\Delta s/\Delta c = -1}$$
$$- \langle K^0 | \mathcal{H}_w^{\text{pc}}(0) | D^0 \rangle^{\Delta s/\Delta c = -1} \}, \tag{22}$$

$$\langle K^+ \eta^0 | \mathcal{H}_w^{\text{pv}}(0) | D^+ \rangle^{\Delta s/\Delta c = -1} = -\frac{\sqrt{3}}{2 f_K} \langle K^+ | \mathcal{H}_w^{\text{pc}}(0) | D^+ \rangle^{\Delta s/\Delta c = -1}.$$

The matrix elements are evaluated by assuming the quark structures

$$D^0 = \frac{1}{\sqrt{2}} \left(\overset{\uparrow}{\bar{u}} \overset{\downarrow}{c} - \overset{\downarrow}{\bar{u}} \overset{\uparrow}{c} \right), \tag{23}$$

$$D^+ = \frac{1}{\sqrt{2}} \left(\overset{\uparrow}{\bar{d}} \overset{\downarrow}{c} - \overset{\downarrow}{\bar{d}} \overset{\uparrow}{c} \right), \tag{24}$$

and taking the parity conserving parts of the Hamiltonians (16) and (17). For the various processes under consideration we get the amplitudes

$$\begin{aligned}
A(D^0 \to K^- \pi^+) &= \left\{ \frac{1}{\sqrt{2}} \cos^2 \theta \right\} k, \\
A(D^0 \to \bar{K}^0 \pi^0) &= (\cos^2 \theta) k, \\
A(D^0 \to K^+ \pi^-) &= (-\sqrt{2} \sin^2 \theta) k, \\
A(D^0 \to \bar{K}^0 \eta^0) &= \left(-\frac{\sqrt{3}}{2} \cos^2 \theta \right) k', \\
A(D^0 \to K^0 \eta^0) &= \left(-\frac{\sqrt{3}}{2} \sin^2 \theta \right) k', \\
A(D^+ \to \bar{K}^0 \pi^+) &= \left(\frac{1}{\sqrt{2}} \cos^2 \theta \right) k, \\
A(D^+ \to K^0 \pi^+) &= (\sqrt{2} \sin^2 \theta) k, \\
A(D^+ \to K^+ \eta^0) &= \left(-\frac{\sqrt{3}}{2} \sin^2 \theta \right) k',
\end{aligned} \tag{25}$$

where

$$k = \frac{G}{\sqrt{2}} \cdot \frac{1}{f_\pi} . I, \tag{26}$$

$$k' = \frac{G}{\sqrt{2}} \cdot \frac{1}{f_K} . I.$$

The pion decay constant $f_\pi = 94$ MeV, the kaon decay constant $f_K \sim 1{\cdot}3\, f_\pi$ and $G = 10^{-5}/M_p^2 = 1{\cdot}132 \times 10^{-5}\,\text{GeV}^{-2}$.

The bag overlap intergral I in (26) is

$$\frac{2N_1N_2N_3N_4}{4\pi}\int_0^R r^2\,dr\,\Big\{\Big(\frac{\omega_1+m_1}{\omega_1}\Big)^{1/2}\Big(\frac{\omega_4+m_4}{\omega_4}\Big)^{1/2} j_0\Big(\frac{x_1r}{R}\Big)\,j_0\Big(\frac{x_4r}{R}\Big)$$

$$+\Big(\frac{\omega_1-m_1}{\omega_1}\Big)^{1/2}\Big(\frac{\omega_4-m_4}{\omega_4}\Big)^{1/2} j_1\Big(\frac{x_1r}{R}\Big)\,j_1\Big(\frac{x_4r}{R}\Big)\Big\}$$

$$\times\Big\{\Big(\frac{\omega_2+m_2}{\omega_2}\Big)^{1/2}\Big(\frac{\omega_3+m_3}{\omega_3}\Big)^{1/2} j_0\Big(\frac{x_2r}{R}\Big)\,j_0\Big(\frac{x_3r}{R}\Big)$$

$$+\Big(\frac{\omega_2-m_2}{\omega_2}\Big)^{1/2}\Big(\frac{\omega_3-m_3}{\omega_3}\Big)^{1/2} j_1\Big(\frac{x_2r}{R}\Big)\,j_1\Big(\frac{x_3r}{R}\Big)\Big\}. \tag{27}$$

This integral has been evaluated numerically on a computer for the following input parameters:

$$m_1 = m_c = 1\cdot 5 \text{ GeV},$$

$$m_2 = m_3 = m_n = 0,$$

$$m_4 = m_s = 0\cdot 3 \text{ GeV},$$

and an average bag radius $R = 3\cdot 3\ \text{GeV}^{-1}$. ω_i, x_i and $N_i (i = 1, 2, 3, 4)$ are solutions of (3), (4) and (5) respectively corresponding to given m_i, R value. The values of the integral is found to be

$$I = 1\cdot 497 \times 10^{-2}.$$

Substituting for I the absolute magnitudes of the decay amplitudes are computed and are presented in table 2.

Table 2. Amplitudes for two body decays

Decay mode	$10^7 \times$ amplitude
$D^0 \to K^- \pi^+$	8·5
$D^0 \to \bar{K}^0 \pi^0$	12·0
$D^0 \to K^+ \pi^-$	−1·22
$D^0 \to \bar{K}^0 \eta^0$	−8·0
$D^0 \to K^0 \eta^0$	−0·5
$D^+ \to \bar{K}^0 \pi^+$	8·5
$D^+ \to K^0 \pi^+$	1·22
$D^+ \to K^+ \eta^0$	−0·5

4. Concluding remarks

A detailed study of the properties of D^0 and D^+ mesons including their principal non-leptonic decay amplitudes has been reported using the MIT bag model. Our estimate of the D meson mass viz., 1·805 GeV should be considered to be in fairly good agreement with the experimental value of 1·865 GeV. An important contribution equal to 165 MeV, coming from 'electric' gluon interaction which is of negligible magnitude (< 5 MeV) for ordinary hadrons, is taken into account in the present calculation. This approach may be contrasted with a recent computation (Szwed 1977) employing a bag model with surface tension (Gnädig *et al* 1976) which reports a value as low as 1·564 GeV for the D meson mass. It may be mentioned that the masses of ordinary as well as charmed hadrons estimated on the MIT bag model are consistently lower than the experimental values. This is a clear indication of the fact that for better accord with observations the MIT bag model needs to be revised.

As for the decay rates we think that the available experimental data are too preliminary to allow a detailed comparison. However, it is instructive to compare our prediction for the rate for $D^0 \to K^- \pi^+$ with other available estimates in the literature. Our prediction, namely, an amplitude of $0{\cdot}85 \times 10^{-6}$ corresponds to a decay rate of $0{\cdot}2 \times 10^{12}$ sec^{-1} which may be compared with the value of 5×10^{12} sec^{-1} (Gaillard *et al* 1975) as well as the amplitude $3{\cdot}9 \times 10^{-6}$ (Donoghue and Golowich 1976). The latter group of authors have employed renormalization group techniques in their calculation. Donoghue and Holstein (1975) in their paper on charmed nonleptonic decay in asymptotically free theories predict on the basis of renormalization group techniques, a sextet enhancement for $\Delta S = \Delta C$ decays similar to the octet enhancement in nonleptonic decays governed by SU(3). This has the consequence of forbidding the $D^+ \to \bar{K}^0 \pi^+$ mode. However, we have obtained equal amplitudes

$$A\,(D^+ \to \bar{K}^0 \pi^+) = A\,(D^0 \to K^- \pi^+)$$

contradicting the speculation regarding sextet enhancement. Cabibbo and Maiani (1977), employing a simple quark recombination scheme, show that $A\,(D^+ \to \bar{K}^0 \pi^+) \sim 0{\cdot}8\ A(D^0 \to K^- \pi^+)$ which is claimed to compare favourably with the recent experimental data (Galtieri 1977). The bag model result reported here is in qualitative agreement with the experimental evidence on the one hand, and on the other, lends indirect support to the proposal that, while the channel $D^+ \to \bar{K}^0 \pi^+$ is a pure '84' in an SU (4) symmetry scheme, the channel $D^0 \to K^- \pi^+$ has a relatively small projection on the '20' and a larger one on the '84'.

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