Pramāṇa, Vol. 9, No. 2, August 1977, pp. 129-140, © Printed in India.

Single particle SU(3) parentage coefficients

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MS received 8 April 1977

Abstract. The single particle SU(3) parentage coefficients are calculated for the case of leading SU(3) representation in the highest orbital symmetry partition, using the method suggested by Hecht. Tabulations are given for all possible cases of identical nucleons in $\eta=3$ and $\eta=4$ shells.

Keywords. Irreducible representation; coefficient of fractional parentage; creation operator; permutation symmetry; unitary group; pseudo SU(3).

1. Introduction

The introduction of the concept of pseudo spin and angular momentum and the classification of the single particle states as pseudo spin-orbit doublets (Arima *et al* 1969, Hecht and Adler 1969) had led to the introduction of a pseudo SU(3) coupling scheme (Arima *et al* 1969, Hecht 1970). In an extremely simple version, taking only the leading pseudo SU(3) representation corresponding to the natural parity nucleons outside the closed shell, this coupling scheme is capable of explaining the ground state properties of heavy deformed nuclei rather well (Ratna Raju *et al* 1973, 1977). Recent investigations show that this coupling scheme could be used to study the spectral properties of heavy deformed even-even nuclei considering only natural parity neutrons to be active (Kota 1976, 1977).

With the availability of general computer codes (Akiyama and Draayer 1973) to make calculations within the SU(3) scheme, studies in the SU(3) model reduces to a calculation of SU(3) parentage coefficients (CFP). It is difficult, if not impossible, to obtain closed expressions for CFP (Moshinisky and Shyamala Devi 1969) as general U(N) algebra is not fully developed. However, one can evaluate the needed CFP by explicit construction of the SU(3) states in terms of single-particle creation operators, as suggested by Hecht (1965).

This technique becomes very laborious when we want to construct the states corresponding to lower SU(3) irreducible representations (IR). So we adopt a slightly modified technique in the present article and this method is explained in detail in section 2.

Single particle CFP are needed to evaluate the static moments, multipole transition probabilities, single particle spectroscopic factors, etc. They could even be used to predict the spectra of nuclei when a quadrupole-quadrupole type of two-body interaction is used (Ratna Raju *et al* 1976, Kota 1976, 1977).

Besides their use to the nuclear physics problems, the parentage coefficients can be used in any other branch of physics where SU(3) group finds its application (Butler and Wybourne 1971, Haskell *et al* 1971). For example, in atomic physics where the spectroscopically active configuration is of the type $(s+d+g)^N$, the chain U(15)⊃SU(3)⊃R(3) (Haskell *et al* 1971) may be used and the calculated CFP for $\eta = 4$ shell can be made use of.

2. Method of calculation

The orbital degeneracy (d) of a given shell η is $(\eta+1)(\eta+2)/2$. Now in the SU(3) scheme one makes use of the chain [U(d)⊃SU(3)⊃R(3)] ⊗ SU(2)* to make a calculation. The parentage coefficients are [U(d)⊃SU(3)] ⊗ SU(2) part of the full Wigner coefficient of [U(d)⊃SU(3)⊃R(3)]⊗SU(2). As the CFP are independent of the subgroup labels, it is advantageous to work them out in the canonical chain U(d)⊃SU(3)⊃[SU(2)⊗U(1)] as these intrinsic states are easy to be constructed. In this chain of subgroups we label our states as

$$|\psi\rangle = |N[F]\; SM_s\,(\lambda\mu)\alpha\epsilon\Lambda M_\Lambda\rangle \tag{1}$$

where $[F]$ is a partition of $U(d)$, S and M_s are the usual spin quantum numbers (if $[F] = (2^a1^b)$, then $s = b/2$), (λ, μ) is an irreducible representation (IR) of SU(3) contained in the partition $[F]$, Λ, M_Λ are the SU(2) quantum numbers, ϵ is the U(1) quantum number and α is an additional label which may be required to specify the states completely.

As the CFP are independent of the subgroup labels, we always choose ϵ, Λ and M_Λ to be maximum, that is we choose the highest weight state corresponding to a given IR of SU(3). Thus our states are

$$|\psi\rangle = |N[F]\; S = M_s\; (\lambda\mu)\; \alpha\; \epsilon_H\; \Lambda H = M_{\Lambda H}\rangle \tag{2}$$

where (Elliott 1958)

$$\epsilon_H = 2\lambda + \mu$$

and

$$\Lambda H = \mu/2.$$

Now to evaluate the single particle CFP, we make use of the expression

$$\begin{aligned}
&\langle N[F]\; S = M_s(\lambda\mu)\; \alpha\; \epsilon_H\; \Lambda H = M_{\Lambda H}\; \big|\, a^{+}{}_{[1]\; 1/2\; m_{s_0}\; (\eta 0)\; \epsilon_0\; \Lambda_0\; m_{\Lambda_0}}\, \big| \\
&\qquad (N-1)\,[F']\; S' = M_s'\; (\lambda'\mu')\; \alpha'\; \epsilon'_H\; \Lambda H' = M'_{\Lambda H}\rangle \\
&= \langle\, N[F]\; S\,(\lambda\mu)\; \alpha\; |||\; a^{+}{}_{[1]\; 1/2\; (\eta 0)}\; |||\; (N-1)\; [F']\; S'\; (\lambda'\mu')\; \alpha'\,\rangle \\
&\quad \langle\, (\lambda'\mu')\; \epsilon'_H\; \Lambda H';\; (\eta 0)\; \epsilon_0\; \Lambda_0 \,\|\, (\lambda\mu)\; \epsilon_H\; \Lambda H\,\rangle \\
&\quad \langle\, S'\; S'\; \tfrac{1}{2}\; m_{s_0}\, |\, SS\,\rangle\; \langle\, \Lambda H'\; \Lambda H'\; \Lambda_0\; m_{\Lambda_0}\, |\, \Lambda H\; \Lambda H\,\rangle
\end{aligned} \tag{3}$$

where a generalized Wigner-Eckart theorem has been used.

*In the present article we deal with only two columned partitions.

In the above expression $a^{+}{}_{[1]\,1/2\,m_{s_0}\,(\eta 0)\,\epsilon_0\,\Lambda_0\,m_{\Lambda_0}}$ is the single particle creation operator, η being the shell number.

The triple barred reduced matrix element in the above equation is related to the more conventional CFP by the relation (Hecht and Braunschweig 1975)

$$\langle (N-1)\,[F']\,S'\,(\lambda'\mu')\,\alpha';\ [1]\ 1/2\ (\eta 0)\,|\}\,N[F]\,S\,(\lambda\mu)\,\alpha\rangle$$
$$= \frac{1}{\sqrt{N}}\langle N[F]\,S\,(\lambda\mu)\,\alpha\,|||\,a^{+}{}_{[1]\,1/2\,(\eta 0)}\,|||$$
$$(N-1)[F']\,S'\,(\lambda'\mu')\,\alpha'\rangle. \tag{4}$$

The double barred coefficient is the reduced SU(3)$\supset$SU(2)$\otimes$U(1) Wigner coefficient. They can be evaluated using the eq. (13) of Hecht's (1965) article. However the Wigners needed for the present calculation can be given in a much simplified form as

$$\langle (\lambda\mu)\,\epsilon_H\,\Lambda_H;\ (\eta 0)\,\epsilon_0\,\Lambda_0\,\|\,(\lambda'\mu')\,\epsilon_H'\,\Lambda_H'\rangle$$
$$= \left[\frac{\lambda!\,(\lambda+\eta+1+K-2\sigma)!\,(\lambda+\mu+1)!\,(\lambda+\mu+2+\eta-\sigma+K)!}{(\lambda-\sigma+K)!\,(\lambda+\eta-\sigma+1)!\,(\lambda+\mu+1-K)!\,(\lambda+\mu+2+\eta-\sigma)!}\right]. \tag{5}$$

The variables σ and K in the above equation are defined through the relations

$$\epsilon'_H = \epsilon_H + 2\eta - 3\sigma$$

and
$$\Lambda'_H = \Lambda_H + \sigma/2 - K$$

where $\sigma = 0, 1, 2, \ldots, \eta$

$$K = 0, 1, 2, \ldots, \sigma. \tag{6}$$

The Wigner coefficients on the r.h.s. of eq. (3) can be evaluated using the relation (Edmonds 1957)

$$\langle j_1 j_1 j_0\ \nu_0\,|\,j_2 j_2\rangle = \left[\frac{(2j_2+1)2j_1!2j_2!}{(j_1+j_2-j_0)!(j_1+j_2+j_0+1)!}\right] \tag{7}$$

Now we are left with the evaluation of the overlap integral on the l.h.s. of eq. (3) to obtain the needed CFP. For this the N and $(N-1)$ particle SU(3) states will be constructed in terms of the single particle creation operators (Hecht 1965) using Elliott's (Harvey 1965) stepdown operators E_+, E_- and W_-, where

$$\begin{aligned} E_+ &= A_{xz} \\ E_- &= A_{yx}A_{xz} - A_{yz}(A_{xx} - A_{yy} + 1) \\ W_- &= A_{yx} \end{aligned} \tag{8}$$

In the above expressions A_{ij} are U(3) shift operators (A_{ij} transform a quanta from the j-th direction to the i-th direction).

The N particle states can always be written down trivially as we are interested in the leading representation. For example the highest weight state corssponding to the leading representation (19 4) of nine particles in the partition (2^4 1) of U(15) can be written as

$$| 9[2^4\ 1]S = M_s = \tfrac{1}{2}(19\ 4)\epsilon = 42\ \Lambda = M_\Lambda = 2\rangle$$

$$= a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})-\frac{1}{2}}$$

$$a^+_{(211)\frac{1}{2}}\, a^+_{(211)-\frac{1}{2}}\, a^+_{(210)\frac{1}{2}}\, | 0 \rangle. \tag{9}$$

In the above equation we have chosen an abbreviated notation $a^+_{(\epsilon_o \Lambda_o m_{\Lambda_o}) m_{s_o}}$ for $a^+_{[1]\frac{1}{2}\, m_{s_o}\, (40)\ \epsilon_o\, \Lambda_o\, m_{\Lambda_o}}$.

Now to obtain the single particle CFP corresponding to the above state, we have to construct the eight particles states which transform according to the SU(3) representations (18 4), (16 5) (17 3) and (15 4) of [2^4] partition and (19 2), (16 5), (17 3) and (15 4) of [2^3 1] partition (Note that only these states can have an overlap with the above nine particle state). The highest weight state corresponding to the leading representation (18 4) can be written down trivially as

$$| 8[2^4]S=M_s=0\ (18\ 4)\ \epsilon=40\ \Lambda=M_\Lambda=2\rangle$$

$$= a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})\frac{1}{2}}$$

$$a^+_{(5\frac{1}{2}-\frac{1}{2})-\frac{1}{2}}\, a^+_{(211)\frac{1}{2}}\, a^+_{(211)-\frac{1}{2}}\, | 0 \rangle. \tag{10}$$

Now we have to construct the highest weight state corresponding to the (16 5) representation of [2^4] partition. This is obtained by making it orthogonal to $| 8[2^4]\ S = M_s = 0\ (18\ 4)\epsilon = 37 \Lambda = M_\Lambda = 3/2\rangle$ state. For this we operate with the operator A_{xz} on the state given in eq. (10). Then we obtain two different pieces

$$\frac{1}{\sqrt{2}} \left\{ \begin{array}{l} + a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})\frac{1}{2}} \\ \quad a^+_{(211)\frac{1}{2}}\, a^+_{(211)-\frac{1}{2}}\, a^+_{(210)-\frac{1}{2}}\, | 0 \rangle \\ - a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})-\frac{1}{2}} \\ \quad a^+_{(211)\frac{1}{2}}\, a^+_{(211)-\frac{1}{2}}\, a^+_{(210)\frac{1}{2}}\, | 0 \rangle \end{array} \right.$$

and

$$\frac{1}{\sqrt{2}} \left\{ \begin{array}{l} + a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})\frac{1}{2}} \\ \quad a^+_{(5\frac{1}{2}-\frac{1}{2})-\frac{1}{2}}\, a^+_{(211)\frac{1}{2}}\, a^+_{(-1\ 3/2\ 3/2)-\frac{1}{2}}\, | 0 \rangle \\ - a^+_{(800)\frac{1}{2}}\, a^+_{(800)-\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})\frac{1}{2}}\, a^+_{(5\frac{1}{2}\frac{1}{2})-\frac{1}{2}}\, a^+_{(5\frac{1}{2}-\frac{1}{2})\frac{1}{2}} \\ \quad a^+_{(5\frac{1}{2}-\frac{1}{2})-\frac{1}{2}}\, a^+_{(211)-\frac{1}{2}}\, a^+_{(-1\ 3/2\ 3/2)\frac{1}{2}}\, | 0 \rangle \end{array} \right. \tag{11}$$

Only the first of the above two pieces can have an overlap with our nine particle state, through a creation operator. And from eq. (3) it is clear that it is enough if we know the coefficient of this piece in the highest weight state corresponding to the (16 5) representation. By operating with the A_{xz} operator on the highest weight state corresponding to the (18 4) representation we get the coefficient of this piece to be $(6/18)^{\frac{1}{2}}$. Hence the coefficient of this piece in the highest weight state corresponding to the (16 5) representation is $(12/18)^{\frac{1}{2}}$. Likewise the coefficients of the pieces in the states corresponding to the other eight particle representations, that can have an overlap with our nine particles state through a certain operators, can be found out. It is important to note that there will always be one and only one piece (Hecht 1974) in the $(N-1)$ particle state that can have an overlap with the N-particle leading representation through a creation operator. So for this special case one need not know the degeneracy α of a particular SU(3) representation in a given partition of a given shell.

The CFP obey the sum rule (Hecht and Braunschweig 1975)

$$\sum_{S'(\lambda'\mu')\alpha'} \langle (N-1)[F']\, S'\, (\lambda'\mu')\alpha';\, [1]\, \tfrac{1}{2}\, (\eta 0) \mid \}\, N\, [F]\, S(\lambda\mu)\alpha \rangle^2$$

$$= \dim\, [F']/\dim\, [F] \tag{12}$$

where dim $[F]$ is the dimensionality of the partition $[F]$ with respect to the permutation symmetry S_N.

The IR of SU (3) needed for the present calculation could be generated in a straightforward way. For example the leading SU(3) representation in a given partition $[F]$ of U(10) where

$$[F] = [f_i],\ i = 1, 10$$

(f_i measure the number of boxes in the i-th row of the Young tableux corresponding to $[F]$) is given by

$$(\lambda\mu) = (3f_1 + f_2 + 2f_3 - f_4 + f_6 - 3f_7 - 2f_8 - f_9,$$
$$f_2 - f_3 + 2f_4 - 2f_6 + 3f_7 + f_8 - f_9 - 3f_{10}).$$

Similarly for a U(15) partition

$$(\lambda\mu) = (4f_1 + 2f_2 + 3f_3 + f_5 + 2f_6 - 2f_7 - f_8 + f_{10} - 4f_{11} - 3f_{12} - 2f_{13} - f_{14},$$
$$f_2 - f_3 + 2f_4 - 2f_6 + 3f_7 + f_8 - f_9 - 3f_{10} + 4f_{11} + 2f_{12} - 2f_{14} - 4f_{15}).$$

Now knowing the N-particle leading SU(3) representations, it is trivial to write down the required $(N-1)$ particle representations using eq. (6).

3. How to use the tables

The single particle CFP $\langle (N-1)\ [F']\ S'(\lambda'\mu')\alpha';\ [1]\tfrac{1}{2}\ (\eta 0) \mid \}\ N\ [F]S(\lambda\mu)\alpha \rangle$ are stored in four tables, corresponding to the four cases; odd N and even N in $\eta = 3$ shell, odd N and even N in $\eta = 4$ shell. We have tabulated only those CFP for which $(\lambda\mu)$ is the leading representation in the highest orbital symmetry partition $[F]$. It is obvious to note that for $N = 2a + 1$ (N is odd), $[F] = [2^a 1]$ and $S = \tfrac{1}{2}$, similarly for $N = 2a$ (N is even, $[F] = [2^a]$ and $S = 0$.

Tables 1-4 consist of several sub-tables, each corresponding to a given number of particles (N). Above each of these sub-tables the value of N and the corresponding leading SU(3) representation $(\lambda\mu)$ [($LM\ MU$) is the symbol used in the tables] are specified.

When N is odd, the sub-tables consists of three columns. First column gives the $(N-1)$ particle SU(3) representation $(\lambda'\mu')$ ((L_1M_1) is the symbol used in the tables). The second column gives the CFP corresponding to the case $S'=0$ and the third column gives the CFP corresponding to the case $S'=1$. (It is obvious to note that only these two cases are possible for $S=\frac{1}{2}$).

When N is even, the tables consist of two columns. The first column gives the $(N-1)$ particle SU(3) representation and the second column gives the CFP corresponding to the case $S'=\frac{1}{2}$ (this is the only possibility for $S=0$).

Table 1. Single particle SU(3) parentage coefficients (Odd-N case)

⟨ (N−1) [F′] S1 (L1 M1); [1] 1/2 (30) | } (N) [F] S = 1/2 (LM MU) ⟩

N=1 (LM MU)=(3 0)

(L1 M1)	S1=0	S1=1
(0 0)	1·000000	0·000000

N=3 (LM MU)=(7 1)

(L1 M1)	S1=0	S1=1
(6 0)	0·707106	0·000000
(4 1)	0·000000	0·707106

N=5 (LM MU)=(10 1)

(L1 M1)	S1=0	S1=1
(8 2)	0·617914	0·000000
(9 0)	0·000000	0·632455
(7 1)	0·134839	0·447213

N=7 (LM MU)=(11 2)

(L1 M1)	S1=0	S1=1
(12 0)	0·443812	0·000000
(9 3)	0·342368	0·592999
(10 1)	0·000000	0·430945
(8 2)	0·207260	0·324799

N=9 (LM MU)=(10 4)

(L1 M1)	S1=0	S1=1
(10 4)	0·476095	0·000000
(11 2)	0·000000	0·486164
(8 5)	0·220388	0·492803
(9 3)	0·153958	0·300499
(7 4)	0·185445	0·311698

N=11 (LM MU)=(11 2)

(L1 M1)	S1=0	S1=1
(10 4)	0.442996	0·000000
(11 2)	0·000000	0·606925
(12 0)	0·204383	0·000000
(9 3)	0·157682	0·427889
(10 1)	0·162054	0·225054
(8 2)	0·170368	0·282347

N=13 (LM MU)=(9 3)

(L1 M1)	S1=0	S1=1
(12 0)	0·316227	0·000000
(10 1)	0·000000	0·290887
(9 3)	0·232896	0·403387
(8 5)	0·275821	0·477737
(7 4)	0·177423	0·307305
(8 2)	0·157116	0·238220
(6 3)	0·145634	0·256029

N=15 (LM MU)=(4 7)

(L1 M1)	S1=0	S1=1
(6 6)	0·388290	0·000000
(7 4)	0·000000	0·399999
(3 9)	0·236832	0·410206
(4 7)	0·091287	0·418330
(5 5)	0·178131	0·259807
(2 8)	0·149649	0·196338
(3 6)	0·144337	0·239792
(1 7)	0·099240	0·182156

N=17 (LM MU)=(1 7)

(L1 M1)	S1=0	S1=1
(2 8)	0·412510	0·000000
(3 6)	0·000000	0·539607
(4 4)	0·240355	0·332105
(0 9)	0·000000	0·402199
(1 7)	0·194452	0·214982
(2 5)	0·168400	0·310521

N=19 (LM MU=(0 3)

(L1 M1)	S1=0	S1=1
(0 6)	0·383885	0·000000
(1 4)	0·000000	0·743391
(2 2)	0·376968	0·000000
(3 0)	0·000000	0·397359

Table 2. Single particle SU(3) parentage coefficients (O-N case)

⟨ (N−1) [F′] S1 (L1 M1); [1] 1/2 (40) | } (N) [F] S=1/2 (LM MU) ⟩

N=1 (LM MU)=(4 0)

(L1 M1)	S1=0	S1=1
(0 0)	1·000000	0·000000

N=3 (LM MU)=(10 1)

(L1 M1)	S1=0	S1=1
(8 0)	0·707106	0·000000
(6 1)	0·000000	0·707106

N=5 (LM MU)=(15 1)

(L1 M1)	S1=0	S1=1
(12 2)	0·616441	0·000000
(13 0)	0·000000	0·626344
(11 1)	0·141421	0·455732

N=7 (LU MU)=(18 2)

(L1 M1)	S1=0	S1=1
(18 0)	0·442807	0·000000
(15 3)	0·339541	0·588102
(16 1)	0·000000	0·422577
(14 2)	0·213953	0·344123

N=9 (LM MU)=(19 4)

(L1 M1)	S1=0	S1=1
(18 4)	0·468835	0·000000
(19 2)	0·000000	0·474495
(16 5)	0·229183	0·486172
(17 3)	0·150588	0·315737
(15 4)	0·195766	0·324755

N=11 (LM MU)=(22 2)

(L1 M1)	S1=0	S1=1
(20 4)	0·436931	0·000000
(21 2)	0·000000	0·592156
(22 0)	0·198379	0·000000
(19 3)	0·168696	0·427723
(20 1)	0·162623	0·233549
(18 2)	0·181695	0·306065

N=13 (LM MU)=(22 3)

(L1 M1)	S1=0	S1=1
(24 0)	0·314347	0·000000
(20 5)	0·268460	0·464987
(21 3)	0·248069	0·384307
(22 1)	0·000000	0·273724
(19 4)	0·189075	0·327488
(20 2)	0·157105	0·273669
(18 3)	0·121807	0·267083

N=15 (L MU)=(19 7)

(L1 M1)	S1=0	S1=1
(20 6)	0·361365	0·000000
(21 4)	0·000000	0·364908
(17 9)	0·227240	0·393591
(18 7)	0·145149	0·360732
(19 5)	0·148318	0·250438
(16 8)	0·169253	0·285238
(17 6)	0·144583	0·254727
(15 7)	0·158616	0·269875

N=17 (LM MU)=(18 7)

(L1 M1)	S1=0	S1=1
(18 8)	0·348570	0·000000
(19 6)	0·000000	0·418379
(20 4)	0·176685	0·244125
(16 9)	0·189001	0·371190
(17 7)	0·135090	0·291192
(18 5)	0·144293	0·257158
(15 8)	0·161327	0·269239
(16 6)	0·142211	0·223958
(14 7)	0·142675	0·244723

N=19 (LM MU)=(19 3)

(L1 M1)	S1=0	S1=1
(18 6)	0·342248	0·000000
(19 4)	0·000000	0·506825
(20 2)	0·228396	0·000000
(21 0)	0·000000	0·229264
(17 5)	0·149369	0·368929
(18 3)	0·167064	0·228763
(19 1)	0·100232	0·195162
(16 4)	0·161578	0·272147
(17 2)	0·113166	0·213715
(15 3)	0.144886	0·234136

N=21 (LM MU)=(16 4)

(L1 M1)	S1=0	S1=1
(20 0)	0·242535	0·000000
(15 7)	0·222428	0·385257
(16 5)	0·195970	0·339430
(17 3)	0·161958	0·280520
(18 1)	0·081563	0·199789
(14 6)	0·166189	0·287849
(15 4)	0·140907	0·268055
(16 2)	0·118681	0·196033
(13 5)	0·147028	0·247556
(14 3)	0·123813	0·207688
(12 4)	0·087305	0·185695

N=25 (LM MU)=(4 12)

(L1 M1)	S1=0	S1=1
(6 12)	0·314626	0·000000
(7 10)	0·000000	0·376651
(8 8)	0·171623	0·254558
(3 15)	0·183809	0·318366
(4 13)	0·081797	0·346932
(5 11)	0·158712	0·230603
(6 9)	0·142013	0·249399
(2 14)	0·122197	0·159538
(3 12)	0·127910	0·206760
(4 10)	0 121807	0·213832
(1 13)	0·081845	0·159739
(2 11)	0·096095	0·166072
(0 12)	0·060483	0·095981

N=23 (LM MU)=(9 10)

(L1 M1)	S1=0	S1=1
(12 8)	0·298026	0·000000
(13 6)	0·000000	0·302166
(8 13)	0·190692	0·330289
(9 11)	0·186870	0·323669
(10 9)	0·092208	0·309276
(11 7)	0·144215	0·218483
(7 12)	0·135331	0·234401
(8 10)	0·141504	0·215586
(9 8)	0·119852	0·220978
(6 11)	0·116534	0·216194
(7 9)	0·117370	0·203249
(5 10)	0·114652	0·174767

N=27 (LM MU)=(1 10)

(L1 M1)	S1=0	S1=1
(2 12)	0·336349	0·000000
(3 10)	0·000000	0·471404
(4 8)	0·233216	0·299572
(5 6)	0·182574	0·333332
(0 13)	0·000000	0·329818
(1 11)	0·188746	0·180648
(2 9)	0·159317	0·290129
(3 7)	0·126157	0·271213

N=29 (LM MU)=(0 4)

(L1 M1)	S1=0	S1=1
(0 8)	0·321633	0·000000
(1 6)	0·000000	0·659153
(2 4)	0·371390	0·000000
(3 2)	0·000000	0·538196
(4 0)	0·185695	0·000000

Table 3. Single particle SU(3) parentage coefficients (even-N case)

⟨ (N−1) [F′] S1 (L1 M1); [1] 1/2 (30) | } (N) [F] S=0 (LM MU) ⟩

N=2 (LM MU)=(6 0)

(L1 M1)	S1=1/2
(3 0)	1·000000

N=4 (LM MU)=(8 2)

(L1 M/1)	S1=1/2
(7 1)	08·45154
(5 2)	0·534522

N=6 (LM MU)=(12 0)

(L1 M1)	S1=1/2
(10 1)	0·912871
(9 0)	0·408248

N=8 (LM MU)=(10 4)

(L1 M1)	S1=1/2
(11 2)	0·617914
(8 5)	0·603558
(9 3)	0·381381
(7 4)	0·329313

N=10 (LM MU)=(10 4)

(L1 M1)	S1=1/2
(10 4)	0·638748
(11 2)	0·376587
(8 5)	0·482848
(9 3)	0·317565
(7 4)	0·340868

N=12 (LM MU)=(12 0)

(L1 M1)	S1=1/2
(11 2)	0·811997
(10 1)	0·487949
(9 0)	0·320255

N=14 (LM MU)=(6 6)

(L1 M1)	S1=1/2
(9 3)	0·451753
(5 8)	0·492145
(6 6)	0·468805
(7 4)	0·305441
(4 7)	0·295742
(5 5)	0·317678
(3 6)	0·228596

N=16 (LM MU)=(2 8)

(L1 M1)	S1=1/2
(4 7)	0·567821
(5 5)	0·377964
(1 10)	0·458964
(2 8)	0·332859
(3 6)	0·345269
(0 9)	0·203644
(1 7)	0·229339

N=18 (LM MU)=(0 6)

(L1 M1)	S1=1/2
(1 7)	0·690065
(2 5)	0·507092
(3 3)	0·516397

N=20 (LM MU)=(0 0)

(L1 M1)	S1=1/2
(0 4)	1·000000

Table 4. Single particle SU(3) parentage coefficients (even-N case)

$\langle$ (N−1) [F′] S1 (L1 M1); [1] 1/2 (40) | $\}$ (N) [F] S=0 (LM MU) $\rangle$

N=2 (LM MU)=(8 0) (L1 M1)	S1=1/2
(4 0)	1·000000

N=4 (LM MU)=(12 2) (L1 M1)	S1=1/2
(10 1)	0·836660
(8 2)	0·547722

N=6 (LM MU)=(18 0) (L1 M1)	S1=1/2
(15 1)	0·907485
(14 0)	0·420083

N=8 (LM MU)=(18 4) (L1 M1)	S1=1/2
(18 2)	0·585778
(15 5)	0·597614
(16 3)	0·381880
(14 4)	0·392286

N=10 (LM MU)=(20 4) (L1 M1)	S1=1/2
(19 4)	0·625131
(20 2)	0·364907
(17 5)	0·487087
(18 3)	0·322317
(16 4)	0·367298

N=12 (LM MU)=(24 0) (L1 M1)	S1=1/2
(22 2)	0·793725
(21 1)	0·497460
(20 2)	0·350046

N=14 (LM MU)=(20 6) (L1 M1)	S1=1/2
(22 3)	0·416754
(18 8)	0·477105
(19 6)	0·429534
(20 4)	0·307805
(17 7)	0·345851
(18 5)	0·318401
(16 6)	0·313766

N=16 (LM MU)=(18 8) (L1 M1)	S1=1/2
(19 7)	0·485327
(20 5)	0.310630
(16 10)	0·435132
(17 8)	0·347652
(18 6)	0·285194
(15 9)	0·319595
(16 7)	0·301356
(14 8)	0·288912

N=18 (LM MU)=(18 6) (L1 M1)	S1=1/2
(18 7)	0·496138
(19 5)	0·317887
(20 3)	0·300097
(16 8)	0·399999
(17 6)	0·292804
(18 4)	0·265849
(15 7)	0·309442
(16 5)	0·270080
(14 6)	0·278616

N=22 (LM MU)=(14 8) (L1 M1)	S1=1/2
(18 4)	0·339339
(13 11)	0·392619
(14 9)	0·368693
(15 7)	0·337293
(16 5)	0·242640
(12 10)	0·300886
(13 8)	0·273632
(14 6)	0·239767
(11 9)	0·269021
(12 7)	0·253696
(10 8)	0·249984

N=24 (LM MU)=(6 12) (L1 M1)	S1=1/2
(9 10)	0·418330
(10 8)	0·280305
(5 15)	0·369556
(6 13)	0·339368
(7 11)	0·283258
(8 9)	0·286186
(4 14)	0·283565
(5 12)	0·251254
(6 10)	0·247108
(3 13)	0·202967
(4 11)	0·219725
(2 12)	0·196141

N=26 (LM MU)=(2 12) (L1 M1)	S1=1/2
(4 12)	0·479017
(5 10)	0·338264
(6 8)	0·356752
(1 15)	0·367302
(2 13)	0·281453
(3 11)	0·320343
(4 9)	0·304341
(0 14)	0·142433
(1 12)	0·216186
(2 10)	0·229022

N=20 (LM MU)=(20 0) (L1 M1)	S1=1/2
(19 3)	0·720749
(18 2)	0·507092
(17 1)	0·396498
(16 0)	0·257226

N=28 (LM MU)=(0 8) (L1 M1)	S1=1/2
(1 10)	0·583503
(2 8)	0·449489
(3 6)	0·511766
(4 4)	0·442242

N=30 (LM MU)=(0 0) (L1 M1)	S1=1/2
(0 4)	1·000000

In the tables $S1$ is the symbol used for S'. In all the tables the degeneracy (α) of the SU(3) representations are not mentioned as it is made clear in the text that the CFP can be made exist for $\alpha=1$ only. As an example, we can read from table 2

$$\langle 8\,[2^4]\,S' = 0\ (18\ 4);\ [1]\,\tfrac{1}{2}\,(40)\,|\}\,9\,[2^41]\,S = \tfrac{1}{2}\,(19\ 4)\rangle = 0{\cdot}468835$$

$$\langle 8\,[2^31^2]\,S' = 1\ (16\ 5);\ [1]\,\tfrac{1}{2}\,(40)\,|\}\,9\,[2^41]\,S = \tfrac{1}{2}\,(19\ 4)\rangle = 0{\cdot}486175.$$

Similarly, we can read from table 4

$$\langle 7\,[2^31]\,S' = \tfrac{1}{2}\ (18\ 2);\ [1]\,\tfrac{1}{2}\,(40)\,|\}\,8\,[2^4]\,S = 0\ (18\ 4)\rangle = 0{\cdot}585178$$

The CFP corresponding to $\eta = 2$ shell are not included in the tables as they can be read out directly from the tables of Akiyama (1966).

A few cases in $\eta = 3$ shell (odd-N) are worked out by Ratna Raju (1972).

4. Conclusions

Now, with the availability of single particle CFP, we can take up the study of the spectral properties of all deformed nuclei using a quadrupole-quadrupole type of two-body interaction. This work is under progress. In the near future, we hope to give tabulations for two particle CFP not only for the case of leading representation but also for a few lower representations. In the present article, all the calculations were done by hand and made a machine print out to bring the results to the required format for publication.

Acknowledgements

The author is very much thankful to Dr R D Ratna Raju for bringing to his notice the reference (Hecht 1974). His thanks are due to Prof. S P Pandya for the excellent working facilities provided in PRL. Financial assistance by CSIR (New Delhi) is gratefully acknowledged.

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