Pramāṇa, Vol. 9, No. 2, August 1977, pp.119-128, © Printed in India.

Neutrino scattering on polarized deuterons

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MS received 31 January 1977; revised 18 April 1977

Abstract. The possible presence of an $I=0$ axial vector piece in the hadronic neutral current may be detected by looking for an asymmetry in the emission of the recoil deuterons in elastic scattering of neutrinos or antineutrinos on polarized deuterons. It is estimated that this asymmetry could be about 40% with the incident neutrinos in the energy range of tens of MeV.

Keywords. Neutrino scattering; neutral current; axial isoscalar current; polarized deuteron; asymmetry.

1. Introduction

Of great importance in the field of neutral current weak interactions is the determination of the space-time structure and isospin structure of the hadronic neutral current. At present due to meagre data only some limited information is available on this question: Dominant scalar and pseudoscalar interactions seem to be ruled out by experiments on inclusive neutrino scattering (Barish *et al* 1975) as well as those on elastic scattering of neutrinos on protons as noted by Fischbach *et al* (1976, 1977) analysing the recently available data (Lee *et al* 1976, Cline *et al* 1976). As for the isospin structure, Gargamelle data on single pion production by neutrinos indicate the presence of a dominant isovector current (Bertrand Coremans *et al* 1976).

A question of special interest relating to the structure of the hadronic neutral current, however, is the possible existence of an isoscalar axial vector term. Some gauge models, for instance, the $SU(3)\times U(1)$ gauge model of weak and electromagnetic interactions (Pandit 1977) proposed to accommodate the Kolar events, need the presence of such a term in the neutral current. Apart from providing a test for such theoretical proposals, the establishment of the presence or absence of the isoscalar axial current is clearly important for weak interaction phenomenology. In this context Pais and Treiman (1974) have suggested that one might look for differences in the two elastic reactions $\nu d\to\nu d$ and $\bar{\nu}d\to\bar{\nu}d$, which could arise due to the simultaneous presence of $I=0$ vector and axial vector currents. Mani and Roy (1975) proposed that one could check for parity violation in the reaction $e^+e^-\to\Lambda\bar{\Lambda}$ by measuring the longitudinal polarization of Λ, which is estimated to be of the order of $10^{-4}\,s$ where s is the square of cm energy measured in $(\text{GeV})^2$.

In this paper we would like to investigate the implications of the presence of the $I=0$ axial vector current in the reaction $\nu(\bar{\nu})+d\to\nu(\bar{\nu})+d$ when the target deuteron is polarized.

This study consists of calculating the up-down asymmetry in the momentum distribution of the recoil deuterons when the incident neutrinos are in the energy range of tens of MeV. In section 2 we discuss the general theory of elastic scattering of neutrinos on polarized deuteron targets under the impulse approximation. Section 3 is devoted to a calculation of the up-down asymmetry and section 4 for discussion of our investigations.

2. Elastic scattering of neutrinos on polarized deuterons

The matrix element for the elastic scattering of neutrino on a deuteron

$$\nu(k_1, \mathbf{k}_1)+d(E_i, \mathbf{P}_i) \rightarrow \nu(k_2, \mathbf{k}_2)+d(E_f, \mathbf{P}_f) \tag{1}$$

(where the quantities in brackets denote energies and momenta), may be written using the impulse approximation as

$$\langle f|T|i\rangle = \langle \mathbf{P}_f, m_f | t_p\, e^{i\mathbf{k}\cdot\mathbf{r}_p}+t_n\, e^{i\mathbf{k}\cdot\mathbf{r}_n} | \mathbf{P}_i, m_i\rangle \tag{2}$$

$$\mathbf{k}=\mathbf{k}_1-\mathbf{k}_2 \tag{3}$$

Here m_i and m_f denote respectively the initial and final magnetic quantum numbers of the deuteron, $\mathbf{k}$ is the momentum transferred to the deuteron, $\mathbf{r}_p$ and $\mathbf{r}_n$ denote the position coordinates of the proton and neutron, and the amplitudes $t_{p,\,n}$ in the impulse approximation represent the amplitudes for a neutrino to scatter elastically. Only minor changes are needed to discuss the case of antineutrino scattering and these shall be indicated at the appropriate places.

Assuming a current-current interaction in which the incident neutrino (either e-type or μ-type) is left-handed and identical to the final state neutrino, and assuming an arbitrary mixture of only vector and axial vector neutral currents to be present at the hadronic vertex, the amplitudes $t_{p,\,n}$ will have the form

$$t_{p,\,n}=\frac{G}{\sqrt{2}} J_\mu{}^o l_\mu \tag{4}$$

where the leptonic current is

$$l_\mu=\bar{u}(\mathbf{k}_2)\gamma_\mu(1\mp\gamma_5)u(\mathbf{k}_1). \tag{5}$$

The u's denote the spinors of the neutrino. Here, as well as in the following expressions, the upper (lower) sign refers to the case of a neutrino (antineutrino) incident on the deuteron. The isoscalar neutral hadronic current is

$$J_\mu{}^o=\overline{U}(\mathbf{p}_2)\gamma_\mu(1-b\gamma_5)U(\mathbf{p}_1) \tag{6}$$

where the U's denote the nucleon spinors with initial and final momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively. The quantity b, which is the ratio of axial vector to vector form factors, shall be assumed to be independent of the momentum transfer. The parameter G which is a measure of the strength of the neutral current weak interaction will also be regarded as a constant and of order 10^{-5} (GeV)$^{-2}$.

The deuteron for simplicity will be described by a pure S-state wave function

$$|\,\mathbf{P},\, m\rangle = \frac{1}{(2\pi)^{3/2}}\, e^{i\mathbf{P}.\mathbf{R}}\, \frac{1}{(4\pi)^{1/2}}\, \psi(r)^3\chi_m \tag{7}$$

$$\mathbf{R} = \tfrac{1}{2}\,(\mathbf{r}_p + \mathbf{r}_n);\; \mathbf{r} = \mathbf{r}_p - \mathbf{r}_n$$

and ${}^3\chi_m$ denotes the triplet-spin state wave function.

The initial state of the polarized deuteron is represented by the density matrix ρ^i which has the form

$$\rho^i = \tfrac{1}{3} \sum_{k=0}^{2} \sum_{q=-k}^{k} (-1)^q J_q^k\, Q_{-q}^k \tag{8}$$

where the spherical tensor operators J_q^k are given by

$$J_0^0 = 1;\; J_q^1 = \mathbf{J}_q;\; J_q^2 = (\mathbf{J} \otimes \mathbf{J})_q^2;$$

in terms of the angular momentum operator **J** of the deuteron. Here the Q's specify the state of the polarization of the target. If the deuteron is purely vector polarized, which will be the only case of interest for us here, then

$$Q_0^0 = 1$$

$$Q^1_{\pm 1} = \mp \frac{1}{\sqrt{2}}(Q_x \pm i\, Q_y);\; Q_0^1 = Q_z$$

$$Q_q^2 = 0$$

where Q_x, Q_y, Q_z are the Cartesian components of the polarization vector **Q** of the deuteron. The final state density matrix of the recoil deuteron is given by

$$\rho^f = \langle f\,|\,T\,|\,i\rangle\, \rho^i\, \langle f\,|\,T\,|\,i\rangle^* \tag{9}$$

and the differential cross section for the process (1) is proportional to the trace of ρ^f.

We note that the amplitude t defined in eq. (4) can be cast for non relativistic nucleons in the convenient form

$$t = \mathscr{L} + i\, \boldsymbol{\sigma} \,.\, \mathscr{K} \tag{10}$$

where $\boldsymbol{\sigma}$ denote the Pauli-spin matrices of the nucleon, **K** and $\mathscr{L}$ denote the spin-dependent and spin-independent amplitudes for the process of neutrino-nucleon elastic scattering

$$\nu(k_1, \mathbf{k}_1) + N\,(E_1, \mathbf{p}_1) \rightarrow \nu(k_2, \mathbf{k}_2) + N\,(E_2, \mathbf{p}_2) \tag{11}$$

where N denotes a nucleon of the deuteron. It is easily calculated from eqs (4)–(6) that

$$\mathscr{L} = \frac{G}{\sqrt{2}}\left[\, l_0 - \frac{1}{2M}\, \mathbf{l}\,.\,(2\mathbf{p}_1 + \mathbf{k})\right] \tag{12}$$

$$\mathscr{K} = \frac{G}{\sqrt{2}}\left[-\, ib\, \mathbf{l} + \frac{1}{2\mathrm{M}}\{\mathbf{l} \times \mathbf{k} + ib\; l_0\,(2\mathbf{p}_1 + \mathbf{k})\}\right] \tag{13}$$

where we neglected terms of the order of $1/4\ M^2$, M being the mass of the nucleon. Here

$$l_0 = [2\,(1+\hat{k}_1 \cdot \hat{k}_2)]^{\frac{1}{2}} \tag{14}$$

and

$$\mathbf{l} = \left[\frac{2}{1+\hat{k}_1 \cdot \hat{k}_2}\right]^{\frac{1}{2}} (\hat{k}_1+\hat{k}_2 \pm i\,\hat{k}_2 \times \hat{k}_1) \tag{15}$$

where the upper and lower signs refer to the cases when the projectiles are neutrinos and antineutrinos respectively.

Defining the momentum space wave function of the deuteron by

$$\phi(p) = \frac{1}{(2\pi)^{3/2}} \int d^3r \frac{\psi(r)}{(4\pi)^{1/2}} e^{-i\,\mathbf{p}\cdot\mathbf{r}} \tag{16}$$

and performing integrations over $\mathbf{r}_p$ and $\mathbf{r}_n$ in eq. (2) we have

$$\langle f\,|T|\,i\rangle = \langle 3\chi_{m_f}\,|L+\tfrac{1}{2}\,i\,(\sigma_p+\sigma_n)\cdot\mathbf{K}|\,3\chi_{m_i}\rangle \times \delta^3\,(\mathbf{P}_i+\mathbf{k}-\mathbf{P}_f) \tag{17}$$

where

$$L \equiv \frac{G}{\sqrt{2}}\left[\left\{ l_0 - \frac{1}{2M}\mathbf{l}\cdot\mathbf{P}_i+\mathbf{k}\right\} F_0 - \frac{1}{2M}\,\mathbf{l}\cdot\mathbf{k}\,F_1\right] \tag{18}$$

$$\mathbf{K} \equiv \frac{G}{\sqrt{2}}\left[\left\{-i\,b\,\mathbf{l} + \frac{1}{2M}(\mathbf{l}\times\mathbf{k} + i\,b\,l_0\,(\mathbf{P}_i+\mathbf{k}))\right\} F_0 + \frac{1}{2M} i\,b\,l_0\,\mathbf{k}\,F_1\right]. \tag{19}$$

The structure functions of the deuteron F_0 and F_1 depend on the momentum transfer k, and are defined by

$$F_0(k) \equiv 2\int \phi^*(\mathbf{p}\pm\tfrac{1}{2}\mathbf{k})\phi(\mathbf{p})d^3p = 2\int_0^\infty r^2dr\psi^*(r)\,j_0(kr/2)\psi(r), \tag{20}$$

$$\mathbf{k}(F_1(k) \equiv 2\int \phi^*(\mathbf{p}\pm\tfrac{1}{2}\mathbf{k})(\pm 2\mathbf{p})\phi(\mathbf{p})d^3p = 4\hat{k}\int_0^\infty r^2dr\psi^*(r)j_1(kr/2)\frac{d\psi(r)}{dr} \tag{21}$$

wherein the space integrals are independent of the sign $\pm$.

Now to calculate the density matrix of the final state ρ^f we proceed as follows. Expressing the transition operator in eq. (17) in terms of spherical tensors S we have

$$L+\tfrac{1}{2}i\,(\sigma_p+\sigma_n)\cdot\mathbf{K} = \sum_{\lambda=0}^{1}\ \sum_{\mu=-\lambda}^{\lambda} (-1)^\mu (i)^\lambda S_\mu^{\ \lambda} K_{-\mu}^{\ \lambda} \tag{22}$$

where

$$S^o=1;\ S_\mu^{\ 1} = \tfrac{1}{2}\,(\sigma_p+\sigma_n)\,\mu;$$

$$K^o=L;\ K_\mu^{\ 1}=\mathbf{K}_\mu. \tag{23}$$

Using the standard techniques of Racah algebra we can then write ρ^f defined in eq. (9) to be

$$\rho^f_{m'm} = \tfrac{1}{3}(-1)^{m-m'} \sum_{k=0}^{2} C(1\,k\,1;\,m\,q\,m')\,\langle\,J^k\,\rangle\,R^k_q \tag{24}$$

where

$$R_q^k \equiv 3 \sum_{\lambda, \lambda', \Lambda, k'} \frac{\langle J^{k'} \rangle}{\langle J^k \rangle} \langle S^{\lambda'} \rangle \langle S^{\lambda} \rangle (2\Lambda+1)^{1/2}(2k+1)^{1/2}$$

$$\times (-1)^{k'+\lambda+\lambda'-k} (i)^{\lambda+\lambda'} \begin{Bmatrix} 1 & \lambda & 1 \\ k' & \Lambda & k \\ 1 & \lambda' & 1 \end{Bmatrix} \left(Q^{k'} \otimes (K^{\lambda'} \otimes k^{*\lambda})^{\Lambda}\right)_q^k. \quad (25)$$

Here the symbol C denotes the Clebsch-Gordan coefficient, the curly bracket denotes the Wigner $9-j$ symbol and quantities enclosed by angular brackets are the reduced matrix elements between states of unit angular momentum

$$\langle S^{\lambda} \rangle \equiv \langle 1 \| S^{\lambda} \| 1 \rangle.$$

As we are interested in the cross section we need to calculate the trace of ρ^f which is given by

$$T_r(\rho^f) = \tfrac{1}{3}[3LL^* + 2\mathbf{K}\cdot\mathbf{K}^* + 2i\mathbf{Q}\cdot(L^*\mathbf{K} - L\mathbf{K}^* - \mathbf{K}\times\mathbf{K}^*)] \quad (26)$$

where $\mathbf{Q}$ is the initial polarization of the deuteron.

3. Calculation of the asymmetry

Substituting the values of L and $\mathbf{K}$ from eqs (18) and (19) in eq. (26) we notice that the cross section depends on the azimuthal angle of the emitted deuteron, leading thereby to an asymmetry with respect to the direction of the target polarization. To see this we shall choose the laboratory frame ($\mathbf{P}_t = 0$), with z axis along the incident neutrino direction and x axis along the polarization of the target deuteron. With respect to this system of axes the polar and azimuthal angles of the final deuteron will be denoted by θ and ϕ. We define the asymmetry function to be

$$A(\theta) = \frac{\int_{-\pi/2}^{\pi/2} d\phi \left[\frac{d\sigma}{d\Omega}(\theta, \phi) - \frac{d\sigma}{d\Omega}(\theta, \pi + \phi)\right]}{\int_{-\pi/2}^{\pi/2} d\phi \left[\frac{d\sigma}{d\Omega}(\theta, \phi) + \frac{d\sigma}{d\Omega}(\theta, \pi + \phi)\right]} \quad (27)$$

and also the integrated asymmetry which will be denoted simply by A:

$$A \equiv \frac{\int_0^{\pi/2} \sin\theta\, d\theta \int_{-\pi/2}^{\pi/2} d\phi \left[\frac{d\sigma}{d\Omega}(\theta, \phi) - \frac{d\sigma}{d\Omega}(\theta, \pi + \phi)\right]}{\int_0^{\pi/2} \sin\theta\, d\theta \int_{-\pi/2}^{\pi/2} d\phi \left[\frac{d\sigma}{d\Omega}(\theta, \phi) + \frac{d\sigma}{d\Omega}(\theta, \pi + \phi)\right]}. \quad (28)$$

The quantity A represents the fractional difference in the number of deuterons going above the yz-plane and the number going below. This will be referred to as the up-down asymmetry.

We shall restrict ourselves to the incident neutrino energies in the range of tens of MeV (at the Los Alamos meson factory the neutrino energies do not exceed 60 MeV) and calculate $A(\theta)$ and A. At such energies, at least to start with, it may be a good approximation to neglect $(k/2M)$ terms in eqs (18) and (19), so that

$$L = \frac{G}{\sqrt{2}} l_0 F_0(k)$$

$$\mathbf{K} = -\frac{G}{\sqrt{2}} i b \mathbf{l} F_0(k). \tag{29}$$

The asymmetry function $A(\theta)$ will of course be independent of the structure functions of the deuteron, and even the integrated asymmetry A will be almost independent of the details of the wave function of the deuteron, because $F_0(k)$ does not vary by more than 1% at these momentum transfers for Hamada-Johnston wave functions (Hamada and Johnston 1962). The expressions for the asymmetry in this approximation, for the case of 100% deuteron polarization ($|\mathbf{Q}| = 1$), are

$$A(\theta) = \frac{8}{\pi} b(\pm b - 1) \frac{\cos\theta \sin\theta}{3\sin^2\theta + 2b^2(1 + \cos^2\theta)} \tag{30}$$

$$A = \frac{1}{\pi} \frac{b(\pm b - 1)}{b^2 + \frac{3}{4}} \tag{31}$$

where the upper (lower) sign corresponds to the case of incident neutrino (antineutrino) and b is the ratio of axial vector to vector couplings.

The fact that the above asymmetries are proportional to the parameter b is easily understood. Since the asymmetry is being defined in relation to the direction of the initially polarized deuteron, its calculation in the impulse approximation will involve the hadronic matrix element of the spin operator $\boldsymbol{\sigma}$ of the nucleon. However, neglecting $k/2M$ terms, the axial vector hadronic current alone contributes to the asymmetry as it gives rise to the desired matrix elements $\langle \boldsymbol{\sigma} \rangle$ (the vector part yields matrix elements of the unit operator and hence does not contribute); and thus the asymmetry is proportional to the axial vector coupling strength b.

The above expressions do not depend on the lepton incident energy as long as $k_1 \ll 2M$. The function $A(\theta)$ in eq. (30) is plotted in figure 1 as a function of the lab scattering angle θ of the final deuteron, for the case of incident neutrinos. In figure 2 the integrated asymmetry A of eq. (31) is plotted wherein the asymmetries for ν and $\bar{\nu}$ are connected by the relation

$$A_\nu(-b) = -A_{\bar{\nu}}(+b). \tag{32}$$

Notice that when $b = 0$ the asymmetry vanishes because of the neglect of the $k/2M$ terms. When $b = 1$ (i.e. the coupling at the hadronic vertex is $V - A$) the asymmetry

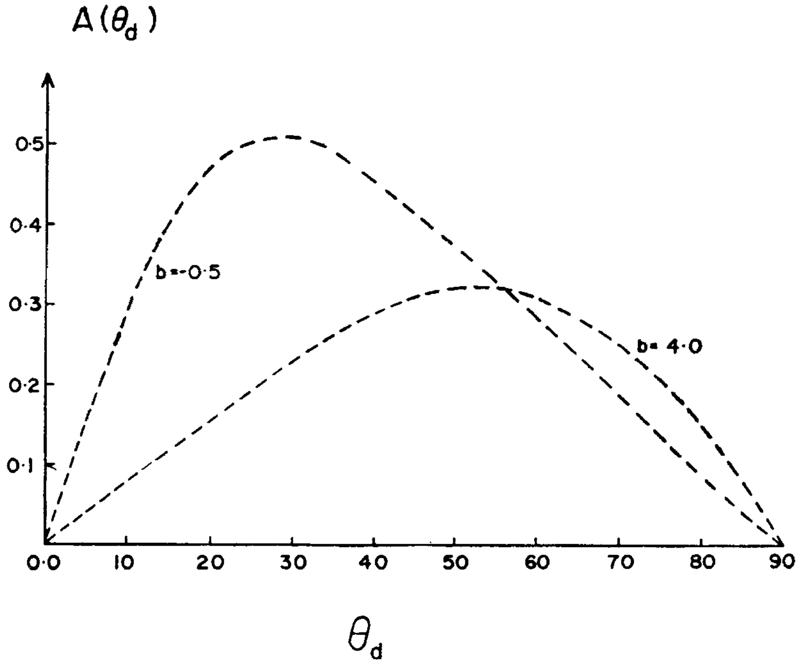

Figure 1. The asymmetry function $A(\theta)$ as a function of the laboratory angle of the recoil deuteron in $\nu d \rightarrow \nu d$, for $b=-0{\cdot}5$ and $b=4{\cdot}0$.

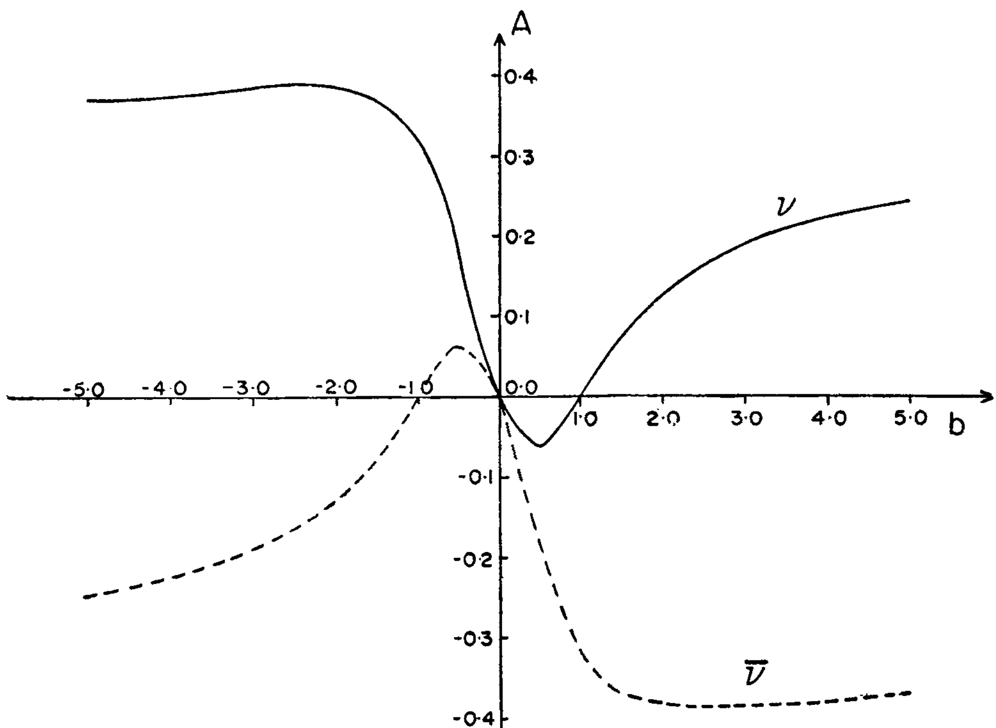

Figure 2. The integrated asymmetry A defined in eq. (31) for incident neutrinos and antineutrinos.

vanishes for the case of ν, while it is about -37% for the case of $\bar{\nu}$. It is evident from eq. (31) that the magnitude of the asymmetry for any value of b cannot exceed about 40%.

However, for a more exact computation one has to retain the $k/2M$ terms in the eqs (18) and (19) for L and **K**. For the calculation of the structure functions F_0 and F_1 we have used the deuteron wave function corresponding to the Hamada-Johnston potential. As to be expected, the inclusion of these recoil terms does not significantly modify the asymmetry at neutrino energies in the range of tens of MeV. For instance, with a 90 MeV neutrino beam, taking $b=-2$ the calculated asymmetry gets reduced by less than 2% as a result of the inclusion of the $k/2M$ terms. These modifications are therefore too small even to be exhibited in figure 2.

4. Discussion

In connection with the question of the existence of an isoscalar axial vector term in the hadronic neutral current the proposed experimental tests consists of looking for effects of parity violation in reactions involving isoscalar hadrons. However, almost all the tests proposed to-date, including the present one, are based upon extremely difficult experiments—because of low counting rates, inevitable systematic errors in comparing two different reactions, poor resolution, back ground contaminations, etc.—and hence the investigation undertaken here may merely be an act of hubris. On the other hand, with the advent of powerful techniques such as dynamic polarization (de Boer *et al* 1974) and the availability of intense neutrino beams at the meson factories, one can be optimistic that the observation of elastic scattering of neutrinos on polarized deuterons may be possible.

Of the many limitations from the experimental point of view, perhaps the most severe one is the fact that the break-up probability of the final deuteron is large as we go to high energies. At low energies, on the other hand, the recoil momentum being too small severely limits the efficiency for detection of the recoiling nucleus. At 10 MeV with equal V and A we estimate the cross section for $\nu d \to \nu d$ to be of the order of 10^{-42} cm^2. It should be noted that from a theoretical viewpoint elastic scattering of neutrinos on any other polarized isoscalar spin-one nucleus would serve the same purpose as the reaction considered here. Another alternative is to look for an appropriate inelastic reaction in which a nucleus is excited by neutrinos and is allowed to decay by photon emission (Sarma 1974). A recent proposal in this connection is the study of the inelastic neutrino scattering on ^{12}C (Donnelly and Peccei 1976) to excite the 12.71 MeV level which has $I=0$ and spin-parity 1^+.

Acknowledgements

One of us (GR) would like to thank Profs K P Sinha, E C G Sudarshan and S Dhawan for the scheme under which his visit to the Centre for Theoretical Studies, Indian Institute of Science, Bangalore was made possible and all the three of us thank Prof N Mukunda for kind hospitality at the Centre during the summer of 1976 when this work was initiated. One of us (MVNM) wishes to gratefully acknowledge the award of a Junior Research Fellowship of CSIR.

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