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Detection of direction of motion and zero order fringe identification in holographic displacement measurement

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Abstract. Two methods have been suggested to detect the direction (forward or backward) of uniform velocity motion along with identification of no motion positions with other intensity maxima positions in the reconstruction. The first method requires two continuous exposures while the other is a combination of one static and one continuous recording.

Keywords. Holography; interferometry; displacement; direction detection; fringe counting.

1. Introduction

Considerable interest is being paid in holographic displacement measurement to detect the direction (positive or negative) of the displacement and to identify no motion fringes. The work of Metherell *et al* (1969a, 1969b, 1970) although intended for detection of very fine motion, it is capable of the detection of the direction as well. They did so by shifting the fringes in the reconstruction using suitable modulation to the reference or the object beam during the recording. Gori and Guattari (1972) suggested differentiation between forward and backward displacements by employing different subject illumination directions in the two exposures while recording the double exposure hologram. Kopf (1973) suggested giving three exposures to the recording medium, one at the beginning, the other while the transformation is taking place (assumed to be with uniform velocity), and the last after the deformation has taken place. This results in a fringe pattern where the brightest zero order fringe corresponding to no motion positions is identified with other maxima. In fact, the identification of the zero order velocity fringe is no problem even with the usual time-average method where we get sinc^2 distribution in the reconstruction (Redman 1967) or with multiple exposure arrangements (Johnson and Holshouser 1972 Pearce 1973; Decker 1975; Miler 1975; Mallick and Malacara 1976). In the method due to Gupta and Aggarwal (1976) which is basically a modification of the Kopf's method, the direction detection and the fringe order determination can be done simultaneously.

The methods reported by Kopf (1973) and Gupta and Aggarwal (1976) assume that the transformation is taking place with uniform velocity and the third exposure is given when the object has stopped suddenly. However, this is rarely possible in actual practice. Employing high intensity pulsed lasers to arrest a given subject position would simultaneously reduce the contribution due to sinc term seriously in the corresponding intensity distribution (Kopf 1973; Gupta and Aggarwal 1976) and hence losing the information regarding the direction and zero order fringe position.

In this connection, we suggest in this paper a two-exposure (while the object is moving) method where static position is not required. Another method is proposed so that the difficulty of rapid decrease in the intensity of fringes against increasing order can be overcome.

2. Two continuous exposures

Suppose that the total phase change of the object beam due to a point in the usual recording time τ is ϕ. Assuming the deformation with uniform velocity, suppose we record a hologram between times 0 and $\rho\tau$ $(0 < \rho < 1)$ and the other on the same medium between the times $T + \rho\tau$ and $\tau + T$ but with increased phase of the object beam by $\pi/2$ (this can be decreased also to get the same result except unimportant change in sign). The normalized reconstructed intensity is

$$I = \left| \frac{1}{[\rho + i(1-\rho)]\,\tau} \left[\int_{0}^{\rho\tau} \exp(i\phi t/\tau)\, dt + \int_{\rho\tau+T}^{\tau+T} \exp(i\phi t/\tau) \exp(\pi i/2)\, dt \right] \right|^2$$

$$= \{\rho^2 \operatorname{sinc}^2(\tfrac{1}{2}\rho\phi) + (1-\rho)^2 \operatorname{sinc}^2[\tfrac{1}{2}(1-\rho)\phi] - 2\rho(1-\rho)\operatorname{sinc}(\tfrac{1}{2}\rho\phi)$$

$$\operatorname{sinc}[\tfrac{1}{2}(1-\rho)\phi] \sin[\tfrac{1}{2}(1 + T/\tau)\phi]\} / [\rho^2 + (1-\rho)^2], \tag{1}$$

where $\operatorname{sinc}(x) = \sin(x)/x$.

The expression (1), in general identifies no motion positions $(\phi = 0)$ with other maxima.

Secondly, positive and negative ϕ give different I due to different signs in the last term of rhs of eq. (1). Thus, positive or negative direction of the displacement can be detected. Let us consider one specific case with no gap between the two exposures $(T = 0)$ and equal exposures during the two recordings $(\rho = 0{\cdot}5)$. The expression (1) thus becomes

$$I = [2^{\frac{1}{2}} \sin(\tfrac{1}{2}\phi + \pi/4) - 1]^2 / (\tfrac{1}{2}\phi)^2. \tag{2}$$

Near no motion positions $(\phi \sim 0)$, the above can be written as

$$I \simeq 1 - \phi. \tag{3}$$

Equation (3) concludes that if we move from no motion position the intensity decreases if ϕ is positive, and it increases if ϕ is negative. This is useful to establish about the direction (forward or backward) of the motion by visual observation of the reconstructed fringes. After a few lower orders the maxima and the minima of the distribution given by eq. (2) correspond to $\sin(\tfrac{1}{2}\phi + \pi/4) = -1$ and $2^{-\frac{1}{2}}$ respectively.

3. One static and one continuous exposure

In the previous section all the terms in the intensity distribution described by (1) are either sinc^2 functions or products of two sinc functions, so that the intensity falls rapidly against increasing order. This makes the upper measurement range very limited because the intensity after a few fringes becomes either nearly zero or comparable to the noise intensity. To overcome this difficulty, let us give the ρ-th portion of the total exposure while the object is static and the remaining $(1-\rho)$-th portion while the deformation is taking place between times 0 and $(1-\rho)\tau$ but with increased phase of the object beam by $\pi/2$. The normalized reconstructed irradiance in this case becomes

$$I = \left| \frac{1}{[\rho + i(1-\rho)\tau]} \left[\rho\tau + \exp(\pi i/2) \int_0^{(1-\rho)\tau} \exp(i\phi t/\tau)\, dt \right] \right|^2$$

$$= \{\rho^2 + (1-\rho)^2 (1-\rho\phi)\, \text{sinc}^2 [\tfrac{1}{2}(1-\rho)\phi]\} / [\rho^2 + (1-\rho)^2]. \tag{4}$$

Here also, the intensity near $\phi = 0$ is

$$I \simeq \frac{\rho^2 + (1-\rho)^2 (1-\rho\phi)}{\rho^2 + (1-\rho)^2} \tag{5}$$

that gives decrease or increase in the intensity if we move from the static region depending on ϕ is positive or negative respectively.

The intensity distribution given by (4) differentiates to motion positions with other maxima, sign of ϕ can be detected, and also the intensity fall is not very rapid due to a constant term $\rho^2/[\rho^2 + (1-\rho)^2]$ in rhs of (4). It is also evident from (4) that the fringe spacing is increased to $1/(1-\rho)$ times the original spacing at $\rho = 0$. This will help to reduce the large number of fringes expected in case of large deformations.

3.1. *Maximum allowable* ρ

As the value of ρ increases, the constant intensity $\rho^2/[\rho^2 + (1-\rho)^2]$ in rhs of (4) increases. However, it is likely to reduce the contrast of the fringes. Here we discuss maximum allowable ρ from this point of view. Equation (4) can be rewritten as

$$I = \frac{1}{\rho^2 + (1-\rho)^2} \left\{ \rho^2 + \frac{4(1-\rho\phi)}{\phi^2} \sin^2 [\tfrac{1}{2}(1-\rho)\phi] \right\}. \tag{6}$$

As we are determining maximum ρ and then, after a few lower order fringes

$$I \simeq \frac{1}{\rho^2 + (1-\rho)^2} \left\{ \rho^2 - \frac{4\rho}{\phi} \sin^2 [\tfrac{1}{2}(1-\rho)\phi] \right\}. \tag{7}$$

Considering positive ϕ, the maxima and minima of (7) in a given region are observed when $\sin^2[\frac{1}{2}(1-\rho)\phi]$ is zero and unity respectively. This means the visibility v is

$$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2}{\rho\phi - 2} \simeq \frac{2}{\rho\phi}. \tag{8}$$

Thus, the visibility decreases when ρ or the displacement increases. Considering the usually acceptable contrast limit for human eye as 0·02 (This assumes zero speckle noise or a very large signal to noise ratio. Anyway, for considerable amount of the speckle noise this limit increases), (8) becomes

$$[\rho\phi]_{max} = 100. \tag{9}$$

With a typical $\rho = 0{\cdot}5$, (9) gives $\phi_{max} = 200$ that roughly corresponds to the 30th order fringe of the usual time-average holography ($\rho = 0$). Reducing ρ further, ϕ_{max} can further be increased.

A little algebra shows that the findings of this section are valid for negative ϕ as well by just replacing ϕ in (9) by $|\phi|$.

3.2. *Intensity difference between positive and negative displacement cases*

The envelope of the intensity given by (4) is

$$E = \frac{1}{\rho^2 + (1-\rho)^2}\left\{\rho^2 + (1-\rho)^2(1-\rho\phi)\frac{4}{[(1-\rho)\phi]^2}\right\}$$

$$= \frac{1}{\rho^2 + (1-\rho)^2}\left[\rho^2 + \frac{4(1-\rho\phi)}{\phi^2}\right]. \tag{10}$$

For a given $|\phi|$, the change in the envelope between positive ϕ and negative ϕ cases is

$$\triangle E = \frac{8\rho}{|\phi|}\cdot\frac{1}{\rho^2 + (1-\rho)^2}. \tag{11}$$

The lower envelope (corresponding to positive ϕ) is

$$E_{lower} = \frac{1}{\rho^2 + (1-\rho)^2}\left[\rho^2 + \frac{4(1-\rho|\phi|)}{\phi^2}\right]$$

$$= \frac{1}{\rho^2 + (1-\rho)^2}\left(\frac{2-\rho|\phi|}{\phi}\right)^2. \tag{12}$$

Equations (11) and (12) give per cent change in the envelope factor as

$$\frac{100\triangle E}{E_{lower}} = \frac{800\,\rho|\phi|}{(2-\rho|\phi|)^2}. \tag{13}$$

The particular situation of $\rho\,|\phi| = 2$ gives infinite increase only because at this, the lower value of the envelope factor is zero. For a typical $\rho = 0{\cdot}5$ and the fifth bright fringe (labelling $\phi = 0$ as the zeroth order), where $|\phi|$ is 22π the change comes out to be about 26%, concluding that sufficient change in the intensity is possible to differentiate between the two cases. Furthermore, for the higher order fringes and considering ρ not very small, we can assume $\rho\,|\phi| \gg 2$ and in this limiting situation

$$\frac{\triangle E}{E_{\text{lower}}} \simeq \frac{8}{\rho\,|\phi|}, \tag{14}$$

concluding that the envelopes corresponding to positive and negative ϕ tend to be the same as the order increases.

4. Conclusion

We have seen in section 2 that two continuous exposures can give information about the direction of the motion, and also the zero order fringe can be identified. This eliminates the requirement of sudden stopping of the object for the third exposure in the method due to Gupta and Aggarwal (1976). The combination of one static and one continuous exposure results in a controllable fall in the reconstructed irradiance so that the higher order fringes are visible and the total number of fringes is reduced. This will help to extend the upper measurement range.

It is worth pointing here that the phase change of $\pi/2$ reported in this article as well as in the paper by Gupta and Aggarwal (1976) is not necessary. Any other angle which is not an integral multiple of π can do the job—although with changed calculations.

Finally, it should be noted that for objects moving with uniform velocity, the suggested method seems better than that due to Metherell *et al* (1970). This is because it is much easier to introduce a fixed phase to one of the beams rather than modulating the phase according to the law of the motion. If our method is applied as such in the double exposure manner (no object motion during each recording), it becomes that due to Metherell *et al* (1969a).

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