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Hadronic final states and sum rules in deep inelastic processes

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Abstract. In order to get maximum information on the hadronic final states and sum rules in deep inelastic processes, we use Regge phenomenology and quarks parton model. The unified picture for the production of hadrons of type i as a function of Bjorken and Feynman variables with only one adjustable parameter is formulated. The results of neutrino experiments and the production of charm particles are discussed in sum rules.

Keywords. Parton; Cabibbo current; charm quark; short-range correlation.

1. Introduction

Deep inelastic weak and electromagnetic processes are considered within the framework of the parton model (Feynman 1972). The main purpose of this investigation is to deal with the description of hadron final states in these processes. The experimental situation of hadron final states are surveyed and then we try to explore in greater detail to correlate and predict our results in various regions of phase in rapidity space, the so-called fragmentation regions and plateau regions. In ordinary hadron-hadron collisions, the masses of the external particles are finite whereas in deep inelastic process, the mass of the projectile tends to infinity. We observe how the results differ for deep inelastic processes as compared to ordinary hadron-hadron collisions.

In order to understand and study deep inelastic processes, we go through a sequence of events. First we look at ordinary hadron-hadron collisions. Let us take as an example $\rho+N\rightarrow$hadron + anything. The distribution in rapidity of one particular type of hadrons produced consists of three regions. The highest rapidity particles are found in the ρ-fragmentation region if ρ is the beam particle and N is the target. Then we have the hadronic plateau of intermediate rapidities and slow particles of lowest rapidity in the target fragmentation region. For the real photon-hadron collision, vector dominance (Sakurai 1960) would imply a similar distribution as in ρ-N interactions. We now replace the real photon by a virtual photon. The nucleon fragmentation region and hadron-plateau region (if ω is large) should be present because what we are changing are the properties of the incident projectile. The ideas of short-range correlation in rapidity (Abarbanel 1971) successively used in ordinary hadron-hadron collisions can be taken over directly to deep inelastic processes and, therefore, the other regions should not be affected by a change in projectile properties. The virtual-photon fragmentation region is different from the real-photon fragmentation region for its length in rapidity space changes with Q^2 (the mass square of the virtual photon). The parton model predicts that the photon fragmentation region itself divides up into three pieces (Bjorken 1973). For the largest rapidity there is the parton fragmentation region and adjacent to it is the current plateau. Also there is a transition region between the current plateau and the hadronic plateau which is the location in longitudinal phase space of the parton constituent of the hadron before it has been struck by the virtual photon. This region is known as the hole fragmentation region. The longitudinal phase in rapidity can, therefore, be divided into following parts: a target fragmentation region (of length ~ 2), a central region for target (of length $\sim \ln\omega$), hole fragmentation region (of length ~ 2), a central region for current (of length $\sim Q^2$) and parton fragmentation region (of length ~ 2). Similar is the situation in deep inelastic neutrino-nucleon collisions. In that case, the virtual photon is replaced by a charged intermediate vector boson. The physics, of course, remains unchanged although we get different, coupling constant at the leptonic vertex in νN collision to that of $e^- N$ collision.

2. Description of model

The main purpose of this investigation is to derive results in different phase space and correlate them with smoothness relations in asymptotic limits. The rate of production of hadrons i with rapidity dN_i/dy to be extracted from theoretical considerations cannot be visualized from any single discipline. The Regge phenomenology, quark parton model and some empirical facts are utilized to extract this quantity. Finally, they are connected by a single unified formula valid for all regions in longitudinal phase space. The only adjustable parameter can, therefore, be extracted from the experimental data.

In the infinite momentum frame, the target hadron is considered to be composed of point-like constituents (quark-partons). The virtual photon interacts with only one parton at a time and others are left undisturbed. The production of hadrons of type i is proportional to the parton distribution function $f_j(\omega)$ of the incident current having struck a parton of type j. The observed hadron is the outcome of the fragmentation of the struck parton together with other undisturbed partons of the target constituents. We call the probability $D_{ji}(Z)$ that all partons fragment into hadron i plus anything. The function $D_{ji}(Z)$ depends only on the observed hadron and Z is the longitudinal fraction $P_{\rm lab}/\nu$ and is usually known as the Feynman variable.

We introduce the following variables:

$$\nu = Pq/M, \; \nu_1 = p.q/m, \; k_1 = P.p/M,$$

$$\omega = 2\, P.q/Q^2 \text{ and } \omega_1 = 2\, p.q/Q^2 \tag{1}$$

where M and m are the masses of the target and the observed hadron respectively and $Q^2 = -q^2$ as usual.

When the high energy virtual photon of energy momentum q hits the target p, it gets excited and fragments into a hadron of energy momentum p plus anything. Kinematic regions of such a hadron are given below: k_1 and ν_1/ν finite when $\nu\to\infty$, ν_1 and k_1/ν finite when $\nu\to\infty$ and the central region can be visualized in the centre of mass frame of the virtual current and the target and this kinematic region is given by $\nu_1 k_1/\nu$ finite and ν, ν_1 and $k_1\to\infty$. We can express dN_i/dy in the following form:

$$dN_i/dy = \sum_j f_j(\omega) D_{ji}(Z) \tag{2}$$

where y is the rapidity of the observed hadron and is defined by the following equation:

$$y = \tfrac{1}{2} \ln (\omega + p_l/\omega - p_l)$$

Here $f_j(\omega)$ is the probability of the incident current to have struck the parton of type j. Also ω and Z are the Bjorken and Feynman variables respectively. Keeping ω fixed, one finds that $D_{ji}(Z)$ tends to zero as Z tends to ± 1. As $Z\to\pm 1$ we will derive explicit forms for $D_{ji}(Z)$. For small Z in the plateau region $D_{ji}(Z)$ behaves like $1/Z$ and this behaviour is in fact necessary in order to get the logarithmic multiplicities expected from the work of Feynman (1972), (Bjorken 1973), Cahn *et al* (1973). The function $D_{ji}(Z)$ defined in this way for small Z i.e. in terms of particle density in rapidity ($dy = dZ/Z$) is not expected to behave as $1/Z$ at small Z but rather as a constant with $1/Z$ already in the definition of rapidity.

According to Feynman (1972) $D_{ji}(Z)$ vanishes as a power of $(1-Z)$ as $Z\to 1$ and for small Z, the universal plateau idea suggests that it should vary inversely with Z like $\mathcal{G}_i/Z$ with the constant $\mathcal{G}_i$ that depends on product hadron only. $D_{ji}(Z)$ should vary as $(1-Z)^{1-2\,\alpha_R(\nu_1)}$ when Z tends to 1. Here $\alpha_R\,(\nu_1)$ is the highest trajectory (excluding the Pomeron) which carries off the quantum numbers needed to change from the virtual projectile to the observed hadrons (see figure 1). We use the general rule for involving Regge behaviour which implies that if anything gets large, then in the channel crossed to that variable there is a leading Regge pole and the asymptotic behaviour is determined by that trajectory. Since Pomeron trajectory involves in diffraction scattering only and, therefore, in our case there will be other leading trajectories excluding the Pomeron. For all values of Z from $Z\simeq 1$ (current fragmentation region) to (plateau region) Z small dN_i/dy can be expressed in the following form:

$$dN_i/dy = \sum_j f_j(w)\; C_{ji}/Z\,(1-Z)^{1-2\,\alpha_R\,(\nu_1)}. \tag{3}$$

In the target fragmentation region, Z tends to -1 and $D_{ji}(Z)$ should behave like $(1+Z)^{1-2\,\alpha'_R(k_1)}$. Here $\alpha'_R\,(k_1)$ is the highest trajectory which can carry off the quantum numbers needed to change from the target to the observed hadron (see figure 1c). Therefore dN_i/dy can be expressed from (target fragmentation region) $Z\simeq -1$ to (central plateau) Z small:

$$dN_i/dy = \sum_j f_j(\omega)\; C_{ji}/(-Z)\,(1+Z)^{1-2\alpha'_R\,(k_1)}. \tag{4}$$

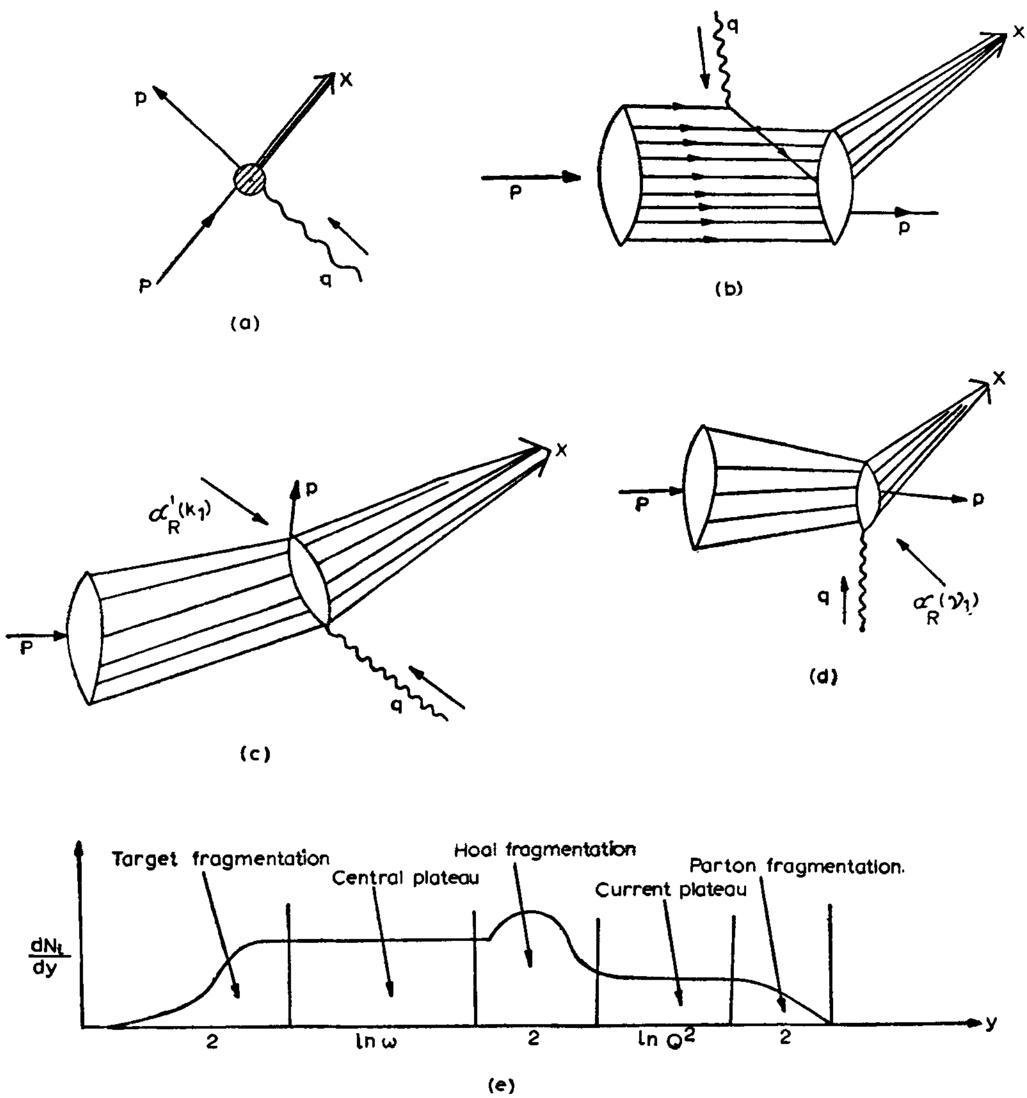

Figure 1 (a) Collision between the virtual projectile q and the target P. Finally one observes a particular type of hadron p plus anything (X). (b) The target in the infinite momentum frame consists of point-like constituents (partons). The virtual projectile q hits one parton and consequently the scattered parton along with undisturbed partons fragments into the hadron p plus anything (X). (c) shows how the hadron p is observed in the target fragmentation region. The Reggeon $\alpha'_R(k_1)$ is exchanged between the target P and the observed hadron p. (d) shows how the hadron p is observed in the current fragmentation region. The Reggeon $\alpha_R(\nu_1)$ is exchanged between the virtual projectile q and the observed hadron p. (e) Distribution of one particular type of hadrons in the longitudinal phases pace in rapidity.

The same C_{ji} is used in eqs (3) and (4) since both of them lead to same plateau region at small Z. The above equations are valid in deep inelastic νN scattering also. Since we are replacing virtual photon by charged intermediate vector bosons in the case of νN deep inelastic process.

3. Sum rules

By invoking charge conservation, we obtain charge sum rules similarly for any other conserved additive quantum number. If, in addition, we employ the fragmentation idea, the sum rule separates into three parts in deep inelastic processes unlike two in ordinary hadron-hadron collisions. In the parton picture for deep inelastic collisions, one views an energetic hadron as equivalent to a beam of partons. The incident hadron beam is replaced by point-like constituents of parton beam. In the target fragmentation region, the high energy partons interact incoherently with virtual current. The struck partons fragment into hadrons plus anything.

$$\sum_i Q_i \int_{-1}^{-Z_1} dN_i/dydZ = \sum_i Q_i \int_{-1}^{-Z_1} \sum_j f_j(\omega)\, C_{ji}/_{(-Z)}\, (1+Z)^{1-2\alpha'_R(k_1)}\, dZ$$

$$= Q_t \tag{5}$$

where Q_t is the total charge of the target.

It has been suggested by Feynman (1972) that the quantum numbers of the parton are measured by the average Q, B, etc. found in the parton fragmentation region. Since the total charge at the target fragmentation is conserved, then one must speculate that the charge found in the parton fragmentation region must come from somewhere. A little reflection convinces one that the missing charge should be located at the lower boundary of the photon fragmentation region which is the longitudinal phase space of the parton before interaction with the virtual photon. In other words, one should consider the virtual photon creating the parton-antiparton pair in the longitudinal phase space of parton and hole fragmentation regions respectively. The average charges found in the parton and hole fragmentation regions should be the same as that of the parton and the antiparton. Therefore, by invoking charge conservation, we should get two more sum rules in parton and hole fragmentation regions respectively. If the initial parton before interaction have got charge Q_p, then after interaction with virtual photon, the average charge in the parton and hole will have Q_p and $-Q_p$. For moderate values of ω, the scattering can take place through valence quarks. When $\omega \to \infty$, the scattering is from sea quarks and hence an equal number of quarks and antiquarks are hit by the incident virtual photon. We will come back to this point later on. In νN scattering, we have W^+ as the incident particle. For moderate values of ω the scattering is mainly due to $W^+ \, d \to u$ where d is the valence quark and consequently one expects $Q = 2/3$ in the parton fragmentation region, and $Q_h = 1/3$ in the hole fragmentation region. When $\omega \to \infty$, the scattering is from sea quarks and hence an equal number of d's and $\bar{u}$'s are hit by the incident W^+. We now write down the sum rules in the parton and hole fragmentation regions respectively.

$$\sum_i Q_i \int \sum_j f_j(\omega)\, C_{ji} / Z\, (1-Z)^{1-2\,\alpha_R(\nu_1)}\, dZ = Q_p \tag{6}$$

and

$$\sum_i \bar{Q}_i \int \sum_j f_j(\omega)\, C_{ji} / Z^{dZ} = Q_h \tag{7}$$

where $\bar{Q}_i$ in (7) is the corresponding antiparton charge of parton charge Q_i in (6). The limits of the integral in eq. (6) over the current fragmentation region are 0·4 and 0·8 respectively (Dakin 1973). The limits of the integral in eq. (7) over the hole fragmentation region are two units in rapidity space. The sign and magnitude of Q_p and Q_h differ to a great extent depending on whether the incident particle is a virtual photon or a charged vector boson.

4. Discussions

The sum rule in the target fragmentation region in deep inelastic processes and in ordinary hadron-hadron collisions is the same. The virtual photon interacts only with valence quarks at small ω in the target fragmentation region. All the partons including the struck one fragment into hadrons plus anything in such a way that the average charge of product hadrons is predicted to be approximately equal to that of the target. Similar interpretation holds good for νN scattering. The other sum rules in deep inelastic processes are entirely dependent on the parton model. The longitudinal phase space of the original parton that interacts with the high energy virtual projectile is known as the hole fragmentation region and that of the struck parton is known as the parton fragmentation region. The total charge of the virtual projectile and that of the struck parton is distributed in the average sense into parton fragmentation and hole fragmentation regions. In the case of e^-N scattering, the charge of the virtual photon is zero. For moderate values of ω only, u and d interact with the virtual photon and, therefore, quark photon interactions are given by $\gamma u \to u$ (i) and $\gamma d \to d$ (ii). The average charges in the parton and hole fragmentation regions for (i) and (ii) are given by

$$Q_p = 2/3, \qquad Q_h = -2/3 \text{ for (i)}$$

and

$$Q_p = -1/3, \qquad Q_h = 1/3 \text{ for (ii)}$$

for large values of ω, when the scattering is from sea quarks, an equal number of u's, $\bar{u}$'s, d's and $\bar{d}$'s are hit by the incident photon. The sum rules for the sea quarks are similar.

In the case of νN scattering before the threshold of charm quarks, the V-A weak current in the 3-quark model due to Cabibbo is

$$(J\mu)_{\text{Cabibbo}} = \bar{u}\gamma_\mu(1+\gamma_5)\,(dC+\lambda S) + \text{h.c} \tag{8}$$

where $C = \cos\,\theta_c$ and $S = \sin\,\theta_c$.

For moderate values of ω in νp scattering, the scattering is mainly due to $W^+d \to u$ and for large values of ω, the number of $\bar{u}$'s hit by the incident W^+ is much greater than the number of λ's hit by the same projectile because of the fact that the probability of W^+ to hit $\bar{u}$'s is C^2 and that of W^+ to hit λ's is S^2 where $C^2=0{\cdot}95$ and $S^2=0{\cdot}05$. The values of the average charges in different regions will remain unchanged. Now we discuss the production of charmed final states in neutrino reactions. Charmed quarks can be produced by partons via the GIM (Glashow 1970) charm-changing V-A weak current.

$$J_\mu{}^c = \bar{c}\gamma_\mu\,(1+\gamma_5)\,(-dS + \lambda C) + \text{h.c.} \tag{9}$$

provided the initial energy exceeds threshold energy for producing charmed particles. For moderate values of ω, the scattering is due to $W^+n \to c$ and for large value of ω, the scattering will be due to $W^+\bar{c} \to \bar{s}$. The above two reactions are of the same order if we take the reasonable choice for $\epsilon \simeq 0{\cdot}06$ (Barger *et al* 1975) where

$$\epsilon = \int_0^1 xN_{\text{sea}}\,(x)\,dx \Big/ \int_0^1 xN_{\text{val}}\,(x)\,dx. \tag{10}$$

Since the probability for $W^+d \to c$ is C^2 ($C^2 = 0{\cdot}95$). The production of charmed particles by partons through an additional $V+A$ charm changing current (Rujula *et al* 1975) is given by

$$J_\mu{}^{c\prime} = \bar{c}\,\gamma_\mu\,(1-\gamma_5)\,d + \text{h.c.} \tag{11}$$

for moderate values of ω, the scattering is due to $W^+d \to c$ and for large value of ω, it is due to $W^+\,\bar{c} \to \bar{d}$. The average charges in different regions are expected to be unchanged. The only thing we require is large enough energy for producing charmed particles. The charm productions for an average nucleon target are

$$\sigma_c{}^{\nu N}\,(x, y)/x = N_{\text{sea}}\,(x) + N_{\text{val}}\,(x)\,S^2 \tag{12}$$

$$\sigma_c{}^{\bar{\nu} N}\,(x, y)/x = N_{\text{sea}}\,(x)$$

Taking $\epsilon = 0{\cdot}06$ and $S^2 = 0{\cdot}05$.

The charm product ratio is then $\sigma_c{}^{\bar{\nu}N} / \sigma_c{}^{\nu N}$ compatible with experimental value (Barger *et al* 1975)

$$\sigma_c{}^{\bar{\nu}N} / \sigma_c{}^{\nu N} = 1{\cdot}0 \pm 0{\cdot}7.$$

We have deduced that the parton fragmentation function should be of the form $1/Z\,(1-Z)^{1-2\,a_R\,(\nu_1)}$. In practice, however, deviations are to be expected in the small Z region. In deep inelastic processes not much is known about the target fragmentation region. The above sum rule will give some clue about parton constituents of hadrons. Figure 1a, b shows that when a virtual projectile hits a nucleon P, one observes a hadron p plus anything. Figure 1d shows the reaction in the current fragmentation region. A Reggeon is exchanged between the virtual projectile of and the observed hadron p . Figure 1c indicates what happens in the target fragmentation region. A Reggeon is exchanged between the target P and the observed hadron p. Finally figure 1e shows the distribution of the observed hadrons in the longitudinal phase space in rapidity.

5. Conclusion

In the limit of very large ω and Q^2 in deep inelastic processes one can contemplate three fragmentation regions and three sum rules rather than two in ordinary hadron-hadron collisions in analysing data along the lines of the parton model. Also our investigation is of significance in view of the new high energy machine ranges like SPS and PEP.

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