Pramāṇa, Vol. 9, No. 3, September 1977, pp. 257–262, © Printed in India.

$N\bar{N}$ resonance and the corrections to the Goldberger-Treiman relation

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MS received 7 May 1977

Abstract. The relevance of the recent experimental observation of possible bound and resonant states in $N\bar{N}$ scattering to the Goldberger-Treiman (GT) relation is examined. It is pointed out that an S-wave resonance in $N\bar{N}$ scattering goes a long way towards accounting for the corrections to the GT relations. Values of the mass and width of the resonance capable of giving a reasonable fit for the GT relation are presented.

Keywords. Dispersion relation; Goldberger-Treiman relation; $N\bar{N}$ resonant intermediate state; elastic unitarity.

1. Introduction

Recent experimental observations on $\bar{p}d$ interactions at 5·55 GeV/c by Braun *et al* (1976) show evidence for the existence of an $N\bar{N}$ resonance at 2·85 GeV with a width $\Gamma = 39$ MeV and probably another one at 3·05 GeV. The existence of such $N\bar{N}$ states (both bound as well as resonant) had been predicted earlier by Kalogeropoulos (1974a) and Shapiro (1972; see also Bogdanova *et al* 1974) and an experimental observation by Kalogeropoulos (1974b, 1975) further supports the existence of two such $N\bar{N}$ states near threshold; one at 1·897 GeV with a width of 25 ± 6 MeV and the other, a much narrower one at 1·932 GeV. That the existence of such resonant states could have an important role in accounting for the corrections to the GT relation (Goldberger and Treiman 1958a; we follow the notation of this article) had been conjectured by one of the authors (Sekhar Raghavan 1974). In this paper we wish to bring out the important role played by the $N\bar{N}$ resonant states in improving the agreement between theory and experiment with reference to the GT relation.

Corrections to the GT relation have claimed the attention of several workers. The works most relevant to our approach are due to Pagels (1969) and Pagels and Zepeda (1972). They have concluded that the corrections to the GT relation, obtained by including the 3π intermediate state in the evaluation of the absorptive part abs $[\langle N | \partial_\mu A_\mu | N \rangle]$ have the wrong sign in addition to being smaller by more than an order of magnitude. In order to get over this difficulty they have conjectured a possible tripion resonance or a subtraction in the dispersion relation. Attempts have also been made (Jones and Scadron 1975 and references therein) in estimating the electromagnetic corrections and contributions from $\rho\pi$ and $\sigma\pi$ intermediate states, which however lead to corrections which are very small.

The main objective of this paper is to bring out the importance of the contribution of a resonant $N\bar{N}$ intermediate state to the pion-nucleon vertex function, in evaluating the pion decay amplitude $F(-m_\pi^2)$, which in turn would account for the corrections to the GT relation. For this purpose we shall adopt the original approach of Goldberger and Treiman (1958a), wherein the role of $N\bar{N}$ intermediate state contribution to abs $F(-m_\pi^2)$ is clearly brought out. $F(-m_\pi^2)$ is related to the pion-nucleon vertex function and the nucleon axial vector form factors a and b. The latter turn out to be real when the contribution of the single pion intermediate state alone is taken into account in estimating them. In a subsequent paper, Goldberger and Treiman (1958b) have pointed out the modification that would result if the $N\bar{N}$ intermediate state contribution are included as well. They conclude however that this modification would not affect their earlier conclusions provided the value of a particular integral (denoted by J) is large compared to 0·1.

2. The method of Goldberger and Treiman

In order to present a cogent and clear picture we repeat here briefly the assumptions made in deriving the GT relation and the important steps of the derivation:

(i) The pion decay amplitude $F(-m_\pi^2)$ is assumed to satisfy an unsubtracted dispersion relation in the variable $\xi = (p_\mu + p_\nu)^2$

$$F(\xi) = \frac{1}{\pi} \int \frac{\mathrm{Im} F(-\xi') d\xi'}{\xi' + \xi - i\epsilon} \tag{1}$$

(ii) In the evaluation of Im $F(\xi)$ all contributions (3π and higher mass intermediate states) other than $N\bar{N}$ are assumed to be negligible. This leads to the following expression

$$\mathrm{Im}\, F(\xi) = \mathrm{Re} \left\{ K^*(\xi) \left[a(\xi) - \frac{\xi}{2m} b(\xi) \right] \right\} \tag{2}$$

where $K(\xi)$ is the pion-nucleon vertex function, m the nucleon mass and a and b are the nucleon axial vector form factors.

(iii) a and b are taken to be real (viz only the pion pole contribution is taken into account in estimating them), and one has

$$a = g_A;\; b[(q_N + q_{\bar{N}})^2] = -\frac{\sqrt{2} G\, F(-m_\pi^2)}{[(q_N + q_{\bar{N}})^2 + m_\pi^2]} \tag{3}$$

where g_A is the axial vector weak coupling constant and G is the strong coupling constant.

(iv) A once-subtracted dispersion relation is assumed to evaluate $K(\xi)$ which turns out to be

$$K(\xi) = K(-m_\pi^2) \exp\left\{-\frac{(\xi+m_\pi^2)}{\pi}\int_{4m^2}^{\infty} \frac{\varphi_0(\xi')d\xi'}{(\xi'-m_\pi^2)(\xi'+\xi-i\epsilon)}\right\} \tag{4}$$

where

$$\tan \varphi_o = \frac{\text{Re}\,(e^{i\delta_0} \sin \delta_0)}{1-\text{Im}\,(e^{i\delta_0} \sin \delta_0)} \tag{4a}$$

and δ_0 is the complex phase shift for $N\bar{N}$ scattering in the $1s_0$ state. Setting the pion mass equal to zero and using equations (1-4), the final expression for $F(0)$ takes the form

$$F(0) = -\frac{m}{2\pi^2}\sqrt{2}\, Gg_A \frac{J}{\left[1+\frac{G^2J}{2\pi^2}\right]} \tag{5}$$

where

$$J = \int_0^\infty dk \frac{k^2}{(k^2+m^2)^{3/2}} \cos \varphi_0(k)$$

$$\times \exp\left\{\frac{2}{\pi}P\int_0^\infty dk'\varphi_0(k')\left[\frac{1}{k'^2-k^2}-\frac{1}{k'^2+m^2}\right]\right\}. \tag{6}$$

The expression for the pion decay rate is then obtained and on comparing it with the experimental value, it is found that J should satisfy the relation

$$\left[\frac{J}{1+\frac{G^2J}{2\pi^2}}\right]_{\text{expt.}} \simeq 0{\cdot}13 \tag{7}$$

3. The effect of $N\bar{N}$ resonant intermediate state

Using the experimental value $G^2/4\pi = 14{\cdot}3$ it can be seen immediately that for $J > 0$ the left hand side of eq. (7) can never exceed 0·1. Therefore to obtain agreement between theory and experiment one has to take J to be negative and it is easily checked that the right hand side of eq. (7) is obtained for $J \simeq -0{\cdot}62$.

Examining the expression for J [eq. (6)] we find that the integral can become negative only if the phase φ_0 increases beyond $\pi/2$. This led us to investigate the effect of a resonant $N\bar{N}$ intermediate state. The mass and width of the resonance may then be varied to obtain the desired value of J. The simplest assumption that one can make in these circumstances is that of elastic unitarity* for $N\bar{N}$ scattering dominated by a resonance. We made use of the Coulter parametrization for the resonant phase shift namely (Coulter 1968; see also Bhamathi *et al* 1974)

$$\delta_R(s) = \tan^{-1}\left[\frac{\gamma\sqrt{s-s_E}}{A^2-s}\right] \tag{8a}$$

where

$$A^2 = [(C-s_E)^2+d^2]^{\frac{1}{2}}+s_E \tag{8b}$$

and

$$\gamma = [2(A^2-C)]^{\frac{1}{2}} \tag{8c}$$

C and d are related to the mass (m_R) and width (Γ_R) respectively of the resonance. The integrals occurring in eq. (6) were evaluated numerically (Bhamathi 1975).

4. Results and discussion

The values of J and $J/[1+(G^2J/2\pi^2)]$ for various values of m_R ranging from 2000 MeV to 2300 MeV and Γ_R in the region 75-225 MeV are presented in table 1. It can be seen that the required value of J namely $\sim -0{\cdot}62$ to account exactly for the corrections to the GT relation, can be obtained if the mass of the resonance lies between 2200 and 2300 MeV and the width between 75 and 225 MeV. The table also includes the J value for the resonances at $m_R = 1932$ and 2850 MeV and $\Gamma_R = 10$ MeV, 39 MeV which seem to be well established by recent experiments. If one of these resonances were taken to dominate the $N\bar{N}$ elastic scattering process, it would then account for about 70% of the total corrections to the GT relation.

Table 1. Variation of J and $J/[1+(G^2J/2\pi^2)]$ with the mass (m_R) and width (Γ_R) of the $N\bar{N}$ resonance.

m_R (MeV)	Γ_R (MeV)	J	$\frac{J}{[1+(G^2J/2\pi^2)]}$
	75	−0·82	0·124
2000	150	−0·77	0·125
	225	−0·72	0·126
	75	−0·75	0·126
2100	150	−0·72	0·127
	225	−0·68	0·128
	75	−0·67	0·128
2200	150	−0·65	0·129
	225	−0·63	0·130
	75	−0·61	0·131
2300	150	−0·59	0·132
	225	−0·57	0·133
1932	10	−0·94	0·122
2835	39	−0·51	0·135

*With this assumption the $\varphi_0(k)$ in eq. (6) should be replaced by $\delta_0(k)$.

In deriving the relation (5) the pion mass was set equal to zero for ease of manipulation. However, this is not necessary and if one carries through the derivation without setting $m_\pi = 0$ one obtains the modified relation

$$F(-m_\pi^2) = -\frac{m}{2\pi^2}\sqrt{2}Gg_A \frac{J_1}{[1+(G^2J_2/2\pi^2)]} \tag{9}$$

where J_1 and J_2 are very similar to J and are given by

$$J_1 = \int_0^\infty dk \frac{k^2 \cos \varphi_o(k)}{(k^2+m^2)^{1/2}[k^2+m^2-(m_\pi^2/4)]}$$

$$\times \exp \left\{\frac{2}{\pi}\left(k^2+m^2-\frac{m_\pi^2}{4}\right) P \int_0^\infty dk' \frac{k'\varphi_o(k')}{(k'^2-k^2)[k'^2+m^2-(m_\pi^2/4)]}\right\} \tag{10a}$$

$$J_2 = \int_0^\infty dk \frac{k^2 (k^2+m^2)^{\frac{1}{2}} \cos \varphi_0(k)}{\left(k^2+m^2-\frac{m_\pi^2}{4}\right)^2}$$

$$\times \exp \left\{\frac{2}{\pi}\left(k^2+m^2-\frac{m_\pi^2}{4}\right) P \int_0^\infty dk' \frac{k'\varphi_0(k')}{(k'^2-k^2)\left(k'^2+m^2-\frac{m_\pi^2}{4}\right)}\right\} \tag{10b}$$

In this case the factor $J_1/[1+(G^2J_2/2\pi^2)]$ should be close to 0·13. On evaluating the integrals J_1 and J_2 in the same way as J we found that modification of the correction factor (7) was by less than one per cent. Therefore, we made use of the factor J in computing the variation of the correction factor with m_R and Γ_R. We conclude, therefore, that with the pion pole dominance approximation for the nuclear axial vector form factors, the inclusion of the contribution from an s-wave $N\bar{N}$ resonant intermediate state goes a long way towards the improvement of the agreement of GT relation with experiment.

Acknowledgements

One of us (SR) would like to thank P M Mathews, M Seetharaman for discussions, and V S Ravikumar for bringing the experimental observation to our notice and the University of Madras for financial assistance in the form of a Fellowship.

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