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Magnetic contributions to high energy inverse bremsstrahlung process

G RAMACHANDRAN and R S KESHAVAMURTHY
Department of Physics, University of Mysore, Manasagangotri, Mysore 570 006

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Abstract. Magnetic contribution to inverse bremsstrahlung is estimated at high energies. It begins to dominate over the Coulomb contribution in the ratio $CE_2^2 \tan^2(\theta_2/2)(1+\sin^2(\theta_2/2))$ where E_2 and θ_2 denote the energy and angle of the accelerated electron and C is essentially a nuclear constant.

Keywords. Inverse bremsstrahlung; magnetic scattering.

We consider here the interesting proposal of Diambrini-Palazzi of accelerating high energy electrons through inverse bremsstrahlung process in the vicinity of nuclei (Diambrini-Palazzi 1975; Barbieri *et al* 1976). Barbieri *et al* (1976) take the nucleus to provide only a Coulomb field in which the energy momentum imbalance of the accelerated electron is made good. It is important to notice that the nucleus could also possess a magnetic dipole moment and its effect has been investigated, as is well known, in the case of electron scattering and also in the case of bremsstrahlung (Ginsberg and Pratt 1964; Goldemberg and Pratt 1966) and found to be quite important especially at backward angles. Consequently the electromagnetic field of the nucleus should be properly represented by the electromagnetic four vector A whose components $(A_t, \mathbf{A})$ are given by

$$A_t = \frac{4\pi Ze}{Q^2}, \quad \mathbf{A} = \frac{4\pi ie}{2M\mathbf{Q}^2} \vec{\mu} \times \mathbf{Q} \tag{1}$$

where Z is the charge on the nucleus and $\vec{\mu}$ is its magnetic dipole moment. For instance if we consider a nucleus like aluminium the dipole moment μ would be as high as 3·639 nuclear magnetons. Here M denotes the mass of the nucleon and $\mathbf{Q}$ the momentum transferred to the nucleus. If $p_1(E_1, \mathbf{p}_1)$ and $p_2(E_2, \mathbf{p}_2)$ denote respectively the initial and final four momenta of the electron and $q(\omega, \mathbf{q})$ denotes the four momentum of the photon absorbed, the four momentum transfer Q to the nucleus is given by $Q = p_1 + q - p_2$. The standard second order Feynman diagrams for the process are shown in figure 1 and following Feynman's conventions and notations (Feynman 1961), the amplitude is written as

$$\tilde{u}_{p_2} M u_{p_1} = (-ie)^2 (4\pi)^{1/2} \tilde{u}_{p_2} \left[\not{A} \frac{i}{\not{p}_1 + \not{q} - m} \not{e} + \not{e} \frac{i}{\not{p}_2 - \not{q} - m} \not{A} \right] u_{p_1} \tag{2}$$

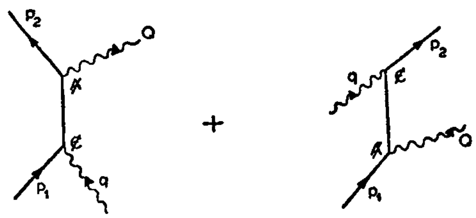

Figure 1.

where $\epsilon\ (0, \vec{\epsilon}\ )$ denotes the polarization vector of the photon. It is interesting to point out that irrespective of what A is, one can evaluate the transition probability, $d\sigma$ after summing and averaging over electron spins to get a master formula.

$$
\begin{aligned}
\frac{1}{4}\sum_{\substack{\text{electron}\\ \text{spins}}} |\tilde{u}_{p_2} M u_{p_1}|^2 = 4\pi e^4 \Bigg[\left(\frac{\epsilon\cdot p_2}{p_2\cdot q}\right)^2 & \{(A.A^*)Q^2+2(A.p_1)(A^*.p_2) \\
& + 2(A^*.p_1)(A.p_2)-2(A.p_1)(A^*.q)-2(A^*.p_1)(A.q)\} \\
& + \left(\frac{\epsilon.p_1}{p_1.q}\right)^2 \{(A.A^*)Q^2+2(A.p_1)(A^*.p_2)+2(A^*.p_1)(A.p_2) \\
& +2(A.p_2)(A^*.q)+2(A^*.p_2)(A.q)\}-\frac{2(\epsilon.p_1)(\epsilon.p_2)}{(p_1.q)(p_2.q)}\times \\
& \{ (A.A^*)Q^2+2(A.p_1)(A^*.p_2)+2(A^*.p_1)(A.p_2)\} - \frac{1}{(p_1.q)\,(p_2.q)}\times \\
& \{(A.A^*)(Q.q)^2+(A.q)(A^*.q)Q^2\}+4(A.\epsilon)(A^*.\epsilon) \\
& +4\left(\frac{\epsilon.p_2}{p_2.q}\right)\{(A.\epsilon)(A^*.p_1)+(A.p_1)(A^*.\epsilon)\} \\
& -4\left(\frac{\epsilon.p_1}{p_1.q}\right)\{(A.\epsilon)(A^*.p_2)+(A.p_2)(A^*.\epsilon)\}\Bigg]
\end{aligned}
\tag{3}
$$

which can be specialised to give the appropriate expressions for all second order quantum electrodynamic processes where at least one real photon is entering into the process. For example the transition probability for Compton scattering is obtained by substituting the polarization vector of the emitted photon, $\epsilon'(0, \vec{\epsilon'}\ )$ for A in (3). Likewise the transition probability for Coulomb bremsstrahlung can be obtained by reversing the sign of the photon four vector q and substituting $((4\pi Ze)/Q^2),0)$ for A in (3). In the problem under investigation we can write $\vec{\mu} = g\mathbf{J}$ where $\mathbf{J}$ is the nuclear spin operator and g is the Lande factor. Assuming that the nucleus remains in the ground state we can sum and average over the $2j+1$ nuclear spin states. Further we shall set $\mathbf{p}_1$ and $\mathbf{q}$ to be in the same direction appropriate to the geometry of the experiment of Diambrini-Palazzi and neglect the effect of nuclear recoil. The assumption that the nucleus is static ensures that there is no interference between the Coulomb and magnetic terms. At high energies, we multiply A_t and A by nuclear form factors $F_C(\mathbf{Q}^2)$ and $F_M(\mathbf{Q}^2)$ respectively which are normalised to unity as $\mathbf{Q}^2 \to 0$. After averaging over the initial photon polarizations, $\vec{\epsilon}$, we have

$$d\sigma = (d\sigma)_{\text{Coul}} + (d\sigma)_{\text{Mag}} \tag{4}$$

where

$$(d\sigma)_{\text{Coul}} = \frac{8\pi^4 D Z^2 \alpha^3 \,|F_C(\mathbf{Q}^2)|^2}{E_1 E_2\, \omega^3\, \mathbf{Q}^4} \Bigg[\frac{p_2^2 \sin^2\theta_2}{(E_2 - p_2\cos\theta_2)^2} (4E_1^2 - \mathbf{Q}^2) + \frac{2\omega^2 p_2^2 \sin^2\theta_2}{(E_1 - p_1)(E_2 - p_2\cos\theta_2)} \Bigg] \tag{5}$$

$$(d\sigma)_{\text{Mag}} = \frac{2\pi^4 D\alpha^3 \,|F_M(\mathbf{Q}^2)|^2\, \mu^2 (j+1)}{E_1 E_2 \omega^3 M^2 \mathbf{Q}^4\, 3j} \Bigg[\frac{p_2^2 \sin^2\theta_2}{(E_2 - p_2\cos\theta_2)^2} \times \left\{ \mathbf{Q}^4 + 2p_2^2 p_1^2 \sin^2\theta_2 \right\} + \frac{1}{(E_1 - p_1)(E_2 - p_2\cos\theta_2)} \times \omega^2 \left\{ \mathbf{Q}^2 + 2(E_1 - p_1)(E_2 - p_2\cos\theta_2) \right\} (2\mathbf{Q}^2 - p_2^2 \sin^2\theta_2) \Bigg] \tag{6}$$

where

$$\mathbf{Q}^2 = p_1^2 + \omega^2 + p_2^2 + 2p_1\omega - 2p_1 p_2 \cos\theta_2 - 2p_2\omega\cos\theta_2$$

in terms of the angle θ_2 made by the accelerated electron with respect to the beam direction. Here D denotes the final two particle density of states. Expression (5) for the Coulomb term agrees with that of Barbieri *et al* (1976). It may be noted that expression (6) could also be obtained by reversing the sign of q in the expression given by Ginsberg and Pratt (1964) for magnetic bremsstrahlung in the same way as the expression of Barbieri *et al* is obtained from that for Coulomb bremsstrahlung. The magnetic contribution is seen to be proportional to $\mathbf{Q}^2/4M^2$ and consequently it should be expected that at high energies and large momentum transfers (especially if $\mathbf{Q}^2 > 4M^2$), the magnetic term would give rise to non-negligible contributions. The ratio of magnetic to Coulomb inverse bremsstrahlung could be approximated by

$$\frac{(d\sigma)_{\text{Mag}}}{(d\sigma)_{\text{Coul}}} = \frac{\mu^2 (j+1)}{6jM^2 Z^2} \frac{|F_M(\mathbf{Q}^2)|^2}{|F_C(\mathbf{Q}^2)|^2} E_2^2 \tan^2 \frac{\theta_2}{2} \left(1 + \sin^2 \frac{\theta_2}{2}\right). \tag{7}$$

This ratio in the case of aluminium is plotted against angle of the accelerated electron in figure 2 for a typical value of E_2 taking the ratio of nuclear form factors to be 1.

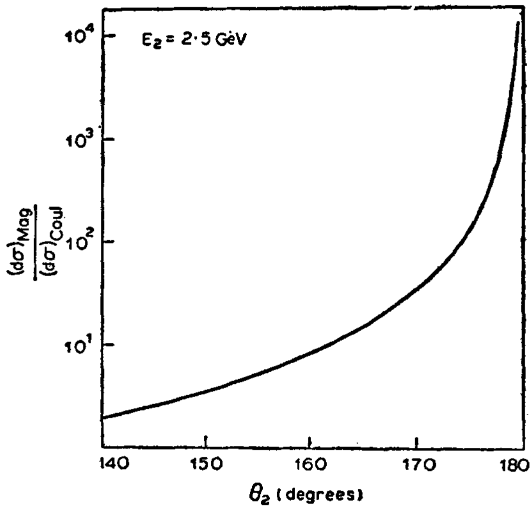

Figure 2. The plot of the ratio of magnetic term to Coulomb term as a function of the angle of the accelerated electron in the case of aluminium.

It is clear from figure 2 that the effect of the magnetic dipole field of the nucleus dominates over that of the Coulomb field at backward angles for inverse bremsstrahlung as is also the case in the case of ordinary bremsstrahlung (Ginsberg and Pratt 1964).

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