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On the symmetries between neutrino and antineutrino-nucleon elastic scattering

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Abstract. Various symmetry relations developed between neutrino-neutron and antineutrino-proton elastic scattering cross sections are surveyed and an identity between scattering amplitudes and a symmetry between cross sections of these processes established by considering *CPT* and *G* conjugation invariance of current matrix elements. A symmetry is obtained giving rise to a theorem on the nature of contribution of form factors to terms in the cross sections.

Keywords. Neutrino-nucleon scattering; polarization of particles; nucleon weak current form factors; scattering amplitude; scattering symmetries.

1. Introduction

Neutrinos experience only weak interactions and hence their processes are useful in studying the weak interaction properties such as hermiticity, *CVC* hypothesis, *G*-conjugation of nucleon weak currents and thereby the properties of first class and second class currents without the obscuring effects of electromagnetic and strong interactions. The elastic scattering processes (hereinafter referred to as (I) and (II))

$$\nu + n \rightarrow l + p \tag{I}$$

and

$$\bar{\nu} + p \rightarrow \bar{l} + n \tag{II}$$

involve respectively *np* and *pn* currents which do not have the same structure. Therefore, in principle, both (I) and (II) should be studied independently of the other. Computational repetition of this sort can be avoided if these processes have symmetry between their scattering amplitudes and hence between their cross sections.

During the last decade (I) and (II) have been related to each other by many workers in a variety of ways each of them different, some incomplete or erroneous. It will be clear from the review given in section 3 that the earlier work does not establish coherent symmetry between amplitudes and cross sections. The present paper aims to establish such a symmetry between (I) and (II) based on *CPT* and *G*-transformation properties of particle currents by neglecting final state electromagnetic corrections.

In section 2 we outline the mathematics used in establishing the symmetry. Section 4 is devoted to establish an identity between the amplitudes of (I) and (II) and a symmetry between their cross sections. By combining it with generalized Bell symmetry (Teli 1974) a symmetry relating a process to itself is obtained leading to a theorem on the contribution of form factors to the cross sections. In section 5, the symmetry developed in section 4 is compared with earlier symmetries and proper corrections are suggested in section 6.

2. Mathematical equipment

We take k_1, p_1, k_2, p_2 and s_ν, s_l, s_1, s_2 respectively as four momenta and polarization four vectors (Okun 1965) of the particles in order of their occurrence in (I). The hamiltonian for (I) is taken as

$$H = (i/\sqrt{2})\, J_\lambda\, j_\lambda \tag{1}$$

$$J_\lambda = \bar{\psi}_p (p_2, s_2)\, O_\lambda\, \psi_n (p_1, s_1) \tag{2}$$

$$O_\lambda = \gamma_\lambda (g_V + g_A \gamma_5) + iP_\lambda (f_V + f_A \gamma_5) + iq_\lambda (h_V + h_A \gamma_5) \tag{2a}$$

$$P = p_1 + p_2,\ q = p_2 - p_1 = k_1 - k_2 \tag{2b}$$

$g_{V,A}, f_{V,A}$ and $h_{V,A}$ are complex form factors and the functions of q^2. The leptonic current is given by

$$j_\lambda = \bar{\psi}_l (k_2, s_l)\, l_\lambda\, \psi_\nu(k_1, s_\nu) \tag{3}$$

$$l_\lambda = \gamma_\lambda (1 + \gamma_5). \tag{3a}$$

The nucleonic current (2) can be separated into first and second class parts of vector and axial vector currents ($V_{\mathrm{I,\,II}}, A_{\mathrm{I,\,II}}$) as

$$\begin{aligned} V_{\mathrm{I}} &= \bar{\psi}_p (g_V \gamma_\lambda + iP_\lambda f_V)\, \psi_n \\ V_{\mathrm{II}} &= \bar{\psi}_p (iq_\lambda h_V)\, \psi_n \\ A_{\mathrm{I}} &= \bar{\psi}_p (g_A \gamma_\lambda + iq_\lambda h_A)\, \gamma_5\, \psi_n \\ A_{\mathrm{II}} &= \bar{\psi}_p (iP_\lambda f_A \gamma_5)\, \psi_n. \end{aligned} \tag{4}$$

The notations $V_{\mathrm{I,\,II}}, A_{\mathrm{I,\,II}}$ will be used in the text to denote the corresponding form factors as

$$V_{\mathrm{I}} \equiv g_V, f_V,\ \ V_{\mathrm{II}} \equiv h_V,\ \ A_{\mathrm{I}} \equiv g_A, h_A,\ \ A_{\mathrm{II}} \equiv f_A. \tag{5}$$

The transition amplitude for (I) from eqs (1)–(3) then becomes

$$\begin{aligned} A_\nu &= \langle lp \,|\, H \,|\, \nu n\rangle = \langle p \,|\, J_\lambda \,|\, n\rangle\, \langle l \,|\, j_\lambda \,|\, \nu\rangle \\ &= (i/\sqrt{2})\, \bar{u}_p (p_2, s_2)\, O_\lambda\, u_n (p_1, s_1)\, \bar{u}_l (k_2, s_l)\, l_\lambda\, u_\nu (k_1, s_\nu). \end{aligned} \tag{6}$$

Under PT-transformation, $PT(q_\lambda)=q_\lambda$. However, under T or PT, $k_1 \longleftrightarrow k_2, p_1 \longleftrightarrow p_2$ and hence q_λ changes sign under PT. Similarly s_λ is PT-odd. According to Muirhead (1968) and Okun (1965) the scattering amplitude A_ν (6) transforms under PT and charge conjugation C respectively as

$$PT(A_\nu)=(i/\sqrt{2})\bar{u}_n(p_1,-s_1)O'_\lambda u_p(p_2,-s_2)\bar{u}_\nu(k_1,-s_\nu)l_\lambda' u_l(k_2,-s_l) \tag{7}$$

where

$$O'_\lambda = \gamma_\lambda(g_V-g_A\gamma_5)+iP_\lambda(f_V+f_A\gamma_5)-iq_\lambda(h_V+h_A\gamma_5) \tag{7a}$$

and

$$l'_\lambda = \gamma_\lambda(1-\gamma_5) \tag{7b}$$

$$C(A_\nu) = (i/\sqrt{2})\, v_n(p_1, s_1)\, O'_\lambda\, v_{\bar{p}}(p_2, s_2)\, \bar{v}_{\bar{\nu}}(k_1, s_\nu)\, l_\lambda'\, v_{\bar{l}}(k_2, s_l) \tag{8}$$

Combining the PT and C transformations (7) and (8) one gets the CPT transformation of A_ν as

$$CPT(A_\nu)=(i/\sqrt{2})v_{\bar{p}}(p_2,-s_2)O_\lambda v_{\bar{n}}(p_1,-s_1)v_{\bar{l}}(k_2,-s_l)l_\lambda v_{\bar{\nu}}(k_1,-s_\nu) \tag{9}$$

G-conjugation (Weinberg 1958) of the nucleonic part of (9) gives

$$G\,[\bar{v}_p(p_2,-s_2)\,O_\lambda\,v_n(p_1,-s_1)] = \bar{u}_n(p_2,-s_2)\,O'_\lambda\,u_p(p_1,-s_1) \tag{10}$$

i.e.

$$GV_{\text{I}}\,G^{-1}=V_{\text{I}},\quad GV_{\text{II}}\,G^{-1}=-V_{\text{II}}$$

$$GA_{\text{I}}\,G^{-1}=-A_{\text{I}},\quad GA_{\text{II}}\,G^{-1}=A_{\text{II}}. \tag{11}$$

The cross section is given by $|A_\nu|^2$ and it involves terms like k_1p_1, k_1s_1, s_1s_2, etc and the angle brackets $\langle p_1p_2k_1s_1\rangle = ip_{1\mu}\,p_{2\nu}\,k_{1\lambda}\,s_{1\rho}\,\epsilon_{\mu\nu\lambda\rho}$. $\langle p_1p_2s_1s_2\rangle$, etc. All the angle brackets are T-odd and the terms without them are T-even. The terms involving odd number of s_μ are PT-odd while those involving even number of them are PT-even. The P-parity nature of the terms follows from their behaviour under T and PT.

3. Earlier work on symmetries

3.1. *Lee and Yang symmetry*

Lee and Yang (1962) were the first to develop theorems for the functional form of the cross sections $d\sigma_\nu$ and $d\sigma_{\bar{\nu}}$ of (I) and (II) respectively by using the matrix elements given by eq. (6). They have given the expressions for cross sections in the laboratory system when the target nucleons are unpolarized and the final particles are longitudinally polarized (eq. 88 of Lee and Yang). These expressions can be summarised as follows;

$$d\sigma_\nu(s_1, s_l)=\alpha\,[A_1X+A_2X^{-1}+A_3Y+A_4Y^{-1}+A_5] \tag{12}$$

$$d\sigma_{\bar\nu}(s_2, s_{\bar l})=\alpha\,[B_1X+B_2X^{-1}+B_3Y+B_4Y^{-1}+B_5] \tag{13}$$

$$A_i\equiv A_i(s_1, s_l),\ B_i\equiv B_i(s_2, s_{\bar l}),\quad i=1, 2, 3, 4, 5. \tag{13a}$$

$\alpha=\frac{1}{2}(1+v_l)\,Kd(q^2)$ and s_1, s_2, s_l and $s_{\bar l}$ are the helicities of the final state particles n, p, l and $\bar l$ in (I) and (II) while v_l is the lepton velocity. The structure functions A_i and B_i are functions of f, g and h and can be related with the Lee and Yang functions $a_\pm$, $b_\pm$, c, $a'_\pm$, $b'_\pm$ and c' as

$$A_{1(3)}=a_+,\ A_{2(4)}=a_-,\ A_{3(1)}=b_+,\ A_{4(2)}=b_-,$$

$$A_5=+(-)c \text{ for } s_l=L(R) \tag{14}$$

$$A_i\equiv A_i\,(s_1, s_l, g_{V,A}, f_{V,A}, h_{V,A}) \text{ and}$$

$$a_\pm\equiv a_\pm(s_1, s_l)\,g_{V,A}, f_{V,A}, h_{V,A}), \text{ etc.} \tag{14a}$$

$$B_{1(3)}=a'_-,\ B_{2(4)}=a'_+,\ B_{3(1)}=b'_+,\ B_{4(2)}=b'_-$$

$$B_5=+(-)c' \text{ for } s_{\bar l}=R(L) \tag{15}$$

$$B_i\equiv B_i\,(s_2, s_{\bar l}),\ a'_\pm\equiv a'_\pm(s_2, s_{\bar l}), \text{ etc.} \tag{15a}$$

Lee and Yang relate the functions $a'_\pm$, $b'_\pm$ and c to the corresponding ones $a_\pm$, $b_\pm$ and c by the following (LY) transformation operator obtained from hermiticity (eq. 92 of them):

$$LY=[g_{V,A}\longrightarrow g^*_{V,A},\ f_V\longrightarrow f^*_V,\ h_V\longrightarrow -h^*_V,\ f_A\longrightarrow -f^*_A,\ h_A\longrightarrow +h^*_A] \tag{16}$$

Thus the Lee and Yang (LY) relations between a'_+ and a_+, etc. are

$$a'_\pm(s_2, s_l)=LY\,(a_\pm)=a_\pm\,(s_1, s_l, g^*_{V,A}, f^*_V, -h^*_V, -f^*_A, h^*_A) \tag{17}$$

etc.

By using eqs (14)–(17) the LY relations between A_i and B_i then become

$$B_{1(2)}=A'_{2(1)},\ B_j=A'_j,\ j=3, 4, 5 \text{ for } s_l=L,\ s_l=-s_{\bar l}=R \tag{18}$$

and

$$B_{3(4)}=A'_{4(3)},\ B_j=A'_j,\ j=1, 2, 5 \text{ for } s_l=R\ s_l=-s_{\bar l}=L \tag{19}$$

where

$$A'_i=LYA_i=A_i\,(s_1, s_l, g^*_{V,A}, f^*_V, -h^*_V, -f^*_A, h^*_A) \tag{20}$$

We note from (18) and (19) that the LY relations do not relate B_1 *directly* to A_1, B_2 to A_2 when $s_l = L$ and also B_3 to A_3 and B_4 to A_4 when $s_l = R$. The lack of *such a direct symmetry* between the respective coefficients of X, X^{-1}, Y and Y^{-1} in eqs (12) and (13) does not, therefore, allow one to obtain $d\sigma_{\bar{\nu}}$ by a mere application of LY (16) to $d\sigma_\nu$ (12) unless X and X^{-1} or Y and Y^{-1} are also interchanged as per the case of lepton polarization in it. When this is done, one has the following LY symmetry between $d\sigma_\nu$ and $d\sigma_{\bar{\nu}}$

$$d\sigma_{\bar{\nu}}(s_2, s_{\bar{l}}) = d\sigma_\nu(s_1, -s_l, g^*_{V,A}, f^*_V, -h^*_V, -f^*_A, h^*_A, X^{-1}, X, Y, Y^{-1})$$
$$\text{for } s_l = L,\ s_{\bar{l}} = -s_l = R, \text{ and} \qquad (21)$$

$$d\sigma_{\bar{\nu}}(s_2, s_{\bar{l}}) = d\sigma_\nu(s_1, -s_l, g^*_{V,A}, f^*_V, -h^*_V, -f^*_A, h^*_A, X, X^{-1}, Y^{-1}, Y)$$
$$\text{for } s_l = R,\ s_{\bar{l}} = -s_l = L. \qquad (21a)$$

3.2. *Adler symmetry*

Starting with the matrix elements A_ν(6), Adler (1963) evaluated covariant expression for $d\sigma_\nu$ for the production of polarized nucleons and leptons. For obtaining $d\sigma_{\bar{\nu}}$, he changed the sign of γ_5 in both the leptonic and nucleonic parts of (6). The result of this mechanism, as Adler obtains, is that $d\sigma_\nu$ and $d\sigma_{\bar{\nu}}$ differ only in the signs of particle polarization four vectors and in the signs of T-odd (angle bracket) terms. Separating $d\sigma_{\nu,\bar{\nu}}$ into T-even ($d\sigma^+$) and T-odd ($d\sigma^-$) parts as

$$d\sigma_{\nu,\bar{\nu}} = d\sigma^+_{\nu,\bar{\nu}} + d\sigma^-_{\nu,\bar{\nu}}. \qquad (22)$$

Adler's symmetry can be written as

$$d\sigma^{\pm}_{\bar{\nu}} = \pm\, d\sigma^{\pm}_{\bar{\nu}}(-s_1, -s_l, g_{V,A}, {}_{V,A}, h_{V,A}) \qquad (23)$$

where the upper sign is for $d\sigma^+$ and the lower one is for $d\sigma^-$ and where s_1 and s_l are respectively the polarization four vectors of p and l in (I).

Because of the assumed equality of the form factors in the matrix elements of (I) and (II) by Adler, eq. (23) gives equal polarization independent cross sections in disagreement with the cross sections of Marshak *et al* (1969).* It then appears that the Adler symmetry (23) is not complete.

3.3. *Sarkar's symmetry*

Sarkar (1966) following the method of Adler considered the process (I) to include target polarization s_2. He, however, neglected the lepton mass and so the form factors $h_{V,A}$ are absent in his expressions. Sarkar's symmetry is, therefore the following

$$d\sigma^{\pm}_{\bar{\nu}} = \pm\, d\sigma^{\pm}_{\nu}(-s_1, -s_2, g_{V,A}, f_{V,A}). \qquad (24)$$

By splitting the cross section into functions of particle momenta and form factors

*Here replacements of h_A by $-f_A$ and of f_A by h_A is to be made in order to agree with our notations. Also compare the results with those given by Smith (1972), his eqs (3.18) and (3.22).

which are independent of particle polarization, A, dependent on s_1 and s_2 separately, $B(s_1)$ and $C(s_2)$ and simultaneously $D(s_1, s_2)$, Sarkar wrote

$$d\sigma_\nu = A + B(s_1) + C(s_2) + D(s_1, s_2) \tag{25}$$

$$A \equiv A(k_1, k_2, p_1, p_2, g_{V,A}, f_{V,A}), \text{ etc.} \tag{26}$$

In terms of the following transformations

$$g_A \to -g_A, \; p_1 \longleftrightarrow p_2, \; M_1 \longleftrightarrow M_2, \; s_1 \to -s_2 \tag{27}$$

Sarkar related $C(s_2)$ with $B(s_1)$ in the same process (I) or (II) as

$$C(s_2) = B(-s_2, k_1, k_2, p_2, p_1, M_2, M_1, g_V, -g_A, f_{V,A}). \tag{28}$$

Eq. (27) will be regarded as Sarkar's internal symmetry operation.

3.4. *Pais symmetry*

The matrix elements (6) used by Pais (1971, 1972) for (I) and (II) differ in the sign of γ_5 only in the leptonic part. As a result, the expressions of $d\sigma_\nu$ and $d\sigma_{\bar\nu}$ differ from each other in the signs of the polarization of the considered final particles and in the sign of the T-odd terms and in addition, all the axial vector form factors have opposite signs. Thus the Pais symmetry is

$$d\sigma_{\bar\nu}^{\pm} = \pm d\sigma_\nu^{\pm}(P, n, q, -s_1, -s_2, g_V, f_V, h_V, -g_A, -f_A, -h_A) \tag{29}$$

$$P = p_1 + p_2, \; n = k_1 + k_2, \; q = k_1 - k_2 = p_2 - p_1. \tag{29a}$$

The Pais symmetry gives correct signs of g_A and h_A but not of f_A when compared with the polarization independent expressions of $d\sigma_{\bar\nu}$ given by Marshak *et al* (1969) where h_V changes sign while f_A (i.e. h_A of Marshak *et al*) does not and hence it is also in error.

3.5. *Wolfenstein's theorems*

Wolfenstein (1972) derived two theorems for relating $d\sigma_{\bar\nu}$ with $d\sigma_\nu$ and one theorem for relating a process to itself. The theorems are in terms of ten bilinear terms a_a obtained from $V_{\mathrm{I,II}}$ and $A_{\mathrm{I,II}}$ which are classified in the following four groups.

$$\underline{A_{\mathrm{I}}A_{\mathrm{I}}}: A_{\mathrm{I}}A_{\mathrm{I}}, V_{\mathrm{I}}V_{\mathrm{I}}, A_{\mathrm{II}}A_{\mathrm{II}}, V_{\mathrm{II}}V_{\mathrm{II}}, \; \underline{A_{\mathrm{I}}A_{\mathrm{II}}}: A_{\mathrm{I}}A_{\mathrm{II}}, V_{\mathrm{I}}V_{\mathrm{II}},$$

$$\underline{V_{\mathrm{I}}A_{\mathrm{I}}}: V_{\mathrm{I}}A_{\mathrm{I}}, V_{\mathrm{II}}A_{\mathrm{II}}, \; \underline{V_{\mathrm{I}}A_{\mathrm{II}}}: V_{\mathrm{I}}A_{\mathrm{II}}, A_{\mathrm{I}}V_{\mathrm{II}}. \tag{30}$$

By applying hermiticity, lepton crossing and simultaneous exchange of initial and final momenta of the particles in (I), Wolfenstein obtains theorem 1

Theorem 1: In going from process (I) to the (II), observables which are P-even and exchange even (odd) and P-odd and exchange odd (even) satisfy

$$a_a^{\nu} = \pm a_a^{\bar{\nu}} \text{ for } \underline{A_{\text{I}} A_{\text{I}}} \text{ and } \underline{A_{\text{I}} A_{\text{II}}}$$

$$= \mp a_a^{\bar{\nu}} \text{ for } \underline{V_{\text{I}} A_{\text{I}}} \text{ and } \underline{V_{\text{I}} A_{\text{II}}}. \tag{31}$$

By applying C-conjugation to (I) and G-conjugation to the nucleon current he obtains theorem 2

Theorem 2: In going from process (I) to the (II) observables which are PT-even (odd) must satisfy

$$a_a^{\nu} = \pm a_a^{\bar{\nu}} \text{ for } \underline{A_{\text{I}} A_{\text{I}}} \text{ and } \underline{V_{\text{I}} A_{\text{II}}}$$

$$= \mp a_a^{\bar{\nu}} \text{ for } \underline{V_{\text{I}} A_{\text{I}}} \text{ and } \underline{A_{\text{I}} A_{\text{II}}}. \tag{32}$$

Adding theorems 1 and 2 he gets theorem 3 relating a process to itself.

Theorem 3: For either reaction (I) or (II), observables which are T-even (odd) and exchange even (odd) must satisfy

$$a_a = 0 \text{ for } \underline{A_{\text{I}} A_{\text{II}}} \text{ and } \underline{V_{\text{I}} A_{\text{II}}} \tag{33}$$

whereas for observables which are T-even (odd) and exchange odd (even)

$$a_a = 0 \text{ for } \underline{A_{\text{I}} A_{\text{I}}} \text{ and } \underline{V_{\text{I}} A_{\text{I}}} \tag{34}$$

Wolfenstein's theorems, though general, contain the following weak points:

(i) No symmetry is implied between scattering amplitudes of (I) and (II).
(ii) Lack of maximum symmetry with minimum factors thus entailing lengthy and difficult statements.
(iii) Their execution is tedious.

3.6. *Bell symmetry*

A symmetry following from hermiticity and CPT invariance of spin averaged lepton tensors in (I) has been obtained between the unpolarized cross sections $d\sigma_{\nu}$ and $d\sigma_{\bar{\nu}}$ in terms of Mandelstam variables s, t, u (Bell 1963, Smith 1972). This is stated as

$$d\sigma_{\bar{\nu}}\,(s, t, u) = d\sigma_{\nu}\,(u, t, s) \tag{35}$$

and shows that while going from (I) to (II) s and u exchange and hence $(s\text{-}u)$ changes sign.

The Bell symmetry (35) has been extended (Teli 1974) for (I) and (II) involving nucleons of unequal masses and polarization of all the particles by using the fact that under *CPT*, the polarization of four vector changes sign and hence the exchange of s and u should be accompanied with the exchange of nucleon masses M_1 and M_2 and their polarizations with sign change. The Bell symmetry generalized in this way then is the following

$$d\sigma_{\bar{\nu}}\,(k_1, k_2, p_1, p_2, M_1, M_2, s_l, s_1, s_2)$$

$$= d\sigma_{\nu}\,(k_1, k_2, -p_2, -p_1, M_2, M_1, -s_l, -s_2, -s_1) \quad (36)$$

or

$$d\sigma_{\bar{\nu}}\,(s, t, u, M_1, M_2, s_l, s_1, s_2)$$

$$= d\sigma_{\nu}\,(u, t, s, M_2, M_1, -s_l, s_2, -s_1). \quad (37)$$

The six types of symmetries between $d\sigma_\nu$ and $d\sigma_{\bar{\nu}}$ surveyed above are completely different from each other with the exceptions of the equivalence between the Adler and Sarkar symmetries (23) and (24) and a partial similarity between the Adler and Pais symmetries (23) and (29), the latter differing from the former two only in the sign of axial vector form factors. However, all the three are in error. The Lee and Yang symmetries (21) and (21a) are obtained under special circumstances viz. by considering (I) and (II) to produce only longitudinally polarized leptons and nucleons in laboratory system of the target. The generalized Bell symmetry (36) or (37) is quite simpler than Wolfenstein's theorems which require knowledge of the transformation properties of the observables under *P*, *PT* and exchange of initial and final momenta.

4. Further symmetries

We now wish to establish an identity between the amplitudes A_ν and $A_{\bar{\nu}}$ of (I) and (II) respectively and a symmetry between their cross sections based on the *CPT* and *G*-conjugation properties of the particle current matrix elements by neglecting final state electromagnetic corrections.

4.1. *An amplitude identity*

Statement: If A_ν is given by eq. (6) as

$$A_\nu = (i/\sqrt{2})\,\bar{u}_p\,(p_2, s_2)\,O_\lambda\,u_n\,(p_1, s_1)\,\bar{u}_l\,(k_2, s_l)\,l_\lambda\,u_\nu\,(k_1, s_\nu) \quad (38)$$

$$= A_\nu\,(k_1, k_2, p_1, p_2, s_\nu, s_l, s_1, s_2, V_{\rm I,II}, A_{\rm I,II}) \quad (39)$$

then

$$A_{\bar{\nu}} = (i/\sqrt{2})\,\bar{u}_n\,(p_2, -s_2)\,O'_\lambda u_p\,(p_1, -s_1)\,\bar{v}_l(k_2, s_l)\,l_\lambda\,v_{\bar{\nu}}\,(k_1, -s_\nu) \quad (40)$$

$$= A_{\bar{\nu}}\,(k_1, k_2, p_1, p_2, -s_\nu, -s_l, -s_1, -s_2, V_{\rm I}, -V_{\rm II}, -A_{\rm I}, A_{\rm II}) \quad (41)$$

with O'_λ as given by eq. (7a).

On squaring the amplitude $A_{\bar{\nu}}(41)$ we get the following symmetry between the cross sections

$$d\sigma_{\bar{\nu}}\pm = d\sigma_{\nu}\pm(k_1, k_2, p_1, p_2, M_1, M_2, -s_{\nu}, -s_l, -s_1, -s_2, V_{\mathrm{I}}, -V_{\mathrm{II}}, -A_{\mathrm{I}}, A_{\mathrm{II}} \quad (42)$$

$$= d\sigma_{\nu}\pm(s, t, u, M_1, M_2, -s_{\nu}, -s_l, -s_1, -s_2, V_{\mathrm{I}}, -V_{\mathrm{II}}, -A_{\mathrm{I}}, A_{\mathrm{II}}). \quad (42a)$$

It is clear from (41) and (42) that the symmetry between the amplitudes is carried over to the cross sections. This coherence between the amplitude identity and the cross sectional symmetry is not present in the symmetries discussed in section 3.

The proof of (40) and hence of (41) directly follows from the combined operation of *CPT* (9) and the *G*-conjugation (10) on $A_{\nu}(6)$.

4.2. *Internal symmetry*

Let us denote by S_1 the generalized Bell symmetry operation which gives (36) when operated on $d\sigma_{\nu}$. The operator S_1 is then

$$S_1 = [p_1 \longrightarrow -p_2, p_2 \longrightarrow -p_1, M_1 \longleftrightarrow M_2, s_{\nu} \longrightarrow -s_{\nu},$$
$$s_l \longrightarrow -s_l, s_1 \longrightarrow -s_2, s_2 \longrightarrow -s_1] \; . \quad (43)$$

Similarly the operator for the symmetry (42) is

$$S_2 = s_j \longrightarrow -s_j, V_{\mathrm{II}} \longrightarrow -V_{\mathrm{II}}, A_{\mathrm{I}} \longrightarrow -A_{\mathrm{I}} \quad (44)$$

with $s_j = s_{\nu}, s_l, s_1, s_2$.

Now if we apply S_1 to $d\sigma_{\nu}$ we get $d\sigma_{\bar{\nu}}$ and when the latter is further subjected to S_2 we get back $d\sigma_{\nu}$. This is also true for obtaining $d\sigma_{\bar{\nu}}$ from itself by the combined operation of S_1 and S_2. The combined operation which relates a cross section to itself is thus

$$S = S_1 S_2 = [p_1 \longrightarrow -p_2, p_2 \longrightarrow -p_1, M_1 \longleftrightarrow M_2, s_1 \longleftrightarrow s_2,$$
$$V_{\mathrm{II}} \longrightarrow -V_{\mathrm{II}}, A_{\mathrm{I}} \longrightarrow -A_{\mathrm{I}}]. \quad (45)$$

We then have the following internal symmetry.

$$S d\sigma^{\pm}_{\nu,\bar{\nu}} = d\sigma^{\pm}_{\nu,\bar{\nu}} \quad (46)$$

or

$$d\sigma^{\pm}_{\nu,\bar{\nu}} + (k_1, k_2, p_1, p_2, M_1, M_2, s_{\nu}, s_l, s_1, s_2, V_{\mathrm{I,II}}, A_{\mathrm{I,II}})$$

$$= d\sigma^{\pm}_{\nu,\bar{\nu}} + (k_1, k_2, -p_2, -p_1, M_2, M_1, s_{\nu}, s_l, s_2, s_1, V_{\mathrm{I}}, -V_{\mathrm{II}}, -A_{\mathrm{I}}, A_{\mathrm{II}}). \quad (47)$$

The internal symmetry operator S (45) relates the term dependent on s_1 [i.e. $B(s_1)$ in eq. (25)] to that dependent on s_2 [i.e. $C(s_2)$] and hence it gives the advantage of evaluating say, $C(s_2)$ from $B(s_1)$ and vice versa.

Thus

$$C(s_2, k_1, k_2, p_1, p_2, M_1, M_2, s_\nu, s_l, V_{\mathrm{I,II}}, A_{\mathrm{I,II}})$$
$$= B(s_2, k_1, k_2, -p_2, -p_1, M_2, M_1, s_\nu, s_l, V_{\mathrm{I}}, -V_{\mathrm{II}}, -A_{\mathrm{I}}, A_{\mathrm{II}}). \quad (48)$$

This internal symmetry (48) is equivalent to that used by Sarkar, eq. (28) since the functions B and C and also A and D in (26) involve the terms like $k_1 p_1 \cdot k_2 p_1$, $p_1 p_2 \cdot k_1 s_1$, etc. which transform to the same respective terms under the operator S (45) as well as under the transformation (27). This shows that the transformation (45) is exactly equivalent to that given by eq. (27). From the internal symmetry we obtain the following theorem.

Theorem: Irrespective of whether T-invariance holds or not, for both the processes (I) and (II) observables which are even under the operation S (45) must satisfy

$$b_a = 0 \text{ for } \underline{A_{\mathrm{I}} A_{\mathrm{II}}},\ \underline{V_{\mathrm{I}} A_{\mathrm{I}}} \quad (49)$$

while for those odd under S

$$d_a = 0 \text{ for } \underline{A^{\mathrm{I}} A_{\mathrm{I}}},\ \underline{V_{\mathrm{I}} A_{\mathrm{II}}} \quad (49a)$$

Proof: Let us write $d\sigma_{\nu,\bar{\nu}}$ (as Wolfenstein 1972) (suffixes ν, $\bar{\nu}$ dropped)

$$d\sigma^{\pm} = \sum_a \left(d_a f_{1a}^{\pm} + b_a f_{2a}^{\pm}\right) \quad (50)$$

$$d_a = \underline{A_{\mathrm{I}} A_{\mathrm{I}}},\ \underline{V_{\mathrm{I}} A_{\mathrm{II}}},\ b_a = \underline{A_{\mathrm{I}} A_{\mathrm{II}}},\ \underline{V_{\mathrm{I}} A_{\mathrm{I}}} \quad (51)$$

$$f_a^{\pm} = f_a^{\pm}(k_1, k_2, p_1, p_2, M_1, M_2, s_j). \quad (51a)$$

Under S_2, $d_a \longrightarrow d_a$ and $b_a \longrightarrow -b_a$ while

$$s_1 f_{ia}^{\pm} = f_{ia}^{\prime\pm} = f_{ia}^{\pm}(k_1, k_2, -p_2, -p_1, M_2, M_1, -s_\nu, -s_l, -s_2, -s_1) \quad (52)$$

Hence under S, eq. (47) becomes

$$\sum_a \left(d_a f_{1a}^{\pm} + b_a f_{2a}^{\pm}\right) = \sum_a \left(d_a f_{1a}^{\prime\pm} - b_a f_{2a}^{\prime\pm}\right)$$

i.e.

$$\sum_a \left[d_a \left(f_{1a}^{\pm} - f_{1a}^{\prime\pm}\right) + b_a \left(f_{2a}^{\pm} + f_{2a}^{\prime\pm}\right)\right] = 0. \quad (53)$$

Now from (52) we have

$$S^2 f_{ia}^{\pm} = S f_{ia}^{\prime\pm} = f_{ia}^{\pm}, \quad i = 1, 2. \quad (54)$$

Hence S has eigenvalues $\lambda = \pm 1$ so that

$$Sf^{\pm}_{ia} = f'^{\pm}_{ia} = \lambda f^{\pm}_{ia} \tag{55}$$

and hence $f^{\pm}_{ia}$ is either even ($\lambda = +1$) or odd ($\lambda = -1$) under S (47). Eqs (53) and (55) then give

$$\sum_{a} \left[(1-\lambda)\, d_a\, f^{\pm}_{1a} + b_a f^{\pm}_{2a} (1+\lambda) \right] = 0. \tag{56}$$

The theorem then follows from (56).

5. Comparison with earlier symmetries

We now show that the LY symmetries (eqs (18) and (19)) and the S_2 symmetry (42) or (44) give identical results (eqs (59) and (59a) below). S_2 gives the following symmetry between the Lee and Yang functions A_i and B_i when applied to (12) to obtain (13).

$$B_i(s_2, s_f) = S_2 A_i = A_i\,(-s_1, -s_f, g_V, f_V, -h_V, -g_A, f_A, h_A). \tag{57}$$

Consider the case of $(-s_i) = L$ and $s_f = R$ and take $B_i = B_1$. Then the combination of eqs (14), (15), (18) and (19) gives eq. (58) below while eqs (14), (15) and (57) give eq. (58a) below.

$$B_1 = a'_- = LY(A_2) = LY(a_-) = a_-(s_1, -s_i, g^*_{V,A}, -h^*_V, -f^*_A, h^*_A) \tag{58}$$

$$B_1 = S_2 A_1 = S_2 a_+ = a_+(-s_1, -s_i, g_V, -g_A, f_{V,A}, -h_{V,A}). \tag{58a}$$

Eqs (58) and (58a) then give

$$\begin{aligned} &a_-(s_1, L, g^*_V, f^*_V, -h^*_V, -f^*_A, g^*_A, h^*_A) \\ &\quad = a_+(-s_1, L, g_V, f_V, -h_V, -g_A, f_A, -h_A) \end{aligned} \tag{59}$$

Similar relations for $b_\pm$ and c are

$$\begin{aligned} &b_j\,(s_1, L, g^*_V, f^*_V, -h^*_V, g^*_A, -f^*_A, h^{*A}) \\ &= b_j\,(-s_1, L, g_V, f_V, -h_V, -g_A, f_A, -h_A) \end{aligned} \tag{59a}$$

$b_j = b_+, b_-$ and c.

Equations (59)–(59a) can be easily verified by using the Lee and Yang expressions (their eqs (90) and (91)) of $a_\pm$, $b_\pm$ and c.

Wolfenstein's theorem 2 also follows from S_2 and hence from the amplitude identity (41).

Let $r = (r_1, r_2)$ where

$$r_1 = (l, s_1 s_2, s_1 p_j, s_2 p_k, \langle p_1 p_2 s_j s_k \rangle, \ldots)$$

and

$$r_2 = (s_1 p_j, s_2 p_j, \langle p_1 p_l p_k s_j \rangle, \ldots)$$

be the rows respectively of the PT-even and PT-odd elements. Let d_a and b_a (51) form a column. Taking A as the matrix of elements which are functions of Mandelstam variables s, t, u and of particle masses only, we write $d\sigma_\nu$ as

$$d\sigma_\nu = (r_1, r_2)\left[\frac{A_1 \mid A_2}{A_3 \mid A_4}\right]\begin{bmatrix} d_a \\ b_a \end{bmatrix}. \tag{60}$$

Then applying the symmetry operator S_2 to (60) we obtain, since A does not change,

$$d\sigma_{\bar{\nu}} = (r_1, -r_2)\left[\frac{A_1 \mid A_2}{A_3 \mid A_4}\right]\begin{bmatrix} d_a \\ -b_a \end{bmatrix}. \tag{61}$$

and this is Wolfenstein's theorem 2.

6. Conclusions

From the foregoing discussion it is concluded that the symmetry S_2 developed in section 4 is quite general and simpler in its form as compared with Wolfenstein's theorems and it gives results (eqs 48, 59a and 61) consistent with those obtained by earlier workers thereby strengthening the validity of the amplitude identity proposed in eqs (40) and (41). The symmetry S_2 when combined with the generalized Bell symmetry (36) gives a theorem on the contribution of form factors to the cross sections. The symmetry S_2 is complete improvement over the symmetries of Adler, Sarkar and Pais. The generalized Bell symmetry S_1 (36) exchanges nucleon polarization while S_2 does not. As a result (i) S_2 gives cross section $d\sigma_{\bar{\nu}}$ from $d\sigma_\nu$ for (II) with polarized nucleons in the same states as those in (I) while the generalized Bell symmetry gives $d\sigma_\nu$ for polarized nucleons in the states opposite to those in (I). For example, if (I) involves unpolarized neutron targets and polarized final state protons then S_2 gives $d\sigma_{\bar{\nu}}$ for (II) involving unpolarized proton targets and polarized final state neutrons while S_1 (36) gives $d\sigma_{\bar{\nu}}$ for (II) involving polarized proton targets and unpolarized final state neutrons. S_1 and S_2, therefore, do not give identical results. (ii) If both the initial and final state nucleons are simultaneously polarized then S_1 and S_2 give identical results.

In view of the accuracy of the symmetry S_2, it follows that in Adler's (and also in Sarkar's) equations of $d\sigma_{\bar{\nu}}$ the signs of the form factors h_V, g_A and h_A should be changed along with the sign change of all the T-odd (angle bracket) terms. Similarly, the equations of $d\sigma\bar{\nu}$ of Pais (1971) require sign change in all the T-odd terms and of h_V and f_A, the sign change of f_A required to correct the minus sign used by Pais. As a result the terms w_5 and w_6 of Pais (his eq. 2.24) should change sign.

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