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Quark charges and colour gluon mass from deep-inelastic bremsstrahlung

P N PANDITA
Tata Institute of Fundamental Research, Bombay 400 005

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Abstract. We derive sum rules for the structure function $V(x)$ for the 'three-photon' process $e^{\pm}+p \to e^{\pm}+\gamma+X$ which can distinguish between various colour models below colour threshold, independently of the quark and gluon distributions. A careful study of the sum rule for $V(x)$ in the broken colour gauge theory model can in principle be used to determine the colour gluon mass. Invoking the specific assumptions of the dominance of p-type quarks and neglecting the sea of quark-antiquark pairs, we also obtain bounds for $V(x)$ in terms of $\nu W_2(x)$ which can distinguish between various colour models below colour threshold.

Keywords. Colour gauge theory; integral and fractional charge quarks; colour gluons; deep-inelastic bremsstrahlung; scaling; quark-parton model.

1. Introduction

There are two main candidates for a unified gauge theory of strong, weak and electromagnetic interactions. One is based on fractionally charged coloured quarks (Gell-Mann 1964; Zweig 1964; Greenberg 1964; Bardeen *et al* 1973) where the strong interactions are derived from an exact colour SU(3) gauge symmetry involving neutral massless colour gluons (Fritzsch *et al* 1973; Gross and Wilczek 1973; Politzer 1973, Weinberg 1973). Colour is supposed to be permanently confined and the colour gauge group commutes with the gauge group of weak and electromagnetic interactions. The theory is asymptotically free and the Bjorken scaling is violated by logarithmic terms (Politzer 1974). The hadronic electromagnetic current in such a theory is a colour singlet and is given by

$$J^{F}_{EM} = J^{\text{flav}} = \sum_i \left(\tfrac{2}{3}\, \bar{p}_i p_i - \tfrac{1}{3}\bar{n}_i n_i - \tfrac{1}{3}\bar{\lambda}_i \lambda_i + \tfrac{2}{3}\,\bar{c}_i c_i \right) \tag{1}$$

in the standard notation.*

The second class of theories are based on integer charge Han-Nambu quark model (Han and Nambu 1965; Pati and Salam 1973, 1974; Rajasekaran and Roy 1975a). In the naive version of Han-Nambu model the electromagnetic current is

$$J^{I}_{EM} = J^{\text{flav}} + J^{\text{col}} \tag{2}$$

*We consider four quark flavours ($q=p, n, \lambda, c$) and three colours ($i=r, y, b$). The flavour charges are ($\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}$) for ($p_i, n_i, \lambda_i, c_i$) and the colour charges are ($-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}$) for ($q_r, q_y, q_b$).

where

$$J^{\rm col} = \sum_q \left(-\tfrac{2}{3}\,\bar{q}_r q_r + \tfrac{1}{3}\,\bar{q}_y q_y + \tfrac{1}{3}\bar{q}_b q_b \right) \tag{3}$$

the summation being over all quark flavours q. $J^{\rm flav}$ transforms as a singlet of colour SU(3) and is given by eq. (1), whereas $J^{\rm col}$ is the colour octet piece. By design, the integer charge model is equivalent to the fractionally charged one in the colour singlet sector (Lipkin 1972, Chanowitz 1977). However, above the threshold for the production of colour nonsinglet states we expect dramatic differences to appear (Greenberg and Nelson 1977). Since no such effects have been observed in the presently accessible energy range, we are led to the conclusion that either the colour threshold is arbitrarily large, or if it is in the presently available energy range then there must be some mechanism which suppresses the colour quantum numbers, and quarks behave as if they have fractional charge. The class of gauget heories based on Han-Nambu quarks precisely achieve the second alternative. Here the strong gauge group is again colour SU(3) which is spontaneously broken. As a consequence the colour gluons acquire mass and also mix with the vector bosons of the weak and electromagnetic gauge group. The colour gauge group does not commute with the physical generators of weak and electromagnetic interactions and hence some of the colour gluons have weak and electromagnetic interactions.* The electromagnetic current contains both a colour singlet and a colour octet part and is given by

$$J^{\rm IGT}_{\rm EM} = J^{\rm flav} + \left(1 - \frac{q^2}{q^2 - m_g^2}\right) J^{\rm col} \tag{4}$$

where q is the four momentum carried by the current and m_g is the colour gluon mass. The current in eq. (4) reduces to the current in the naive Han-Nambu model, eq. (2), for $q^2 \ll m_g^2$. The q^2 dependence of the current in eq. (4) arises due to the phenomenon of photon-gluon mixing and is a general property of a class of spontaneously broken colour gauge theories based on integer charge quarks (Pati and Salam 1975; Rajasekaran and Roy 1975b). For electromagnetic interactions involving colour singlet states $J^{\rm col}$ is inoperative and the fractional charge model is equivalent to the integer charge model (with or without gauge theory). On the other hand, the dramatic differences expected between fractional charge colour model and the naive Han-Nambu model above colour threshold are damped in the gauge theory version of integer charge model for $q^2 \gg m_g^2$. Thus far above the colour threshold with $q^2 \gg m_g^2$ the gauge theory version of the Han-Nambu model behaves like the fractional charge model. If the colour threshold is low, as has been suggested by Pati and Salam (1975), it would be difficult to distinguish the integer charge model from fractional charge model even above colour threshold. Nevertheless, colour may manifest itself through nonvanishing gluonic contribution to the deep inelastic lepton hadron processes in the asymptotic region (Pati and Salam 1975, Rajasekaran and Roy 1975). But this requires both a knowledge of colour threshold as well as the colour gluon mass. It has been suggested that the colour gluon mass might be

*Asymptotic freedom can still be maintained except to the order of weak and electromagnetic coupling strengths which are taken to be much smaller than the strong colour gauge coupling constant.

as low as 1 to 2 GeV and the colour threshold about 2 to 3 times the free colour gluon mass (Pati *et al* 1977). However, it is difficult to know these values without actually producing and observing a colour gluon as a peak in the invariant mass spectrum, all efforts toward which have as yet been unsuccessful. The status of observability of coloured quarks and gluons in broken colour gauge theories remains somewhat unclear at present. On the other hand, above colour threshold but with $q^2 \ll m_g^2$ the electromagnetic current in the naive and gauge theory version of the integer charge model are equivalent and it should in principle be possible to differentiate the integral and fractional charge colour models in deep inelastic experiments in this threshold region. This again requires a knowledge of colour threshold and colour gluon mass and given this, the colour production might be suppressed dynamically relative to flavour production in this kinematic region (Pati and Salam 1975). Further, the parton model calculations are not reliable in the threshold region. For these very reasons the colour gluon mass cannot be obtained from a quantitative study of departures from Bjorken scaling near and above colour threshold.

In view of all this, it is clearly desirable to have a direct probe of the quark charges as well as an estimate of colour gluon mass, without invoking any assumptions about the colour threshold. In this paper we discuss the deep inelastic inclusive bremsstrahlung process

$$e^{\pm} + p \to e^{\pm} + \gamma + X \tag{5}$$

in an appropriate scaling region as a possible experiment for distinguishing between various possibilities for quark charges as well as for a determination of colour gluon mass without having to ensure colour excitation explicitly. This type of process has previously been discussed (Brodsky *et al* 1972, Pandita 1975) as a possible source of information about fractional versus the integral charge quark model above colour threshold. Here we shall discuss the possibility of measuring the quark charges below the colour threshold via the process (5). More precisely, the difference of inclusive positron and electron bremsstrahlung cross sections on a proton target in an appropriate scaling region measures a structure function $V(x)$ which depends on charge cube of quark-partons. We obtain sum rules and bounds for this structure function which can distinguish between various possibilities for quark charges even below colour threshold. This process avoids complications of Pomeranchuk subtractions and hadronic decay backgrounds. The structure function is odd under charge conjugation and hence should have a quasielastic peak; sum rules involving integral of the structure function can be expected to converge in a finite experimentally accessible region. Further, we have the advantage that in the derivation of sum rules for structure function $V(x)$ we do not have to make assumptions about the distributions of partons; sum rules are derived only from normalization of charge, baryon number, etc., and hence should provide a definitive test of various quark-parton models. Also the sum rule for the structure function in the broken colour gauge theory model can provide an indirect way of estimating the colour gluon mass.

In section 2 we first briefly discuss the kinematics for this process and then obtain the sum rules which are independent of any assumptions about quark distributions. In section 3 we obtain bounds for the structure function $V(x)$ in terms of electroproduction structure function $\nu W_2(x)$ using the popular assumption of valence quark dominance. These bounds can also be used to distinguish between various

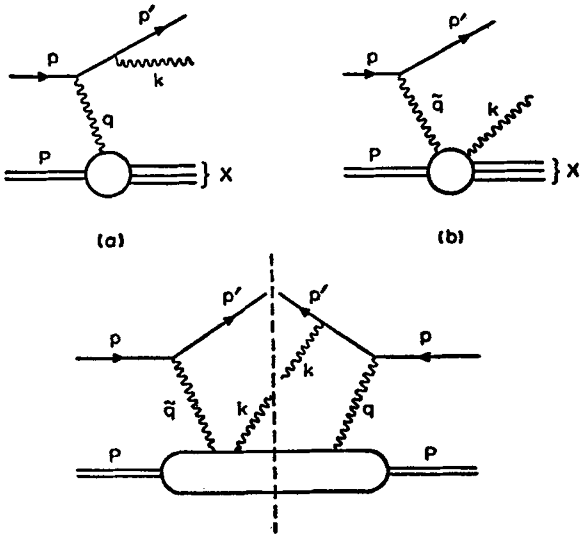

Figure 1. Feynman diagrams contributing to inclusive bremsstrahlung $e^{\pm} p \to e^{\pm} \gamma X$, (a) Bethe-Heitler, (b) Compton and (c) Interference. The Bethe-Heitler amplitude also receives a contribution from the amplitude in which the photon is emitted from the incident lepton. The Compton amplitude changes sign with the lepton charge.

possibilities for quark charges. We conclude the paper with a discussion about the experimental situation vis-a-vis this process.

2. Sum rules

The diagrams contributing to process (5) are shown in figures 1a and 1b. In general there are contributions from both Bethe-Heitler as well as Compton diagrams. However, the difference of the positron and electron inclusive cross sections is due only to the interference of the two types of diagrams, figure 1c, and can be written as ($s = 2P.p$)

$$\frac{d^6\sigma(e^+)}{(d^3p'/p'_0)\,(d^3k/k_0)} - \frac{d^6\sigma(e^-)}{(d^3p'/p'_0)\,(d^3k/k_0)}$$

$$= \frac{\alpha}{\pi^2 s}\,\frac{[-L^{\mu\nu\lambda}V_{\mu\nu\lambda}]}{q^2\tilde{q}^2} \qquad (6)$$

where the hadronic structure tensor $V_{\mu\nu\lambda}$ is a particular discontinuity of 'three-photon' Compton amplitude

$$V_{\mu\nu\lambda} = \frac{4\pi^2 E_p}{M}\int d^4x d^4y \exp(iq.y + ik.x)$$

$$\times \langle P\,|\,J_\nu(y)\,T^*(J_\lambda(o)\,J_\mu(x)\,|\,P\rangle \qquad (7)$$

$L_{\mu\nu\lambda}$ is a known leptonic structure part. The notations are shown in figure 1. All the hadronic physics is contained in $V_{\mu\nu\lambda}$. If we work in the Bjorken kinematic region*

$$\begin{aligned} &2P.q = 2P.(\tilde{q}-k) \gg M^2 \\ &Q^2 \equiv -q^2 = -(\tilde{q}-k)^2 \gg M^2 \\ &x = Q^2/2\,P.q \text{ fixed} \end{aligned} \tag{8a}$$

while requiring in addition that

$$\begin{aligned} &\tilde{Q}^2 \equiv -\tilde{q}^2 \gg M^2 \\ &2\,P.\,\tilde{q} \gg M^2 \\ &Q^2 - \tilde{Q}^2 = 2\,k.q \gg M^2 \end{aligned} \tag{8b}$$

then the process admits a parton model description and the leading contribution to $V_{\mu\nu\lambda}$ arises when all the three photons scatter from an individual parton, figure 2 (Brodsky *et al* 1972, Pandita 1975). The leading contribution to $V_{\mu\nu\lambda}$ is given by kinematical factors multiplying a scale invariant function $V(x)$:

$$\begin{aligned} V_{\mu\nu\lambda} &= \frac{1}{2\,P.q}\,\frac{1}{x}\,M_{\mu\nu\lambda}\,V(x) \\ V(x) &= \Sigma_i\,U_i(x)\,e_i^3 \end{aligned} \tag{9}$$

where e_i is the charge of the ith parton and $U_i(x)$ is the corresponding longitudinal momentum distribution function of partons. $M_{\mu\nu\lambda}$ is a known kinematical structure tensor (Brodsky *et al* 1972, Pandita 1975) which depends on the type of partons (spin-$\frac{1}{2}$ or spin-0). The structure function $V(x)$ can be directly obtained from the experimental measurement of the difference cross section through the relation (Brodsky *et al* 1972, Pandita 1975)

$$V(x) = \frac{\dfrac{d^6\sigma(e^+)}{(d^3p'/p'_0)\,(d^3k/k_0)} - \dfrac{d^6\sigma(e^-)}{(d^3p'/p'_0)\,(d^3k/k_0)}}{\left[\dfrac{\alpha^3}{\pi^2 s Q^2}\,|\,\mathrm{T_{int}}\,|^2\right]} \tag{10}$$

where

$$|\,T_{\rm int}\,|^2 = \frac{-M_{\mu\nu\lambda}L^{\mu\nu\lambda}}{q^2\tilde{q}^2}$$

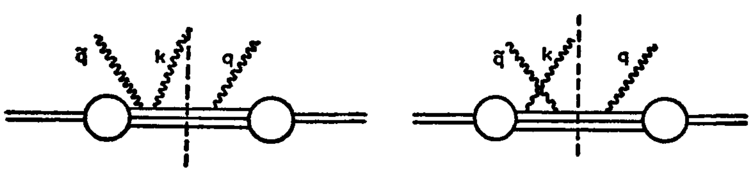

Figure 2. The dominant single-parton contribution to the interference amplitude in the scaling limit. The kinematical restrictions require that all three photons interact with the same parton.

*From our experience in deep-inelastic scattering it is sufficient that various kinematic invariants be $\gg$ 1 GeV^2.

is a known function of kinematical variables. The relation (10), by demanding that the right hand side be a function of x alone, serves as a crucial test of parton model.

Since the structure function $V(x)$ depends on the cube of parton charge, it is possible to obtain sum rules which do not depend on the parton distribution functions and are solely determined by quantum number conservation (Brodsky *et al* 1972, Pandita 1975). For example, in the fractional charge quark-parton model (e_q and b_q refer to charge and baryon number of the quark with flavour q; e and B are the charge and baryon number of target hadron)

$$e_q^3 = \tfrac{1}{3} e_q + \tfrac{2}{9} b_q$$

so that

$$\begin{aligned}\int_0^1 V(x)\, dx &= \int_0^1 dx \sum_q \left(\tfrac{1}{3} e_q + \tfrac{2}{9} b_q\right) U_q(x) \\ &= \tfrac{1}{3} e + \tfrac{2}{9} B \\ &= \tfrac{5}{9} \quad \text{for protons.}\end{aligned} \tag{11}$$

For Han-Nambu model $e_q^3 = e_q$ and

$$\begin{aligned}\int_0^1 V(x)\, dx &= \int_0^1 dx \sum_q e_q U_q(x) \\ &= e \\ &= 1 \quad \text{for protons.}\end{aligned} \tag{12}$$

In eqs (11) and (12) we have used the charge and baryon number conservation relations

$$\int_0^1 dx \sum_q e_q U_q(x) = e$$

$$\int_0^1 dx \sum_q b_q U_q(x) = B.$$

The sum rules (11) and (12) provide striking test for fractional versus integral charge model. Since $V(x)$ has a quasielastic peak ($V(x) \to 0$ as $x \to 0$) the integrals over $V(x)$ can be expected to converge in a finite experimentally accessible region. Since the sum rules are independent of parton distributions, these can be tested on nuclear targets as well with the additional benefit of large cross sections (Brodsky *et al* 1972, Pandita 1975). However, the sum rule (12) for the integer charge partons assumes that both colour singlet as well as colour octet pieces of the current in eq. (2) are operative. Thus to test this sum rule we shall have to cross the colour threshold. We shall now derive sum rules for the structure function $V(x)$ which can distinguish between the various possibilities for quark charges even below colour threshold.

Since the electromagnetic current in the fractionally charge colour model is a colour singlet, we have

$$\begin{aligned}\int_0^1 V(x)\, dx &= \int_0^1 dx \sum_{q,i} e_{q,i}^3 U_{q,i}(x) \\ &= \int_0^1 dx \sum_{q,i} \left(\tfrac{1}{3} e_{q,i}^{\text{flav}} + \tfrac{2}{9} b_{q,i}\right) U_{q,i}(x) \\ &= \tfrac{1}{3} e + \tfrac{2}{9} B \\ &= \tfrac{5}{9} \quad \text{for protons}\end{aligned} \tag{13}$$

where $e^{\text{flav}}_{q,\,i}$ is the flavour charge of quark q and colour i. As expected this is the same as in the fractional charge model without colour, because the proton and electromagnetic current are colour singlets. Note that the flavour charge of the quark q is the same for all colours.

For the naive integral charge model we have a colour singlet as well as a colour octet part in the current. For single current matrix elements between colour singlet states the colour octet current J^{col} does not contribute. However, in the three current amplitude (7), the colour octet part of the current does make a contribution even if the intermediate states are restricted to be colour singlets. This is so because in the two-photon amplitude $\langle X \,|\, T^*(J_\lambda J_\mu) \,|\, P \rangle$ with X a colour singlet state corresponding to the Compton diagram figure 1b, the $J^{\text{col}}\,J^{\text{col}}$ can contribute since it has a colour singlet projection. Since we require that all the photons in the three-photon amplitude couple to the same parton in the scaling region, we have the effective operator for $J^{\text{col}}\,J^{\text{col}}$ as (Chanowitz 1976)

$$J^{\text{col}}\,J^{\text{col}} = \sum_q \left(\tfrac{4}{9}\,\bar{q}_r\,q_r + \tfrac{1}{9}\,\bar{q}_y\,q_y + \tfrac{1}{9}\,\bar{q}_b\,q_b\right)$$

$$= \tfrac{2}{9} \sum_{q,\,i} \bar{q}_i\,q_i - \tfrac{1}{3} \sum_q \left(-\tfrac{2}{3}\,\bar{q}_r\,q_r + \tfrac{1}{3}\,\bar{q}_y\,q_y + \tfrac{1}{3}\,\bar{q}_b\,q_b\right) \tag{14}$$

where the first term in the last line is a colour as well as a flavour singlet, and the last term is colour octet piece. The colour octet piece does not contribute to the matrix element $\langle X | T^*(J_\lambda J_\mu) | P \rangle$ if X is a colour singlet state. The colour singlet piece in eq. (14) is to be combined with $J^{\text{flav}}\,J^{\text{flav}}$ to obtain the complete contribution of $\langle X | T^*(J_\lambda J_\mu) | P \rangle$ to the three-photon amplitude below colour threshold.* In this manner we find the effective operator for two-photon amplitude below colour threshold as

$$\sum_i \left(\tfrac{2}{3}\,\bar{p}_i\,p_i + \tfrac{1}{3}\bar{n}_i\,n_i + \tfrac{1}{3}\,\bar{\lambda}_i\,\lambda_i + \tfrac{2}{3}\,\bar{c}_i\,c_i\right).$$

To compute $e^{3}_{q,i}$ we must multiply this effective operator by the operator for $\langle P | J^{\text{flav}}_\nu \,|\, X \rangle$ yielding

$$e^{3}_{q,i} = \tfrac{1}{3}\left(1 + \tfrac{2}{3}\right) e^{\text{flav}}_{q,\,i} + \tfrac{2}{9}\, b_{q,i} \tag{15}$$

Note that J^{col} does not contribute to single photon matrix elements below colour threshold. From (15) we obtain the sum rule**

$$\int_0^1 V(x)\,dx = \tfrac{5}{9}\,e + \tfrac{2}{9}\,B$$

$$= \tfrac{7}{9} \quad \text{for protons} \tag{16}$$

*Terms like $J^{\text{flav}}J^{\text{col}}$ in the two photon amplitude do not contribute since they have no colour singlet projection.

**The total charge of a colour singlet object like proton is given by the flavour charges only.

below colour threshold to be compared with the value 1 obtained above colour threshold. Thus the sum rules (13) and (16) for the structure function $V(x)$ can distinguish between the fractional and the integral charge colour models even below colour threshold.

Although the above calculation is simple and just uses the group theoretical result that the product of two colour octet currents has a colour singlet piece, it involves an assumption about the dynamics. On the intuitive level the point is the following. In electroproduction process $e + p \to e + X$, to get scaling, it is necessary to have a kinematic condition such that an impulse approximation can be applied. Such condition is obtained in the deep inelastic region where the four momentum and the energy of the virtual photon is large so that the time of interaction between the virtual photon and the parton system is small compared to the lifetime of the virtual partons. The struck parton scatters as a free particle without having any interaction with the remaining fragment quarks. On such a short time scale of photon parton interaction the colour charges can be freely separated and the quark partons are in a state of net colour. It is only on a time scale long enough compared to the interaction time that the partons rearrange themselves to form colour singlet hadrons below colour threshold (Bjorken 1973). In the present case of bremsstrahlung process if the kinematic condition (8) is satisfied, then an impulse approximation can be applied to this process. The colour charges are freely separated during the short time in which all the photons interact with the same parton. It is only on a longer time scale that the struck parton and the spectator partons recombine to form colour singlet hadrons in the final state. In this sense the interaction is instantaneous and we can include the colour singlet projection of $J^{\text{col}}\ J^{\text{col}}$ below the colour threshold while calculating the Compton amplitude.

In the gauge theory version of the integral charge model the argument proceeds essentially along the same lines as in the naive version. However, due to photon-gluon mixing, we have to include a factor $-m_g^2/(\tilde{q}^2 - m_g^2)$ with the octet part of the current associated with the photon $\tilde{q}$ while forming the colour singlet projection to the intermediate state in the three-photon amplitude. For photon k this factor is unity ($k^2 = 0$). We thus obtain

$$e^3_{q,i} = \frac{1}{3}\left[1 + \frac{2}{3}\left(\frac{m_g^2}{m_g^2 - \tilde{q}^2}\right)\right] e^{\text{flav}}_{q,i} + \frac{2}{9} b_{q,i}$$

$$\int_0^1 V(x)\, dx = \frac{5}{9} + \frac{2}{9}\left(\frac{m_g^2}{m_g^2 - \tilde{q}^2}\right) \quad \text{for proton.} \tag{17}$$

The right hand side of the sum rule (17) lies between 5/9 and 7/9 corresponding to $|\tilde{q}^2| \gg m_g^2$ and $|\tilde{q}^2| \ll m_g^2$. The sum rules (13), (16) and (17) thus provide tests for different versions of colour models. However, it is important to realize that the broken colour gauge theory model predicts a scaling violation for $\int_0^1 V(x)dx$ even below colour threshold by requiring it to be $\tilde{q}^2$ dependent. By studying the $\tilde{q}^2$ dependence of the sum rule (17) one can obtain an estimate of the colour gluon mass. For this purpose we can write eq. (17) as

$$\frac{2}{3}\left[\int_0^1 V(x)\, dx - \frac{5}{9}\right]^{-1} = \frac{-\tilde{q}^2}{m_g^2} + 1 \tag{18}$$

which is a plot of a straight line with slope $1/m_g^2$. The quantity in the square bracket measures the deviation of $\int_0^1 V(x)dx$ from its value in the fractional charge model.

In the broken colour gauge theory model there are four charged colour gluons (two positively and two negatively charged) in addition to the quarks. However, since these couple only through J^{col}, these do not contribute below colour threshold. Further, since proton is a colour singlet (i.e. all colour gluon distribution functions for a proton are equal), these do not contribute to its charge, so that the total charge is still given by

$$1 = e = \int_0^1 dx \sum_{q,i} e_{q,i}^{\text{flav}} \; U_{q,i}\ (x) \tag{19}$$

where the sum is over quarks and antiquarks only. Thus the sum rule (17) is independent of the presence of the colour gluons. The sum rules (13), (16) and (17) are obviously independent of the quark and gluon distribution, since we have used only the normalization of conserved quantities like charge and baryon number in their derivation. Because of this fact the tests of these sum rules can be performed on nuclear targets with the benefit of large cross sections.

3. Bounds on V(x)

In this section we shall derive bounds on $V(x)$ in terms of the structure function $\nu W_2(x)$ measured in inclusive electroproduction experiments. These bounds can be used to distinguish between various possibilities for quark charges.

For the case of fractional charge model with colour, we can write

$$\begin{aligned} V(x) &= \sum_{q,i} e_{q,i}^3 \; U_{q,i}\ (x) \\ &= \sum_{q,i} \left(\tfrac{1}{3}\, e_{q,i}^{\text{flav}} + \tfrac{2}{9}\, b_{q,i} \right) \left[U_{q,i}\ (x) - \bar{U}_{q,i}\ (x) \right] \end{aligned} \tag{20}$$

where $U_{q,i}\ (x)$ is the distribution function for antipartons. The sum is now over partons only. Due to odd power of charge, $V(x)$ does not receive a contribution from parton-antiparton sea. The structure function for inelastic electron scattering can be written as

$$\frac{\nu W_2(x)}{x} = \sum_{q,\, i} \left(e_{q,\, i}^{\text{flav}} \right)^2 \left[(U_{q,i}\ (x) + \bar{U}_{q,i}\ (x) \right]. \tag{21}$$

The ratio $V(x)/(\nu W_2/x)$ has an upper bound

$$\begin{aligned} R\ (x) &= \frac{V\ (x)}{(\nu W_2(x)/x)} \\ &= \frac{\sum_{q,\, i} \left(\tfrac{1}{3}\, e_{q,\, i}^{\text{flav}} + \tfrac{2}{9}\, b_{q,i} \right) \left[U_{q,i}\ (x) - \bar{U}_{q,i}\ (x) \right]}{\sum_{q,\, i} \left(e_{q,i}^{\text{flav}} \right)^2 \left[U_{q,i}\ (x) + \bar{U}_{q,i}\ (x) \right]} \\ &\leqslant 2/3 \end{aligned} \tag{22}$$

given by the assumption of no antiquarks and the dominance of p type quarks (Morrison 1977).

For the case of Han-Nambu model below colour threshold we have

$$R(x) = \frac{\sum_{q,i} \left[\frac{5}{9} e_{q,i}^{\text{flav}} + \frac{2}{9} b_{q,i}\right] \left[U_{q,i}(x) - \bar{U}_{q,i}(x)\right]}{\sum_{q,i} \left(e_{q,i}^{\text{flav}}\right)^2 \left[U_{q,i}(x) + \bar{U}_{q,i}(x)\right]}$$

$$\leqslant 1. \tag{23}$$

Finally for the gauge theory version of the integer charge model below colour threshold, we have

$$R(x) = \frac{\sum_{q,i} \left[\frac{1}{3} e_{q,i}^{\text{flav}} \left(1 + \frac{2}{3} \frac{m_g^2}{m_g^2 - \tilde{q}^2}\right) + \frac{2}{9} b_{q,i}\right] \left[U_{q,i}(x) - \bar{U}_{q,i}(x)\right]}{\sum_{q,i} \left(e_{q,i}^{\text{flav}}\right)^2 \left[U_{q,i}(x) + \bar{U}_{q,i}(x)\right]}$$

$$\leqslant \frac{2}{3} + \frac{1}{3} \frac{m_g^2}{m_g^2 - \tilde{q}^2}. \tag{24}$$

The upper bound in this case lies between 2/3 and 1 corresponding to $|\tilde{q}^2| \gg m_g^2$ and $|\tilde{q}^2| \ll m_g^2$, respectively. Note that, below colour threshold, the electron scattering structure function $\nu W_2(x)$ is the same for all the three versions of colour models. Values of the ratio $R(x)$ obtained from electron data up to $x \simeq 0{\cdot}6$ are still well below the fractional charge model upper bound of 2/3 (Morrison 1977). However, since the bounds for $R(x)$ are obtained under the assumption of the dominance of p quarks, which may be correct only near $x \simeq 1$, it will be interesting to watch the behaviour of $R(x)$ close to $x \simeq 1$.

4. Concluding remarks

We now come to the question of the experimental evaluation of sum rules. Recent experimental data give (Fancher *et al* 1977)

$$\int_0^1 V(x)\, dx = 0{\cdot}89 \pm 0{\cdot}34. \tag{25}$$

This result although not statistically precise enough to distinguish between various colour models or provide an estimate of the colour gluon mass, does support parton model with partons of low charge. Obviously the first thing to be confronted with the experiment is the scaling result (10). If scaling is verified then the application of parton model to this process is justified. On the other hand since $V(x)$ is expected to have a quasielastic peak ($V(x) \to 0$ as $x \to 0$) the low x region should not be important in the evaluation of the integral over $V(x)$. Thus in order to experimentally evaluate $\int_0^1 V(x)dx$ it is only necessary to measure $V(x)$ for all values of x where $V(x)$

is large. However, it has been shown (Morrison 1977) on the basis of electroproduction data from SLAC that substantial part of $\int_0^1 V(x)dx$ comes from $x < 0{\cdot}1$, and thus $V(x)$ may not have the anticipated quasielastic peak.* In order to satisfy the scaling criterion the magnitudes of invariants q^2, $\tilde{q}^2$, $2k.q$, etc. must be at least as large as $1(\mathrm{GeV}/c)^2$. As Fermilab and SPS energies these conditions can be met for small x but the luminosities are marginally low for a precise experiment. At SLAC the practical limit is $x \gtrsim 0{\cdot}1$. Since a sizeable fraction of the integral is thus kinematically inaccessible, an accurate evaluation of the sum rules for $V(x)$ may not be possible at SLAC. However, using specific assumptions about quark-parton distributions, practical upper bounds of $V(x)$ in terms of $\nu W_2(x)$ can still distinguish between various colour quark models.

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*Note however that Fancher *et al* (1977) do not use method of Morrison (1977) to obtain information about quark charges from $V(x)$.