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Nuclear matter test for a separable non-local N-N interaction

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Abstract. A nuclear matter test has been conducted on a separable non-local N-N interaction proposed by Sirohi and Srivastava. The potential had been constructed by fitting the phase-shifts. The results obtained compare with those obtained by Tabakin in each partial wave. The binding energy per particle and the symmetry energy coefficient obtained are 19·4 MeV and 66·5 MeV respectively at $k_F = 1{\cdot}65$ fm^{-1}, while the best estimated values for these quantities are 16 MeV and 30–35 MeV at $k_F = 1{\cdot}5$ fm^{-1}. Single particle potential generated out of this NLSI has in general the same shape as the one obtained through Tabakin potential. Finally as a test of non-locality the integrated photoabsorption cross-section parameter h has been calculated and the value of 1·03 agrees with other reported values.

Keywords. Non-local separable interaction; nuclear matter; binding energy; symmetry energy coefficient; Fermi momentum; single particle potential.

1. Introduction

In continuation of our investigations regarding a separable non-local N-N interaction (Sirohi and Srivastava 1972, 1973) we report in this paper nuclear matter calculations with this potential. The earlier calculations for nuclear matter are due to Brueckner and Gammel (1958) using reaction-matrix formalism. They obtained a binding energy per particle of 15·2 MeV. Results which were more unreasonable were obtained by Brueckner and Masterson (1962) viz. E/A=8 MeV and a similar value was obtained by Razavi (1963). According to Bethe, the best estimated value for binding energy may be taken as 16 MeV at $k_F = 1{\cdot}5$ fm^{-1} (Kallio 1975). In all the above calculations, hard core potentials where one has to resort to very crude approximations, have been used.

Contrary to the above methods, Hartree-Fock type calculations with an effective interaction are much simpler. The possibility of using non singular effective interactions for nuclear matter calculations had earlier been shown by Pieschl and Werner (1963). They obtained almost similar results using simpler calculations.

Further progress in this direction was made following the calculations of Tabakin (1964). Constructing a non-local separable potential model for various partial waves he used them in nuclear matter calculations and the BE/A obtained by him was only 8 MeV. Other investigations on similar lines were made by Elbaz *et al* (1972) using Tabakin's approach but the results were not very satisfactory. Nuclear matter calculations using Volkov type potential having a density dependent shape have been reported by Khanna and Barhai (1973) who obtained nuclear matter parameters (BE, compressibility, nuclear density) very close to the best estimates for these parameters.

Sirohi and Srivastava (1972) constructed a rank-two non-local separable *N-N* potential model for various partial waves. The potential model has the advantage of having simple calculational features with no significant difference in off-shell behaviour as compared to the usual local potential. In our earlier investigations (Sharma and Sirohi 1976) we had studied the accuracy of this potential in the unitary pole approximation both at positive and negative energies and it was found that the quality of UPA fits was promising. This may inspire one to use this potential in three body calculations. In the present paper, however, the other aspect of this potential model viz. nuclear matter calculations has been reported. Our main purpose is to demonstrate the power and utility of this potential model. Besides fitting phase shifts which is a common feature in most of the potentials, giving accurate off-shell behaviour in UPA, it can also lead to correct saturation properties of nuclear matter. We have tested the separable potential model of Sirohi and Srivastava (1972) in nuclear matter calculations using Tabakin's approach only to the first order perturbation term.

Section 2 contains the interaction to be studied while section 3 deals with the basic formalism. Section 4 gives the results while our conclusions are given in section 5.

2. Separable non-local *N-N* interaction

A rank-two separable non-local *N-N* interaction in momentum representation can be written as

$$\langle \mathbf{k} \mid V_{ll'} \mid \mathbf{k}' \rangle = \frac{2\hbar^2}{\pi m} i^{l-l'} \sum_{\sigma M l l'} \left[- g_l^\sigma (k)\ g_{l'}^\sigma (k) + h_l^\sigma (k) h_{l'}^\sigma (k') \right]$$

$$\tilde{Y}_{\sigma l}^{M} (\hat{k})\ \tilde{Y}_{\sigma l'}^{*M} (\hat{k}') \qquad (1)$$

where σ denotes quantum numbers J, T and S. The functions $\tilde{Y}_{\sigma l}^{M} (\hat{k})$ are normalised eigenstates of total angular momentum J and its Z component. The form factor $g(k)$ corresponds to attraction and $h(k)$ to repulsion. The form factors and the best fit parameters for various partial waves are given in Sirohi and Srivastava (1972, 1973).

3. Formalism

3.1. *Binding energy*

Nuclear matter consists of an infinite number of interacting nucleons whose average energy per particle should equal the volume term in semi-empirical mass formula. In this hypothetical configuration it is assumed that no coulomb repulsion exists and that surface effects are negligible. It is evident that single particle states are plane waves. The total energy of the system is given by well known expansion

$$E = \frac{3}{5} A \epsilon_f + \frac{1}{2} \sum_{\mu\nu<F} \langle \mu\nu | V | \mu\nu - \nu\mu \rangle + \frac{1}{2} \frac{m^*}{m}$$

$$\sum_{\substack{\mu\nu<F \\ \sigma\tau>F}} \frac{\langle \mu\nu | V | \sigma\tau \rangle \langle \sigma\tau | V | \mu\nu - \nu\mu \rangle}{\left(\frac{\hbar}{2}\right) k_\mu^2 + k_\nu^2 - k_\sigma^2 - k_\tau^2} . \tag{2}$$

Here the first two terms are simply the Hartree-Fock solutions while the third and higher terms are a measure of the inter particle correlation energy. Using the interaction under study and the Tabakin (1964) approach the final expression for binding energy per particle in first order perturbation calculation becomes,

$$\frac{E}{A} = \frac{3}{5} \epsilon_f + \left(\frac{4\hbar}{\pi}\right) \sum_{\sigma l} (2J+1)(2T+1)$$

$$\times \int_0^{k_F} k^2 \left(1 - \frac{3}{2} k k_F^{-1} + \frac{1}{2} k^3 k_F^{-3}\right) f_l^\sigma(k) dk \tag{3}$$

with $$f_l^\sigma(k) = -g_{\sigma l}^2(k) + h_{\sigma l}^2(k).$$

3.2. *Symmetry energy coefficient*

An important property acting as a test of perturbation theory with model potential is the symmetry energy of infinite nuclear matter. The fact that nuclei with equal number of neutrons (N) and protons (Z) tend to be more stable is a result of the exclusion principle and the nature of two-body interaction. The symmetry energy of infinite nuclear matter is calculated by considering a system of two coexisting Fermi gases, consisting of neutrons and protons. Neutron and proton values are taken to be equal. The expression for the symmetry energy coefficient can be written as

$$a_\tau(k_F) = \frac{1}{3} \epsilon_f + \left(\frac{2\hbar}{\pi}\right) \sum_{\sigma l} (2J+1)(2T+1) \left[\int_0^{k_F} k k_F^{-1} (k^2 - k_F^2) f_l^\sigma(k) dk \right]. \tag{4}$$

Here $\frac{1}{3}\epsilon_f$ is the kinetic energy contribution to the symmetry energy.

3.3 *Integrated photoabsorption cross-section*

The non locality of the potential can be tested by determining thei ntegrated photoabsorption cross-section,

$$\sigma_{\text{int}} = 60 \frac{NZ}{A} (1+h). \tag{5}$$

Here h vanishes for local potentials and takes a non negligible value for NLSI. The expression for h can be written as

$$h=\left(\frac{4}{\pi}\right)\sum_{\sigma l}(2J+1)\,(2T+1)\int_0^{k_F}(2k^3k_F^{-3}-kk_F^{-1})\,f_l^{\sigma}(k)dk. \tag{6}$$

3.4. *Single particle potential*

In Hartree-Fock formalism a single particle average potential is defined as

$$\langle\omega\mid u\mid\omega'\rangle=\sum_{\mu<F}\langle\omega\mu\mid V\mid\omega'\mu-\mu\omega'\rangle. \tag{7}$$

Nuclear matter is a translation invariant system, hence u must be a function of momentum only. In the light of the present approach we have

$$u(k_\omega)=\left(\frac{2\lambda}{\pi}\right)\sum_{\sigma l}(2J+1)\,(2T+1)\int_{|2\mathbf{k}+\mathbf{k}_\omega|<k_F}dkk^2d\Omega_{\hat{k}}\,f_l^{\sigma}(k) \tag{8}$$

with a Fermi sea restriction on the domain of integration. The angular integration gives

$$\int_{|2\mathbf{k}+\mathbf{k}_\omega|<k_F}d\Omega_{\hat{k}}=\begin{cases}4\pi \text{ for } 0\leqslant k\leqslant\tfrac{1}{2}(k_F-k_\omega) \text{ and } |k_\omega|\leqslant k_F\\ [k_F^2-(2k-k_\omega)^2]\,\dfrac{\pi}{2kk_\omega} \text{ for } \tfrac{1}{2}|k_F-k_\omega|\leqslant k\leqslant\tfrac{1}{2}(k_F+k_\omega).\end{cases} \tag{9}$$

The final expression for single particle potential can be written as

$$u(k_\omega)=\left(\frac{2\lambda}{\pi}\right)\sum_{\sigma l}(2J+1)\,(2T+1)\left[\int_0^{\frac{1}{2}(kF-k_\omega)}4\pi k^2f_l^{\sigma}(k)dk\right.$$

$$\left.+\int_{\frac{1}{2}(k_F-k_\omega)}^{\frac{1}{2}(k_F+k_\omega)}[k_F^2-(2k-k_\omega)^2]\,\frac{\pi k}{2k_\omega}\,f_l^{\sigma}(k)\,dk\right]. \tag{10}$$

The single particle potential in Hartree-Fock definition is therefore related directly to parameters of the model potential and to the Fermi momentum (k_F). All the integrals, involved in the expressions for BE/A, symmetry energy coefficients, single particle potential and parameter h have been numerically calculated.

4. Results and discussion

We have calculated BE/A, symmetry energy coefficient, parameter h appearing in integrated photoabsorption cross-section and single particle potential using Sirohi and Srivastava (1972) NLSI.

The potential energy curves for various partial waves have been plotted along with the Tabakin curves and are given in figures 1–4. It is seen that the general behaviour is similar to that of Tabakin (1964) but the overall result is considerably better in our case. The present results have also been compared with other calculations using different types of *N-N* interaction i.e., local soft core, hard core, non local separable, etc. The results for various partial waves are also compared with a few other calculations (Kallio 1975). In general we find that our results are quite comparable to earlier investigations suggesting the usefulness of this potential model in nuclear matter calculations.

Binding energy obtained in our case has been compared with values obtained by Tabakin (1964), Elbaz *et al* (1973) and Khanna and Barhai (1973) (figure 5). The bind-

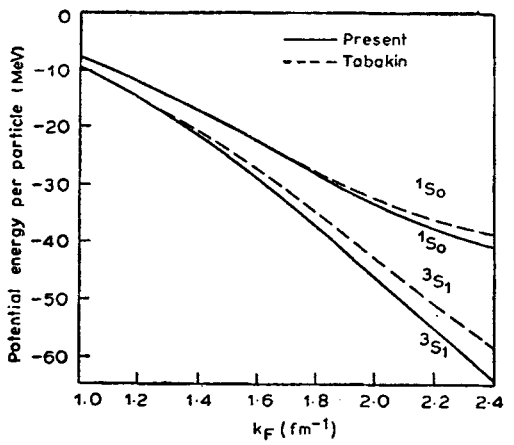

Figure 1. Potential energy per particle for $^1S_0$ and $^3S_1$ waves vs Fermi momentum

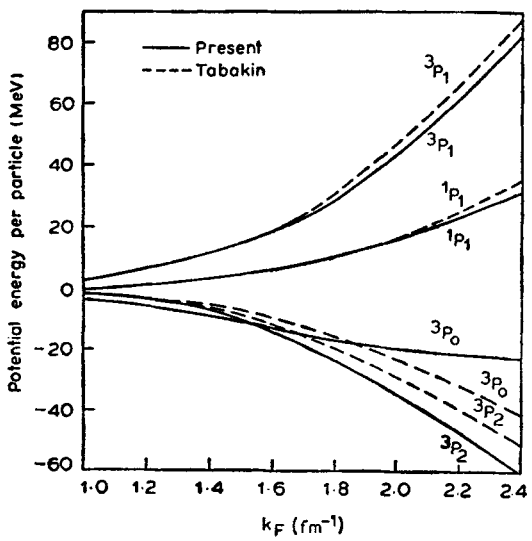

Figure 2. Potential energy per particle for $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$ waves vs Ferm momentum.

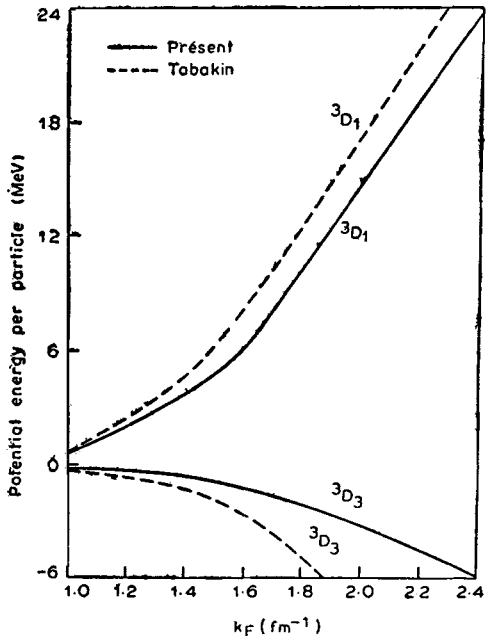

Figure 3. Potential energy per particle for $^3D_1$ and $^3D_3$ waves vs Fermi momentum.

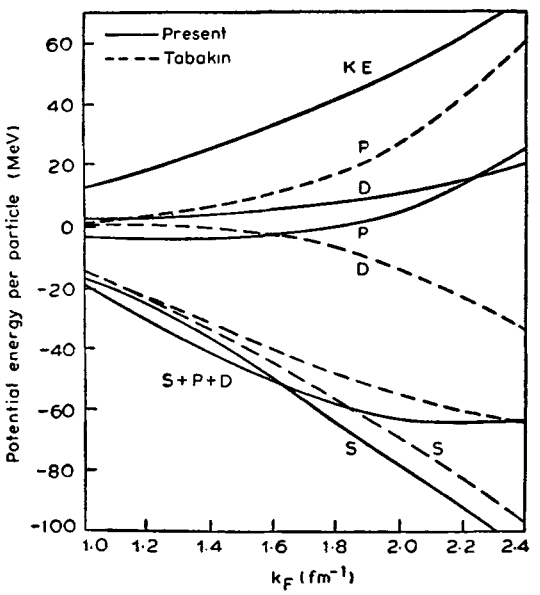

Figure 4. Potential energy per particle for S, P, and D waves vs Fermi momentum.

ing energy obtained by Tabakin is 8 MeV, at k_F=1·6 fm^{-1}, which is too small a value. Elbaz *et al* (1973) have used the modified version of Mongan IV NLSI and the value of BE obtained in the first order calculation is 10 MeV per particle. Khanna and Barhai (1973) using local potential obtained a BE per particle of 15·5 MeV at k_F= 1·35 fm^{-1}. It may be noted that our results are very close to the best estimated values, given by Bethe BE/A=16 MeV at k_F=1·5 fm^{-1} and a_τ=30–35 MeV). The binding energy in our case is 19·4 MeV at k_F=1·65 fm^{-1} which shows the saturation at a reasonably correct density.

In the case of symmetry energy coefficient our value is 66·5 MeV as against the estimated value of 30–35 MeV. This is not a good result but when compared with others, it is not unreasonable. Strong dependence of a_τ on density, suggests that if coulomb term lowers the equilibrium value of k_F, the symmetry energy coefficient would approach the estimated value.

The single particle potential has been calculated using eq. (10) and is presented in figure 6 for various Fermi momenta. The single particle total energy is also presented for k_F=1·65 fm^{-1}. The Tabakin results are also plotted in the figure. A comparison shows that the nature of potential obtained in our case is similar to that of Tabakin but with different parameters.

As a test of non-locality we have added calculations of integrated photoabsorption cross-section parameter h. This value comes to 1·03 and agrees well with the experimental one as well as with the other calculations using Tabakin, Hammann, and Mongan modified (MV) potentials. It may be pointed out that for NLSI this parameter must have a non-negligible value, whereas for local potentials it must be zero.

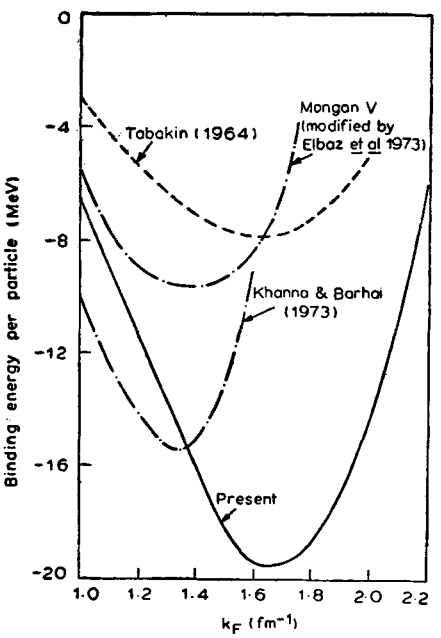

Figure 5. Binding energy per particle vs Fermi momentum.

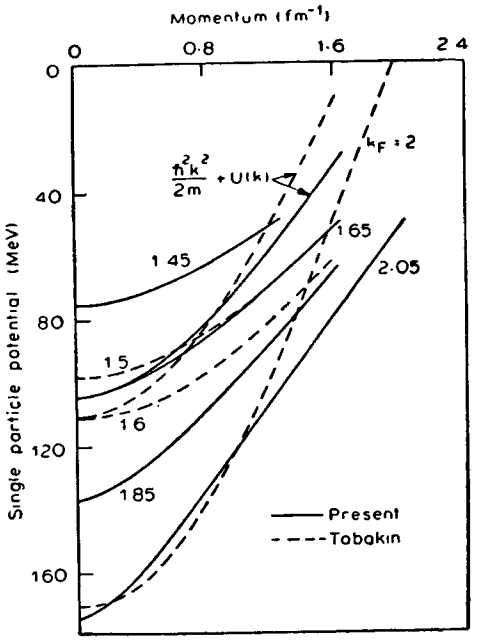

Figure 6. Single particle potential vs momentum in Fermi inverse. The strength of the potential in our case has been plotted by reducing it to one fifth of its actual value. (Note: Single particle potential, MeV. Values are negative, i.e. −40, −80, −120, −160.)

5. Conclusions

The following conclusions can be drawn regarding the potential model on the basis of the present and earlier studies;

(a) Sirohi-Srivastava potential fits well with the phase-shifts in each partial wave up to 400 MeV. (Sirohi and Srivastava 1973).

(b) The potential is very accurate in UPA both at positive and negative energies and therefore it can be used in three-body calculations. (Sharma and Sirohi 1976).

(c) It has oscillatory off-shell behaviour which compares well with that for local potentials and hence the present calculations can be compared with those using local potentials (Sirohi and Srivastava 1973).

(d) In nuclear matter calculations, it reproduces the binding energy, nuclear matter density, symmetry energy coefficient, integrated photoabsorption cross-section parameter h and single particle potential comparable to other calculations (e.g. present calculations).

In view of the above findings, it is suggested that Sirohi-Srivastava potential is a better choice in nuclear structure calculations. Although this study is restricted to first order due to the shape of form factors of the potential, the suitability of the potential is well established.

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