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A quark-parton description of the deep inelastic scattering processes

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Abstract. A consistent description of various deep inelastic processes in a quark-parton model is presented. The valence quark probability distribution and the form of core quark probability distribution is fixed from the deep inelastic electroproduction data. Langacker and Suzuki prescription is used to fix the p and n quark core distribution. The differential excitation of quark currents similar to the Harari model of $e^+ e^-$ annihilation process is invoked in deep inelastic electroproduction and neutrino reactions. An effective phenomenological form of the weak currents associated with new quarks and the associated nucleon structure is determined.

Keywords. Quark-parton model; deep inelastic processes.

1. Introduction

The deep inelastic electroproduction results gave birth to the parton structure of the nucleon. The success enjoyed by the quark models helped in postulating quarks as partons. Accordingly various quark parton models were constructed to reproduce the behaviour of deep inelastic structure functions $F_2^{ep}(x)$ and $F_2^{en}(x)$. However, when these models were used for the description of electron positron annihilation and deep inelastic neutrino processes, various difficulties came up. In addition, the discovery of Ψ particles required the existence of other varieties of quark-partons, normally ignored in the interpretation of electroproduction results. So much so, that people began to wonder whether it is at all possible to explain various deep inelastic processes in a consistent way, within an unified frame work of any quark parton model. In this paper, we present such a framework that explains all the characteristic features of the various deep inelastic scattering phenomena.

The model employed (Bajpai and Mukherjee 1974b) is a modified version of the quark-parton model of Kuti and Weiskopff (1971). In this model, the structure of nucleon is expressed in terms of valence and core quarks. The functions $G_{1v}{}^p(x)$ and $G_{2v}{}^p(x)$ give respectively the probability distribution of the valence p and n quark partons, with a momentum fraction x of the nucleon. The core of sea quarks probability distribution functions will now be expressed as $g_i/2V_c(x)$, with a view to accommodate the various varieties of quarks. The constant g_i determines the relative abundance of the i-th quark or antiquark. The functional forms of $G_{1v}{}^p(x)$, $G_{2v}{}^p(x)$ and $V_c(x)$ are determined in the model (Bajpai and Mukherjee 1974b) in terms of a phenomenological pair correlation parameter ϵ. The parameters of the model—namely the constants g_i's and ϵ are to be determined by the experimental data. We, therefore, discuss in sections 2 and 3 the salient features of the experimental data for the different scattering processes and their relevance in determining the quark parton structure of the nucleon.

2. Deep inelastic electroproduction

The data determine reasonably well, the scale invariant structure functions $F_2^{ep}(x)$ and $F_2^{en}(x)$ in the range $0{\cdot}1<x<0{\cdot}8$. The threshold behaviour of the structure functions further suggests

$$\lim_{x\to 1} \frac{F_2^{en}(x)}{F_2^{ep}(x)}=\frac{1}{4}. \tag{1}$$

One can express various structure functions in quark-parton models (Bajpai-Mukherjee 1974a) in the following way:

$$F_2^{ep}(x)-F_2^{en}(x)=\tfrac{1}{3}x\left[G^p_{1v}(x)-G^p_{2v}(x)\right] \tag{2}$$

and

$$F_2^{ep}(x)=\frac{4xG^p_{1v}(x)+xG^p_{2v}(x)}{9}+\sum_i g_i Q_i^2 xV_c(x) \tag{3}$$

where Q_i is the charge of the i-th quark-parton in electronic unit. The threshold condition (1) determines the values $\epsilon=1$, which then fixes various probability distribution functions uniquely. The experimental data are well reproduced if one requires the following condition for the core contribution (Kuti and Weiskopff 1971).

$$\sum_i g_i Q_i^2=\frac{2}{9} \tag{4}$$

which expresses the fact that the deep inelastic electroproduction data cannot specify the types and relative abundance of the various quarks present inside the core.

Anticipating the quark-parton description of neutrino processes, we give in table 1, the integrated values of the various probability functions in different ranges of x.

Table 1. Integrated values of the various probability functions: for different ranges of x

S. No.	$f(x)$	$\int_0^{0\cdot 1} f(x)dx$	$\int_0^{0\cdot 6} f(x)dx$	$\int_0^{1} f(x)dx$
1.	$xG^p_{1v}(x)$	0·036	0·20	0·25
2.	$x[G^p_{1v}(x)+G^p_{2v}(x)]$	0·057	0·27	0·34
3.	$xV_c(x)$	0·084	0·13	0·22

It may be mentioned that the integrated values are almost model independent, so that the arguments and the conclusions, presented henceforth will be valid for any quark parton model, that reasonably reproduces the electroproduction data.

3. Deep inelastic neutrino processes

The total cross-sections σ_ν and $\sigma_{\bar{\nu}}$ of the neutrino and antineutrino reactions have a typical scaling behaviour. The ratio of the two cross-sections.

$$R=\sigma_{\bar{\nu}}/\sigma_\nu \tag{5}$$

is a well established quantity. It has an interesting energy dependence on the neutrino or antineutrino energy E_ν. For $E_\nu<20$ GeV (Benvenuti *et al* 1974, Eichen *et al* 1974), one has

$$R=0{\cdot}37\pm0{\cdot}03 \tag{6}$$

while for $E_\nu>20$ GeV observed values (Benvenuti *et al* 1976) after a transition region approximated as

$$R=0{\cdot}6. \tag{7}$$

The deep inelastic differential cross-sections can be expressed as

$$\frac{d^2\sigma^\nu}{dxdy}=\sigma_0[W_-^\nu(x)+(1-y)^2W_+^\nu(x)] \tag{8}$$

and

$$\frac{d^2\sigma^{\bar{\nu}}}{dxdy}=\sigma_0[W_+^{\bar{\nu}}(x)+(1-y)^2W_-^{\bar{\nu}}(x)] \tag{9}$$

where

$$x=\frac{Q^2}{2M\nu};\ y=\frac{\nu}{E_\nu}\ \text{and}\ \sigma_0=\frac{G^2M\varepsilon_\nu}{\pi}$$

in the standard notation (Bargar and Phillips 1974). The functions $W_\pm^{\nu,\bar{\nu}}(x)$ are related to the usual structure functions $F_2^{\nu,\bar{\nu}}(x)$ and $F_3^{\nu,\bar{\nu}}(x)$, (Kuti and Weiskopff 1971) in the following manner

$$W_\pm^{\nu,\bar{\nu}}(x)=\tfrac{1}{2}[F_2(x)^{\nu,\bar{\nu}}\pm xF_3^{\nu,\bar{\nu}}(x)]. \tag{10}$$

The experimental data give the average of the various structure functions for the nucleon target. The available y distribution in the smaller neutrino or antineutrino energy E_ν in the range $1<E_\nu<11$ GeV (Aubert *et al* 1974, Dedan *et al* 1975 and Derrick *et al* 1976) corresponds to the situation where $W_-^{\nu,\bar{\nu}}\gg W_+^{\nu,\bar{\nu}}$ where $W_\pm^{\nu,\bar{\nu}}$ gives the integrated values of the structure functions

$$W_\pm^{\nu,\bar{\nu}}=\int W_\pm^{\nu,\bar{\nu}}(x)dx.$$

At energies above 30 GeV, the pattern of ν data does not change, though the $\bar{\nu}$ data show a remarkable change and requires $W_-^{\bar{\nu}} \simeq W_+^{\bar{\nu}}$. The effect-called $\bar{\nu}$ anamoly is much more pronounced in the small x region.

The structure functions $W_\pm^{\nu\bar{\nu}}(x)$ can be related to the probability distribution functions in quark parton models, if the form of the weak current is known. Restricting ourselves to the usual current-current interactions, the contributing partons to the various structure functions for the $(V-A)$ and $(V+A)$ types of current are listed in table 2. The quantity $W_\pm^{\nu,\bar{\nu}}(x)/2x$ will simply be the sum of the probabilities of different partons in the target excited by the weak currents.

The energy dependent behaviour of R and the $\bar{\nu}$ anomaly suggest that the entire weak current is not excited at low energies. Let us suppose that the only Cabibbo current J_μ^{V-A} is excited at low energies, where

$$J_\mu^{V-A} = \bar{p}\,\gamma_\mu(1-\gamma_5)\,(n\cos\theta_c + \lambda\sin\theta_c) \tag{12}$$

with $\cos^2\theta_c = 0{\cdot}95$. As is well known (Glashow *et al* 1974), this current can reproduce small energy data if sea contribution is ignored i.e. $g_i = 0$. It may be emphasized at this stage that the available differential cross section $d\sigma/dx$ data, for $x > 0{\cdot}2$ is not very sensitive to the sea contribution; though the situation is quite different with the ratio R, which cannot accommodate any significant sea contribution. A zero or small sea contribution will contradict the electroproduction data, so that eq. (4) determining the sea/core contribution to the electro-production data is crucial for our future arguments. It does not determine the individual values of g_i's. However if one invokes an SU(3) symmetric core one gets $g_p = g_n = g_\lambda = \frac{1}{3}$. Anticipating the possibility of other quark varieties, it is safe to conclude that it is an overestimate.

Following Langacker and Suzuki (1973) one can estimate the value of g_p from the measured total pion nucleon cross-section $\sigma_{\rm tot}^{\pi N}$ at energy ε. The $\sigma_{\rm tot}^{\pi N}$ can be related to the scale invariant axial vector structure function $F_2{}^A(x)$ by PCAC in the following way

$$\sigma_{\rm tot}^{\pi N}(\varepsilon) = \tfrac{1}{2}\,\pi\, f_\pi^{-2}\,(\cos\theta_c)^{-2}\,F_2{}^A(x), \tag{13}$$

Table 2. Contributing quark-partons to the different structure functions for $(V-A)$ and $(V+A)$ weak currents

S. No.	Structure function	$(V-A)$ current	$(V+A)$ current
1.	$W_+^{\bar{\nu}}(x)$	$\bar{n}, \bar{\lambda}, \bar{b}$	p, c, t
2.	$W_-^{\bar{\nu}}(x)$	p, c, t	$\bar{n}, \bar{\lambda}, \bar{b}$
3.	$W_+^{\nu}(x)$	$\bar{p}, \bar{c}, \bar{t}$	n, λ, b
4.	$W_-^{\nu}(x)$	n, λ, b	$\bar{p}, \bar{c}, \bar{t}$

where $\sqrt{2}\, f_\pi = 0{\cdot}96\, m_\pi$ is the pion decay, constant. Taking the pomeron contribution to $\sigma_{\text{tot}}^{\pi N}$ as 21·3 mb and the axial vector current given in eq. (12) one obtains

$$g_p = g_n = 0{\cdot}30. \tag{14}$$

The estimate of g_p given in eq. (14) is susceptible to criticism because of the not-so-strong theoretical foundation of the Langacker-Suzuki relation (13). It is therefore pointed out that we are using only the pomeron contribution, which may not be so objectionable. Beside the relation has been used earlier (Schwinger 1977) and is known to give an excellent description of the pion nucleon scattering, once a suitable generalized scaling variable is defined (Bajpai and Mukherjee 1974a). In any case the value of g_p obtained above differs slightly with the one obtained in the only SU(3) symmetric core of the Kuti-Weiskopff model and suggests only a small population of the other exotic varieties of quark partons in the core.

The effect of $g_p = 0{\cdot}30$ in the differential cross-section data for $x > 0{\cdot}2$ will only be marginal and can be masked by the experimental error bars. The situation is quite different for the ratio R. In the presence of only Cabibbo current R can expressed as

$$R = \frac{\int_0^1 x\,[G^p_{1v}(x) + G^p_{2v}(x)]\,dx + [3g_p \cos^2\theta_c + 3g_\lambda \sin^2\theta_c + g_p] \int_0^1 x\, V_c(x)\,dx}{3\cos^2\theta_c \int_0^1 x[G^p_{1v}(x) + G^p_{2v}(x)]\,dx + [3g_p \cos^2\theta_c + 3g_\lambda \sin^2\theta_c + g_p] \int_0^1 x\, V_c(x)\,dx}. \tag{15}$$

Using eq. (14) and table 1, one gets $R = 0{\cdot}48$, so that some other current should also be excited along with the Cabibbo current. Such a current should increase σ_ν without significantly affecting $\sigma_{\bar{\nu}}$. It can be accomplished only if the new current involves a valence quark parton in interactions, for the sea partons contribution to σ_ν and $\sigma_{\bar{\nu}}$ is identical. Taking the phenomenological form of additional weak current as

$$J_\mu^c = \bar{c}\, \gamma_\mu(1 - \gamma_5)\,(n \cos\theta + \lambda \sin\theta) \tag{16}$$

with the angle θ kept arbitrary, one obtains $R = 0{\cdot}37$ for θ in the range $0{\cdot}62 < \sin^2\theta < 0{\cdot}77$ (Bajpai 1976), corresponding to the values of g_c in the range $0{\cdot}12 > g_c > 0$ required by eqs (14) and (4). For an SU(3) symmetric core one gets $\sin^2\theta = 0{\cdot}74$ and $g_c = 0{\cdot}04$. The differential cross section data can again be accommodated in any solution. The presence of J_μ^c at low energies raises questions regarding the mass of particles with c quantum number. It is generally believed that the D meson with the mass around 1·8 GeV is the lowest mass charmed particle. Such a charmed current will be kinematically forbidden at low energies and cannot be responsible for $R = 0{\cdot}37$ at energies around 2 GeV. We therefore emphasize the phenomenological nature of the current J_μ^c and suggest that the quantum number c represents a generic charm quantum number. The presence of anamolous $\mu^- e^+ V^0$ events seen at Gargamelle further points towards such a possibility.

The additional changes in the characteristics of deep inelastic neutrino processes with the rise of energy are again to be interpreted as arising due to the excitation of some other quark degree of freedom say t or b quarks of Harari (1975). Absence of any observable effects in F_2^{ep} indicating the excitation of t and b quarks at high energies suggests, that like g_c, g_t or g_b should also be small compared to g_p.

The form of weak current involving t and b quarks has to be such as to explain the change of R from 0·37 to 0·6 as well as the $\bar{\nu}$ anomaly. As discussed earlier such a large change in R cannot be attained by invoking weak currents involving only the sea quark partons. The $\bar{\nu}$ anomaly further requires that the new weak current involving a valance quark should contribute significantly only to $W_+^{\bar{\nu}}(x)$. Table 2 can now be used to conclude that such a current can only be $(V+A)$ type involving p and b quark partons. It will considerably increase $W_+^{\bar{\nu}}(x)$ without effecting any significant changes in other structure functions. Absence of any appreciable changes in the neutrino structure functions suggests that the current involving t and n quark is small and can be ignored. We shall again take the current $J_\mu{}^B$ involving b quark as

$$J_\mu^{+B} = \bar{b}\gamma\mu(1+\gamma_5)\,(p\cos\alpha + c\sin\alpha) \tag{17}$$

where α is arbitrary. The value $R=0{\cdot}6$ will require $\cos^2\alpha=0{\cdot}49$ for an SU(3) symmetric core.

With the help of table 1 it is now easy to calculate the changes in the differential cross-section due to the excitation of the current $J_\mu{}^B$. The antineutrino cross-section

$$\frac{d\sigma^{\bar{\nu}N}}{dy} = 0{\cdot}31\,\sigma_0\,[1+3{\cdot}1(1-y)^2] \tag{18}$$

before the excitation, will change to

$$\frac{d\sigma^{\bar{\nu}N}}{dy} = 0{\cdot}31\sigma_0\,[1+1{\cdot}31(1-y)^2] \tag{19}$$

while the neutrino cross section before excitation

$$\frac{d\sigma^{\nu N}}{dy} = 0{\cdot}71\sigma_0[1+0{\cdot}1(1-y)^2] \tag{20}$$

will change only to

$$\frac{d\sigma^{\nu N}}{dy} = 0{\cdot}74\sigma_0[1+0.1(1-y)^2]. \tag{21}$$

The effect is much more marked for a smaller x range say $0<x<0{\cdot}6$ where $[1+3{\cdot}04(1-y)^2]$ behaviour of $(d\sigma/dy)^{\bar{\nu}N}$ changes to $[1+1{\cdot}28(1-y)^2]$. A detailed fit is not called for in the present state of experimental data. Even the numbers given should only be considered as indicative. The importance of the current $J_\mu{}^c$ and $J_\mu{}^B$ is a significant feature of the neutrino data.

4. Conclusions

The nucleon structure emerging out of the various deep inelastic processes is that of valence plus core quarks. The valence quark distribution is very well given by various quark-parton models. Inside the core the p and n population is fixed by $g_p = g_n = 0{\cdot}30$. The quarks c, t and b have much smaller population inside the nucleon core and could be ignored. However the currents involving c(or t) and b quarks partons —namely $J_\mu{}^c$ and $J_\mu{}^B$, are significant. The current $J_\mu{}^c$ should be effective even at lower energies. The smallness of g_i's for the new quarks tells us that the right place to look for the new physics associated with $J_\mu{}^c$ and $J_\mu{}^B$ lies in neutrino and annihilation processes.

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