Pramāṇa, Vol. 9, No. 1, July 1977, pp. 27–40.

Universality in weak interactions—The Cabibbo suppression

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MS received 15 March 1977

Abstract. An SU_3 gauge model of weak interactions, which generates the Cabibbo suppression spontaneously and consistently, is presented. Out of the eight currents, a triplet couples to relatively light gauge bosons and satisfies the commutation relations of the SU_2-universality algebra of Gell-Mann. The other five couple to necessarily very massive gauge bosons. The leptons have a realistic mass spectrum and the (bare) quarks are massless. The theory as it stands does not encompass charm and cannot suppress strangeness changing neutral currents; both these deficiencies require a larger gauge group for their correction.

Keywords: Universality; Cabibbo angle; SU_3-gauge model; ultraweak interactions.

1. Introduction

The term weak interaction encompasses a variety of processes marked by a whole hierarchy of effective measurable strengths. This is in strong contrast with the strong interactions and, even more, electromagnetic interactions. Indeed, about the only thing weak interactions appear to have approximately in common is the sets of selection rules they obey and disobey. In spite of this, there has been for quite some time a general hope (or, at least, a wish) that this wide variation of the effective strength hides a fundamental simplicity of this class of interactions. The cause of this optimism lies in the theory of spontaneously broken non-abelian gauge theories. If the gauge group is a simple one (one without an invariant subgroup) all couplings of the gauge bosons to appropriate fermion (lepton and quark) currents are of equal strength g. But when the gauge invariance is spontaneously broken, the gauge bosons not only " mix " among themselves to form eigenstates of the mass matrix, they also acquire a mass spectrum, non-degenerate in general. At energies much lower than the masses of the physical gauge bosons, the effective Fermi coupling between two currents coupling to the same boson W_i is given by $G_i = g^2/m^2{}_{W_i}$ and can therefore vary with i.

This very well-known line of reasoning merits being restated here if only because of the negligible role it has so far played in the construction of theories of weak interactions. The situation is best illustrated by the (" weak " part of the) Weinberg-Salam (Weinberg 1967, Salam 1968) class of models. The best documented deviation from universality is that described by the Cabibbo angle θ (Cabibbo 1963). The Weinberg-Salam model accommodates this by assuming that, for some as yet unknown reason, the single quark states which belong to irreducible representations of the gauge group (SU_2) come already Cabibbo-rotated, (q_p, $q_n \cos\theta + q_\lambda \sin\theta$), ($q_{p'}$, $-q_n \sin\theta + q_\lambda \cos\theta$) being the doublets. The second doublet was introduced because, having rotated q_n and therefore the charged current, it then becomes necessary to un-rotate the total neutral strangeness-changing current to accord with experiment (Glashow *et al* 1970, Weinberg 1972). Since the two doublets couple entirely independently to the gauge bosons, one is obliged to postulate (at least at the back of one's mind) an all-knowing force which chooses the two orthogonal combinations and puts them in different multiplets.

Ironically, the main reason for this negligence of the universality problem in weak interactions is the very success of the Weinberg-Salam model itself, especially the unification of the weak and electromagnetic forces. Having understood and appreciated the many beautiful properties of gauge theories from the detailed development of that model over the past decade, it is perhaps time to turn our attention to the problem of unifying the weak interactions themselves. The present paper describes the first steps in such an attempt. It focusses, as a beginning, on the generation of the Cabibbo suppression through spontaneous symmetry breakdown in a consistent way.

The paper begins by discussing in some, and sometimes obvious, detail the problem of universality (section 2). This discussion highlights the primary concern of the paper and sets down the point of view which guides the whole approach (for this reason, a sympathetic reading of this section is an absolute prerequisite to the understanding of the rest of the paper). The technical part of the paper then deals (in sections 3, 4 and 5) with an SU_3 gauge model with a triplet of quarks and two triplets of leptons. It turns out that if the fermions are not allowed to couple to the Higgs bosons so that they remain massless, the Cabibbo rotation of the charged weak currents comes about in a fairly straightforward manner. Fermion-Higgs couplings impose a number of non-trivial constraints on this procedure all of which are found not only not to change Cabibbo universality but to be essential in making all unwanted gauge-bosons very massive. This point is to be stressed: the mass spectrum of the bosons is not forced at any point.

The major results of the paper are summarised in the last section. Two points bear being stated here. The first is that this model does not get rid of strangeness changing neutral currents—the gauge group is too small for that, as will become obvious. Also for obvious reasons, no attempt has been made to incorporate electromagnetism, even though the extension of the SU_3 model to an $SU_3 \times U_1$ model in the manner of Weinberg and Salam is direct.

2. General guidelines—Universality and symmetry in weak interactions

There are two manifestations of the universality of weak interactions which are part of the traditional wisdom on the subject. These are (1) $e-\mu$ universality, i.e., the equality of the couplings of the $\bar{\nu}_e e$ and $\bar{\nu}_\mu \mu$ currents to any other given current and (2) the very near equality of the vector (V) coupling constants describing μ-decay and nuclear β-decay, i.e., those describing the interaction $(\bar{\nu}_\mu \gamma_\alpha \mu)$ $(\bar{e}\gamma_\alpha \nu_e)$ and, at the quark level, the interaction $(\bar{q}_n \gamma_\alpha q_p)$ $(\bar{e}\gamma_\alpha \nu_e)$. Since the renormalization of the corresponding axial vector (A) coupling constant due to strong interaction effects is well understood (Adler 1965, Weisberger 1965), we may consider (2) to hold for the A couplings as well, as long as we are speaking of quark couplings. With this proviso, we may add a third aspect of universality to this list: (3) the effective V and A couplings are equal.

Out of the large class of weak processes now accessible to experimental study, this list exhausts the cases in which universality is evident at the (low energy) effective 4-fermion level. As against this, there is first of all Cabibbo suppression—the coupling of the charged, strange, hadronic current to the leptonic current is a factor $\simeq \frac{1}{4}$ smaller than that of the charged non-strange current. This is well described by assuming that the effective quark current which couples universally to leptons is proportional (Cabibbo 1963, Gell-Mann 1964) to

$$\cos\theta\,(\bar{q}_p\gamma_{\alpha L}q_n)+\sin\theta(\bar{q}_p\gamma_{\alpha L}q_\lambda)\quad \text{with } \tan\theta\simeq\tfrac{1}{4},$$

where, and throughout this paper, $\gamma_{\alpha L}=\gamma_\alpha\frac{1}{2}(1+\gamma_5)$ and $\frac{1}{2}(1+\gamma_5)$ is the left-handed projection on a spinor. Next, there is the enormous suppression of the strange, neutral hadronic current as evidenced in the branching fractions $\mathrm{Br}(K_L\to\mu\bar{\mu})=(1{\cdot}0\pm0{\cdot}3)\times10^{-8}$, $\mathrm{Br}(K\to\pi e\bar{e})=2{\cdot}6\pm0{\cdot}5)\times10^{-7}$ (only upper limits being known in the case of other relevant strange particle decays) and the absence of a large $|\triangle S|=2$ contribution to the K_L-K_S mass difference. There are also a number of qualitatively different processes for which information on the effective strength is either just beginning to emerge, as in the case of the $\bar{\nu}_\mu\nu_\mu$ part of the leptonic neutral current which presumably couples dominantly to the non-strange neutral hadronic current, or is still anticipated, as in the case of charm changing currents.

The point of view that motivates this paper is that this hierarchy of effective coupling strengths must faithfully reflect the mass spectrum of the gauge vector bosons. In other words, a method of violating gauge symmetry is sought in which all weak currents, even though they couple to the corresponding gauge bosons with the same strength, are seen in their low energy couplings to other currents with a realistic spectrum of coupling constants. This is of course a large undertaking and the first hesitant approach to the problem described here is primarily exploratory. Specifically, we confine attention to understanding the best documented suppression mechanism, that denoted by the Cabibbo angle θ. Apart from its value as a test case for the idea described above, this problem is of interest even in isolation. The Weinberg-Salam model for example starts by assuming that the Cabibbo angle is given, the tan θ factor being already present in the couplings of the quarks to the gauge bosons before gauge invariance is broken. Later attempts to gauge-generate this suppression have generally taken the path of trying to obtain a rotation of the appropriate single-quark states induced by the weak interactions and so are formulated in the context of a unified theory of weak and strong (and of course electromagnetic) interactions. The procedure we follow is almost completely orthogonal to this. The picture which emerges attributes the rotation to the gauge bosons and it is the considerable freedom that remains in defining the lepton states within this framework that leads to the emergence of the Cabibbo form of the Hamiltonian.

In doing this, it is clearly necessary to have a standard against which to measure the rotation. Since strangeness and its conservation are properties which characterise the strong interactions, this standard is taken to be provided by the (quark) eigenstates of the strong interactions—i.e., we assume that strong interactions have already acted among the quarks and that we have already made a choice, to be rigidly adhered to in all that follows, of what are to be called q_p, q_n and q_λ The corresponding states are then uniquely characterised by a complete set of commuting observables pertaining to the strong (global) symmetry group, which is, naturally, taken to be SU_3 and to be an exact symmetry. Every SU_3 current bilinear in quarks is then a candidate to be a weak current and so the weak gauge group is necessarily also SU_3—more generally, our point of view obliges us to consider, even if we seek to generate only the Cabibbo suppression, a weak gauge group larger than SU_2. There must then be at least three leptons of each (e and μ) type together with their antiparticles, and the economical choice is to assign the left-handed projection of each multiplet to a triplet **3**: ν_{eL}, e_L and a "heavy" electron E_L and $\nu_{\mu L}$, μ_L and a "heavy" muon M_L.

Even though the weak gauge group is necessarily the "flavour" group, it is to be emphasised that there is no advantage in carrying the analogy further and assigning flavours to leptons. It is true that the quantum numbers arising from the gauge group, the weak isospin and the weak hypercharge, are identical with the corresponding strong quantum numbers where the two can be simultaneously defined, namely on the left-handed quarks. But for leptons, the standard frame of reference, which is provided by strong interactions in the case of quarks, is lacking and there is nothing against which to measure the tilts caused by the weak interactions. The electromagnetic interactions do provide a standard frame but since the only quantum number it provides, the electric charge Q, is strictly conserved also by weak interactions, it is obligatory to design the theory so as to produce no tilt with respect to Q; Q is therefore not a useful standard. There is thus a measure of freedom in defining leptonic states and this freedom will turn out to be crucial.

Closely related to the question of universality is that of the symmetries of weak interactions. Thus, the Cabibbo rotation may alternatively be looked upon as the violation of the conservation of weak isospin and weak hypercharge for the leptons and, therefore, that of *strong* isospin and *strong* hypercharge for the quarks. The question of whether all symmetry violation, especially that of discrete symmetries like P and CP, can be understood in this picture immediately raises itself. In the case of parity, the decision to treat the left and right-handed fermions differently has already closed this option to us. The problem of parity appears to be a tricky one to deal with in our universal framework, and at present it seems difficult to avoid more or less arbitrary devices such as the Weinberg-Salam trick of distinguishing between left and right-handed fermions. This goes against the basic prejudices of this paper since it allows the weak and strong groups to be identified only when they act on left-handed quarks. For this reason, a natural solution of the parity problem is of importance for the ultimate viability of our approach.

The status of CP-violation in gauge theories is also not much clearer. The recognition (Kobayashi and Maskawa 1973; more recent discussions of this problem may be traced from Weinberg 1976) that not all phases on current operators which look as though they might generate CP-violation have, in fact, this property, has underlined the importance of understanding when a theory is invariant under independent phase changes of individual fields. We choose here not to face this problem by limiting our discussion to real vacuum expectation values for the Higgs scalars and, thereby, to a CP-conserving model. Here again, a natural way of producing a realistic violation of CP would be a critical test of the correctness of our philosophy. Throughout this paper, the word " natural ", when qualifying a physical effect, means that the effect is produced by spontaneous symmetry breakdown and is not present in the fundamental Lagrangian of the theory.

There are also other unnatural features in the model, chief among them the doubling of leptons to accommodate the muon multiplet. It is deficient also in leaving the whole world of charm untouched. Experimentally, the model cannot accommodate the smallness of the strangeness-changing neutral current. In spite of all this, the success of the model in meeting the basic demand made of it, that of selfconsistently generating the Cabibbo suppression (and further work) has convinced me that a unified picture that achieves all these aims is certainly not impossible.

There are several reasons for not attempting to bring electromagnetism into the picture being developed here. Firstly, the time to do that would be when the model begins to become more comprehensive and realistic—it is not unreasonable to want to unify weak interactions first, before worrying about electromagnetism. Especially, as long as the parity problem is not solved, any unification is likely to remain provisional since electromagnetism is parity conserving. In any case, the Weinberg-Salam method of unification can easily be adopted here. Secondly, the logical case for this " second unification " is not very compelling; it has of course the advantage of making the whole theory renormalisable, but at our present stage of understanding of when a theory is renormalisable, this is perhaps a luxury which can be postponed. As far as the present paper is concerned, the advantage of unifying weak interactions takes precedence over the desirability of calculating higher order corrections (for an earlier discussion of this and related points, see Divakaran 1975).

3. The model: One Higgs multiplet, massless fermions

In accordance with the remarks in sec. 2, the gauge group is taken to be SU_3, the same as the global strong symmetry group, and the left-handed quark triplet $q_L=(q_{pL}, q_{nL}, q_{\lambda L})$ belongs to the representation **3**. More precisely, the quarks q_p, q_n and q_λ whose left-handed projections transform as **3** are exactly those which transform as the representation **3** under the strong symmetry group, and with same assignments of I and Y. The strong interactions are thus already assumed to have acted (as also the electromagnetic, so that we know what the charges are) and left the quarks labelled, a labelling we are not to depart from. They are also assumed to have left them massless, although all fields are 4-component Dirac spinors.

The left-handed leptons also belong to a **3**, $l_L=(\nu_L, e_{oL}, E_{oL})$, where E_0 is a new " heavy " electron. The muon multiplet merely duplicates the electron multiplet and can be added on at will—so it will not even be mentioned from now on. The subscript 0 denotes that the particles are bare in the sense that after the symmetry is spontaneously broken, these particles will have to be redefined to take account of the resulting weak interaction effects. This is important for the leptons since they are defined by the weak and electromagnetic interactions alone, and electromagnetism allows mixing between the similarly charged e_o and E_o—hence the lack of a subscript on ν. It is negligible for the quarks since their quantum numbers are already fixed by the strong SU_3 and electromagnetic and weak admixtures are expected to be small—the " physical " hadronic states are very nearly the same as the eigenstates of the strong SU_3. The right-handed projections of all fermions are, as conventionally, to transform as singlets. The leptons are also assumed to be described by massless Dirac spinors.

The choice of the Higgs multiplets is strongly constrained by the guidelines described in the previous section. Since all weak effects come from gauge-symmetry and its spontaneous violation, and since they must arise dominantly at the tree graph level (for one thing, we cannot calculate loop corrections yet), fermion masses in particular must be generated by gauge-invariant couplings of the fermions to the Higgs multiplets Φ of the form $\bar{q}_L q_R f(\Phi)+\bar{q}_R q_L f^{\dagger}(\Phi)$ (plus similar terms for the leptons) through the vacuum expectation values of Φ. The function f and, therefore, Φ itself must then transform as the representation **3** to ensure gauge invariance. It is of course possible to have Higgs multiplets which couple only to the gauge bosons without having a mass-generating coupling to the fermions, but that possibility certainly is not in the spirit of the approach and so is to be rejected. The Higgs particles thus come in two triplets:

$$\Phi=(\Phi_1, \Phi_2, \Phi_3),\ \Phi'=(\Phi'_1, \Phi'_2, \Phi'_3). \tag{1}$$

Making Φ_1 and Φ'_1 charged ($Q=+1$) and all the other Φ's neutral, the vacuum expectation values are

$$\langle\Phi\rangle=(o, \eta_2, \eta_3),\ \langle\Phi'\rangle=(o, \eta'_2, \eta'_3), \tag{2}$$

the η's and η's being taken to be real. This is sufficient to ensure that all gauge bosons acquire mass.

We shall carry out our study of symmetry breaking in two stages. In the first, to be described in the rest of this section and the next, we shall turn on only $\langle\Phi\rangle$, assuming $\langle\Phi'\rangle=0$. This procedure displays clearly some interesting properties of the complete theory in the limit $\eta'_2, \eta'_3\to 0$. In this limit, there is a triplet of gauge bosons which remain massless, corresponding to exact gauge invariance under an SU_2 subgroup. This subgroup is in fact generated by Gell-Mann's universality algebra (Gell-Mann 1964). Nor can the electron acquire a mass in this limit (the neutrino always remains massless). As $\langle\Phi'\rangle$ is turned on, both the electron and this triplet of bosons become massive together, the other five (and the heavy electron) remaining still very heavy. Thus, the smallness of the mass of e and the dominance of the Gell-Mann-Cabibbo currents are intimately tied together.

Denote the eight gauge boson fields by U_a, $a=1, ..., 8$ in the usual hermitian basis for the adjoint representation of SU_3 and by U^Q_I in the isospin-charge basis:

$$\begin{aligned} &U^{\pm}_{1}=\frac{1}{\sqrt{2}}(U_1\pm iU_2),\ U^{\pm}_{\frac{1}{2}}=(U_4\pm iU_5),\\ &U^{0}_{\frac{1}{2}}=\frac{1}{\sqrt{2}}(U_6+iU_7),\ \overline{U}^{0}_{\frac{1}{2}}=\frac{1}{\sqrt{2}}(U_6-iU_7),\\ &U^{0}_{1}=U_3,\qquad U^{0}_{0}=U_8 \end{aligned} \tag{3}$$

The U-boson mass term generated by the gauge-invariant $U-\Phi$ interaction is then

$$M^2_{ab}U^{\mu}_{a}U_{\mu b}=g^2\langle\Phi\rangle^{\dagger}\lambda_a s\lambda_b\langle\Phi\rangle U^{\mu}_{a}U_{\mu b},$$

where g is the gauge coupling constant and $\{\lambda_a\}$ are the Gell-Mann matrices. So

$$M^2_{ab}=\tfrac{2}{3}g^2\langle\Phi\rangle^\dagger\langle\Phi\rangle\delta_{ab}+g^2 d_{abc}\langle\Phi\rangle^\dagger\lambda_c\langle\Phi\rangle,$$

d_{abc} being the symmetric d-coefficients. Confining attention to the charged sector, we have then

$$M^2_{ch}=g^2\begin{pmatrix}\eta_2^2 & \eta_2\eta_3\\ \eta_2\eta_3 & \eta_3^2\end{pmatrix} \tag{4}$$

in the $(U^\pm_1, U^\pm_{\frac{1}{2}})$ basis. The corresponding physical bosons which diagonalise M^2_{ch} are

$$W^\pm=\cos\theta\, U^\pm_{\frac{1}{2}}+\sin\theta\, U^\pm_{\frac{1}{2}} \tag{5}$$

and

$$X^\pm=-\sin\theta\, U^\pm_1+\cos\theta\, U^\pm_{\frac{1}{2}}, \tag{6}$$

with

$$\tan\theta=-\eta_2/\eta_3. \tag{7}$$

The physical masses are

$$M^2(W^\pm)=0,\ M^2(X^\pm)=g^2(\eta_2^2+\eta_3^2)=g^2\eta_3^2\sec^2\theta \tag{8}$$

The fermion currents which couple to $W^\pm$ and $X^\pm$ are mixtures of J_1, J_2, J_4, and J_5 (in the hermitian basis). They are given by

$$J^+_{al}(W)=\cos\theta\,\bar{\nu}_o\,\gamma_{aL}e_o+\sin\theta\,\bar{\nu}_o\,\gamma_{aL}E_o, \tag{9}$$

$$J^+_{aq}(W)=\cos\theta\,\bar{q}_p\gamma_{aL}q_n+\sin\theta\,\bar{q}_p\gamma_{aL}\bar{q}_\lambda, \tag{10}$$

$$J^+_{al}(X)=-\sin\theta\,\bar{\nu}_o\gamma_{aL}e_o+\cos\theta\,\bar{\nu}_o\gamma_{aL}E_o, \tag{11}$$

$$J^+_{aq}(X)=-\sin\theta\,\bar{q}_p\gamma_{aL}q_n+\cos\theta\,q_p\gamma_{aL}q_\lambda, \tag{12}$$

with the interaction Lagrangian for the charged currents written as

$$\mathscr{L}_{ch}=g[J^+_a(W)W^{a-}+J^+_a(X)X^{a-}+\text{h.c.}],$$

$$J_a=J_{al}+J_{aq},$$

and dropping the cubic gauge boson terms (not involving fermions) in the gauge currents.

The masslessness of $W^\pm$ reflects the fact that $\langle\Phi\rangle$ is annihilated by two linear combinations of the generators:

$$(\eta_3\lambda_1-\eta_2\lambda_4)\langle\Phi\rangle=(\eta_3\lambda_2-\eta_2\lambda_5)\langle\Phi\rangle=0.$$

Obviously their commutator also has this property:

$$[(\eta_3^2+\tfrac{1}{2}\eta_2^2)\lambda_3+\frac{\sqrt{3}}{2}\eta_2^2\lambda_8+\eta_2\eta_3\lambda_6]\langle\Phi\rangle=0.$$

The three (hermitian) matrices which annihilate $\langle\Phi\rangle$ generate an SU_2 algebra, denoted $SU_2(W)$. Thus, the neutral gauge boson mass matrix has one massless eigenvector which we call W^0 and which in terms of θ is

$$W^0=(\cos^2\theta+\tfrac{1}{2}\sin^2\theta)U_3+\frac{\sqrt{3}}{2}\sin^2\theta\, U_8-\sin\theta\cos\theta\, U_6,\ M^2(W^0)=0 \tag{13}$$

corresponding to the neutral annihilator of $\langle\Phi\rangle$. Of the other neutral bosons, U_7 remains unmixed, but acquires a mass. The complete (U_3, U_8, U_6, U_7) mass matrix, with rows and columns labelled in that order, is

$$M^2_{\text{neut}}=g^2\begin{pmatrix} \eta_2^2 & -\frac{1}{\sqrt{3}}\eta_2^2 & -\eta_2\eta_3 & 0 \\ -\frac{1}{\sqrt{3}}\eta_2^2 & \frac{1}{3}\eta_2^2+\frac{4}{3}\eta_3^2 & -\frac{1}{\sqrt{3}}\eta_2\eta_3 & 0 \\ -\eta_2\eta_3 & -\frac{1}{\sqrt{3}}\eta_2\eta_3 & \eta_2^2+\eta_3^2 & 0 \\ 0 & 0 & 0 & \eta_2^2+\eta_3^2 \end{pmatrix} \tag{14}$$

the eigenvectors and eigenvalues being, apart from W^0,

$$Y^0\equiv\frac{\sqrt{3}}{2}\sin^2\theta\ U_3+(\cos^2\theta-\tfrac{1}{2}\sin^2\theta)U_8+\sqrt{3}\sin\theta\cos\theta\ U_6, \tag{15}$$

$$X_1^0\equiv-\sin\theta\cos\theta\ U_3+\sqrt{3}\sin\theta\cos\theta\ U_8+(\sin^2\theta-\cos^2\theta)U_6, \tag{16}$$

$$X_2^0\equiv U_7; \tag{17}$$

$$M^2(Y^0)=\tfrac{4}{3}g^2\eta_3^2\sec^2\theta \tag{18}$$

$$M^2(X_1^0)=M^2(X_2^0)=g^2\eta_3^2\sec^2\theta. \tag{19}$$

The pattern of mass breaking is thus the one familiar in other SU_3 contexts: $W^\pm$ and W^0, which have the quantum numbers of the pions (with $SU_2(W)$ taken as " weak isospin ") are degenerate and massless, $X^\pm$, X_1 and X_2 whose weak isospin and weak hypercharge are the same as the corresponding strong quantum numbers of $K^\pm$, K_1 and K_2, are degenerate and massive and Y, the weak η, is still more massive in a way which satisfies the Gell-Mann-Okubo formula.

The fermion currents which couple to these neutral physical bosons are the same combinations of $J(U_3)$, $J(U_8)$, $J(U_6)$ and $J(U_7)$ as these bosons themselves are of U_3, U_8, U_6 and U_7 and are easily written down in terms of the lepton and quark fields. But, as far as the Cabibbo suppression mechanism is concerned, it is the charged currents coupling to $W^\pm$, eqs (9, 10) which are the objects of primary interest. As they stand, they do not have the Cabibbo form because of the unwanted Cabibbo-like rotation of the leptonic part of the currents. The key idea now is that we have so far no way of telling what the physical lepton operators are and are therefore at liberty to redefine them in such a way that it does not change the leptonic part of the Lagrangian. Since all fermions are as yet massless and since e_o and E_o have the same electric charge, any orthogonal mixing of e_o and E_o will leave the free lepton Lagrangian as well as their electromagnetic current invariant. We thus define the physical leptons to be

$$\nu_e\equiv\nu_{eo},\ e\equiv e_o\cos\theta+E_o\sin\theta,\ E\equiv-e_o\sin\theta+E_o\cos\theta \tag{20}$$

for both the left-handed and the right-handed projections. In terms of these, the weak-interaction Lagrangian arising from W-boson couplings is

$$\mathscr{L}_W=g[(W_a^+J_a^-(W)+W_a^-J_a^+(W)+W_a^0J_a^0(W)], \tag{21}$$

where

$$J_a(W)=J_{al}(W)+J_{aq}(W)$$

for all components and

$$J^{+}_{al}(W)=\bar{\nu}_e\gamma_{\ aL}e, \tag{22}$$

$$J^{+}_{aq}(W)=\cos\theta\,\bar{q}_p\gamma_{aL}\,q_n+\sin\theta\,\bar{q}_p\gamma_{aL}\,q_\lambda, \tag{23}$$

$$J^{0}_{al}(W)=\bar{\nu}_e\gamma_{aL}\,\nu_e-\bar{e}\gamma_{aL}\,e, \tag{24}$$

$$J^{0}_{aq}(W)=\bar{q}_p\gamma_{aL}q_p-\cos^2\theta\,\bar{q}_n\gamma_{aL}\,q_n-\sin^2\theta\,\bar{q}_\lambda\,\gamma_{aL}\,q_\lambda- \\ -\sin\theta\,\cos\theta(\bar{q}_n\gamma_{aL}\,q_\lambda+\bar{q}_\lambda\gamma_{aL}q_n). \tag{25}$$

We conclude from all this that the structure of the dominant weak interactions is precisely that described by Gell-Mann's SU_2 algebra of universality, for both the leptonic and the hadronic currents. The weak isospin group $SU_2(W)$ is identical with the SU_2 part of the $SU_2\times U(1)$ gauge group of Weinberg and Salam. But before this identification is accepted we must still worry about the problem of fermion masses: what is the mass spectrum generated by the Yukawa coupling of the fermions to Φ, in particular, does the electron remain massless? In our context, this question has an importance beyond that of the identification mentioned above. The way the Cabibbo rotation came about, through the redefinition of the leptons, depended very crucially on ignoring the $\bar{l}l\Phi$ couplings since mass terms arising from these couplings are not, in general, invariant under the orthogonal transformation from (e_o, E_o) to (e, E). This is the question addressed in the next section.

4. Fermion masses

The most general SU_3 invariant coupling of the fermions to Φ is of the form

$$\mathscr{L}_{\Phi f}=(\bar{\nu}_L\Phi_1+\bar{e}_{oL}\,\Phi_2+\bar{E}_{oL}\Phi_3)\,(f_e e_{oR}+f_E\,E_{oR}+f_\nu \nu_R) \\ +(\bar{q}_{pL}\Phi_1+\bar{q}_{nL}\,\Phi_2+\bar{q}_{\lambda L}\Phi_3\,(f_n q_{nR}+f_\lambda q_{\lambda R}+f_p q_{pR})+\text{h.c.} \\ \equiv L_{\Phi l}+L_{\Phi q},$$

where the coupling constants f are arbitrary, but taken to be real. The fermion mass Lagrangian which arises from this at the tree graph level is then

$$\mathscr{L}_{mf}=\mathscr{L}_{ml}+\mathscr{L}_{mq},$$

$$\mathscr{L}_{ml}=(\eta_2\bar{e}_{oL}+\eta_3\bar{E}_{oL})\,(f_e e_{oR}+f_E E_{oR})+\text{h.c.}, \tag{26}$$

$$\mathscr{L}_{mq}=(\eta_2\bar{q}_{nL}+\eta_3\bar{q}_{\lambda L})\,(f_n q_{nR}+f_\lambda q_{\lambda R})+\text{h.c.} \tag{27}$$

Evidently, a Yukawa interaction with the Higgs multiplet still leaves the neutrino and the proton quark massless.

Let us consider the lepton masses first. It is imperative for us that the mass matrix given by $\mathscr{L}_{ml}$ must be diagonal in the physical (e, E) basis, i.e. $\mathscr{L}_{ml}$ must be of the form

$$\mathscr{L}_{ml}=-m_l\bar{e}_L e_R-m_E\bar{E}_L E_R+\text{h.c.} \tag{28}$$

where e_L, e_R and E_L, E_R are given by eq. (20) and m_e and m_E are non-negative. Equating the two forms of $\mathscr{L}_{ml}$, we have the following conditions to be satisfied:

$$-\eta_2 f_e=m_e\cos^2\theta+m_E\sin^2\theta, \\ -\eta_3 f_E=m_e\sin^2\theta+m_E\cos^2\theta, \\ -\eta_2 f_E=-\eta_3 f_e=(m_e-m_E)\sin\theta\cos\theta. \tag{29}$$

These equations immediately imply the constraint

$$m_e m_E = 0. \tag{30}$$

In fact, they have the following two sets of solutions: §

$$\eta_2/\eta_3 = -\tan\theta = f_e/f_E, \tag{31}$$
$$m_e = 0,\ m_E = -f_E\eta_3 \sec^2\theta;$$

or

$$\eta_2/\eta_3 = \cot\theta = f_e/f_E,$$
$$m_e = -f_E\eta_3 \operatorname{cosec}^2\theta,\ m_E = 0; \tag{32}$$

with, in both cases,

$$f_E\eta_3 < 0.$$

The first solution gives the same value to $\tan\theta$ as determined from the gauge boson mixings, eq. (7), while the second contradicts it. Thus a consistent generation of the Cabibbo suppression *and* lepton masses is possible, but only when the electron mass is zero. In other words, the leptons which diagonalize the lepton mass matrix couple to the W-bosons (which diagonalise the gauge-boson mass matrix) in the desired way when $m_e=0$ and $M(W)=0$. In the lepton sector, the fermion mass problem solves itself in a most satisfactory way.

In the quark sector, however, we have quite a different situation. Within the framework of section 2, it is axiomatic that the quark states are not appreciably shifted with respect to the " axes " defined by the strong SU_3 group by any weak interaction effects, including interactions with the Higgs multiplets—the basis which diagonalises the quark mass matrix must not be very different from the strong SU_3 eigenstates. For the $q_n q_\lambda$ mixing term in $\mathscr{L}_{mq}$ to vanish, the quark-Higgs coupling, $\mathscr{L}_{\Phi q}$ itself must vanish: $f_n = f_\lambda = 0$. Expressed differently, our requirement is that no interaction of the quarks must deviate significantly from strong SU_3 invariance, which rules out any bilinear, P-conserving coupling of the quarks with Φ. Consequently, in our picture, all the bare (current) quarks remain massless even after the gauge symmetry is broken. We may note here that recent (inevitably model-dependent) work makes such a situation entirely acceptable (see, e.g., Georgi and Politzer 1976).

The theory as developed so far thus leads us to the precise point at which the Weinberg-Salam model begins, in every respect: (i) our (so far conserved) $SU_2(W)$ is the same as the Weinberg-Salam SU_2; (ii) the lepton doublet is formed of ν_e and e and the quark doublet, effectively, of q_p and $q_n\cos\theta + q_\lambda\sin\theta$; and (iii) all these four fermions are massless. While the Cabibbo-orthogonal combination also remains massless, the heavy lepton E acquires mass along with the five gauge bosons X and Y.

5. The second Higgs multiplet

As we turn on the second set of vacuum expectation values, $\langle\Phi'\rangle = (o, \eta'_2, \eta'_3)$, we expect the residual $SU_2(W)$ symmetry to be completely broken. But because this second stage of symmetry breaking is embedded within the larger SU_3 framework,

§ The relation $f_e/f_E = \eta_2/\eta_3$ is a consequence of the hermiticity of the lepton mass matrix.

there are interrelations in its consequences which are absent in the Weinberg-Salam theory.

We consider the contribution of $\langle\Phi'\rangle$ to the lepton masses first. $\langle\Phi'\rangle$ modifies the lepton mass Lagrangian eq. (26) to

$$\mathcal{L}_{ml}=(\vec{\eta}_2\,\bar{e}_{oL}+\vec{\eta}_3\,\bar{E}_{oL})\cdot(\vec{f}_e\,e_{oR}+\vec{f}_E\,E_{oR})+\text{h.c.} \tag{33}$$

in a vector notation where $\vec{\eta}_2=(\eta_2,\eta'_2)$, $\vec{\eta}_3=(\eta_3,\eta'_3)$, $\vec{f}_e=(f_e,f'_e)$ and $\vec{f}_E=(f_E,f'_E)$ are all real 2-dimensional vectors. The neutrino is thus still massless. As for e and E, eqs. (29) get modified in an obvious way and the constraint (30) becomes

$$(\vec{\eta}_2\cdot\vec{f}_e)(\vec{\eta}_3\cdot\vec{f}_E)-(\vec{\eta}_2\cdot\vec{f}_E)(\vec{\eta}_3\cdot\vec{f}_e)\equiv(\vec{\eta}_2\times\vec{\eta}_3)\cdot(\vec{f}_e\times\vec{f}_E)=m_e\,m_E. \tag{34}$$

If $\vec{\eta}_2$ is parallel to $\vec{\eta}_3$ (or $\vec{f}_e$ parallel to $\vec{f}_E$, which amounts to the same) m_e remains zero, a possibility we must avoid (think of the muon). Since the converse is also true, i.e., a non-zero m_e implies that $\vec{\eta}_2$ cannot be parallel to $\vec{\eta}_3$, there is no loss of generality* in assuming that $\vec{\eta}_2$ and $\vec{\eta}_3$ are mutually perpendicular and therefore also that $\vec{f}_e$ and $\vec{f}_E$ are mutually perpendicular:

$$\eta'_2/\eta'_3=-\eta_3/\eta_2=\cot\theta, \tag{35}$$

$$f'_e/f'_E=-f_E/f_e=\cot\theta. \tag{36}$$

As a result of Φ' couplings, the (e_o, E_o) mass matrix

$$-f_E\,\eta_3\begin{pmatrix}\tan^2\theta & -\tan\theta\\ -\tan\theta & 1\end{pmatrix}$$

acquires an additional part

$$-f'_E\,\eta'_3\begin{pmatrix}\cot^2\theta & \cot\theta\\ \cot\theta & 1\end{pmatrix}.$$

The physical leptons $e=e_o\cos\theta+E_o\sin\theta$ and $E=-e_o\sin\theta+E_o\cos\theta$ *are still the eigenstates of the complete mass matrix* (the old and new parts of the mass matrix commute), but with masses

$$m_e=-f'_E\,\eta'_3\operatorname{cosec}^2\theta,\ f'_E\,\eta'_3<0, \tag{37}$$

$$m_E=-f_E\,\eta_3\sec^2\theta,\ f_E\,\eta_3<0, \tag{38}$$

m_E remaining unchanged. Since we wish to make E much heavier than e, we must have

$$\frac{f_E\eta_3}{f'_E\eta'_3}\tan^2\theta=\frac{m_E}{m_e}\gg 1 \tag{39}$$

and, so,

$$|f_E\,\eta_3|\gg|f'_E\,\eta'_3|.$$

In particular, if the coupling constants f_E and f'_E are of comparable magnitude, the approximate relation

$$\eta'_3/\eta_3\simeq m_e\tan^2\theta/m_E$$

follows.

* At least for the purpose of the present discussion.

The quarks are, of course, assumed not to couple to Φ' either.

We turn next to the effect of $\langle \Phi' \rangle$ on the gauge bosons. In the charge-1 sector, the additional contribution to the mass matrix is easily calculated to be

$$g^2 \eta_3^2 \begin{pmatrix} \cot^2 \theta & \cot \theta \\ \cot \theta & 1 \end{pmatrix}$$

so that, again, *the physical $W^\pm$ and $X^\pm$ are unchanged as linear combinations of U_1, U_2, U_4* and *U_5 and couple to the same fermion currents as before.* The $W^\pm$ bosons, however, are now massive:

$$M^2(W^\pm) = g^2 \eta'^2_3 \operatorname{cosec}^2 \theta, \tag{40}$$

$M^2(X^\pm)$ remaining the same as before:

$$M^2(X^\pm) = g^2 \eta_3^2 \sec^2 \theta. \tag{41}$$

The mass ratio can then be expressed, using eq. (39), as

$$\frac{M^2(X^\pm)}{M^2(W^\pm)} = \frac{\eta_3^2}{\eta'^2_3} \tan^2 \theta = \frac{m_E^2}{m_e^2} \frac{f'^2_E}{f_E^2} \cot^2 \theta \simeq 10^8 \frac{f'^2_E}{f_E^2} \tag{42}$$

if we take $\tan \theta \simeq \frac{1}{4}$ and, for illustration only, $m_E \sim 1{\cdot}2$ GeV. Thus, unless for some totally mysterious reason f'_E/f_E is very small, the correction to the Cabibbo Hamiltonian which comes from the $X^\pm$-exchange (and which can come from nowhere else) is going to be exceedingly small.

For the neutral gauge bosons, things are slightly more complicated. The boson X_2^0 continues to be an exact eigenvector of the mass matrix, with mass

$$M^2(X_2^0) = g^2 \eta_3^2 \sec^2 \theta + g^2 \eta'^2_3 \operatorname{cosec}^2 \theta. \tag{43}$$

But none of the other three, W^0, Y^0, X_1^0 as given by eqs. (13, 15, 16), is an eigenvector of the total mass matrix. But with our knowledge that the contribution of $\langle \Phi' \rangle$ can be treated as a small perturbation, this is not a serious problem—the masses $M(Y^0)$ and $M(X_1^0)$ and the states W^0, Y^0 and X_1^0 change negligibly, by terms of order η'^2_3/η_3^2. The only interesting change is in $M^2(W^0)$ from zero to

$$M^2(W^0) = g^2 \eta'^2_3 \operatorname{cosec}^2 \theta \cos^2 2\theta; \tag{44}$$

the neutral boson is lighter than the charged one by a factor $M^2(W^0)/M^2(W^+) = \cos^2 2\theta \simeq 0{\cdot}8$.

6. Concluding remarks

The picture that has emerged from the work described above is a rather satisfactory one in most respects. Indeed, within its own limitations, embodied in the model from the outset, the only unsatisfactory feature is the not unexpected difficulty with the strangeness changing neutral currents. We summarise the more interesting aspects:

(1) At the fundamental level, the theory is truly uninversal - all fermion-gauge boson couplings are the same. At the effective 4-fermion level seen at relatively low energies, the totality of weak interactions separate themselves into two regimes, the conventionally weak and the " ultraweak ". The former describes, in a qualitatively satisfactory way, most of the by now common experimental manifestations

of the weak force. The ultraweak forces are mediated by gauge bosons which couple to exotic currents. At low energies, the ultraweak processes are very strongly suppressed.

(2) The dominant weak interactions are described by an SU_2 gauge theory which is, modulo the electromagnetic unification and the charm modification, indistinguishable from the Weinberg-Salam theory. In contrast with it, however, the Cabibbo rotation is not incorporated from the outside, but genuinely produced by spontaneous violation of gauge invariance. Furthermore, the Cabibbo suppression does not result from a rotation of the quark states at all, but from a rotation of the gauge bosons and a redefinition of the leptons, both arising from the diagonalisation of the total effective weak Hamiltonian. The Cabibbo-orthogonal current couples ultraweakly and so is negligible.

(3) Since our (almost conserved) $SU_2(W)$ is the same as the Weinberg-Salam SU_2, incorporation of electromagnetism in the manner adopted there is a trivial matter, if it is felt to be desirable. This will change the properties of W^0 and the $\triangle S=0$ part of the normal neutral current in the familiar way.

(4) The neutrinos are strictly massless and the electron and the muon relatively light in comparison with their heavy partners. Whether these new heavy leptons are an uncalled for luxury is a question which experiment can be expected to answer conclusively in the near future. All quarks have mass zero.

(5) The embarrassing presence of $\triangle S=\pm 1$ neutral currents with an effective $g^2 \sin\theta / M^2(W^0)$ strength is hardly a surprise. What is much more interesting is the fact that the charm trick, including the pairing of the charmed quark with the Cabibbo-orthogonal combination of q_n and q_λ is not immediately available to us. Firstly, the quarks are not rotated in the present theory. Far more importantly, the larger SU_3 framework which subsumes the weak $SU_2(W)$ theory imposes strong conditions on this type of modification—the Cabibbo-orthogonal current *is* one of the SU_3 currents and its transformation properties under the larger group is determined.

(6) Perhaps connected with this is the problem of the muon multiplet. Just adding on another triplet not only leaves the mystery of the muon unexplained but becomes even more arbitrary if we cannot, as seen above, add on a similar quark multiplet to take care of the neutral $\triangle S=\pm 1$ current. In fact, there is every hope that these two problems will solve themselves together in an intrinsic way in a more realistic model.

(7) There are many other directions along which progress is needed and perhaps attainable, chief among them a better understanding of *P* and *CP* violation within an integral picture. Consideration of these questions was rigorously excluded here, from the beginning. There have, of course, been many attempts to understand discrete symmetries from viewpoints similar to that adopted in this work. The continuing uncertainties, either experimental or theoretical, that they have met certainly underline the subtlety of the points at issue. A definitive resolution of these difficulties would be strong support indeed for the viewpoint advocated here.

(8) And, finally, electromagnetism. The point has already been made that, as far as basic issues are concerned, it is a much less urgent task to unify weak and electromagnetic forces than to unify weak and ultraweak forces. There is, of course, the practical difficulty of calculating in a sensible way corrections, e.g.,

electromagnetic, to the basic weak interactions in an apparently unrenormalisable theory. But, logically, this problem can wait for the day when we understand the weak processes themselves in the lowest order, better. The history of field-theoretic particle physics has many cases to offer of theories which at first were not known to be renormalisable, but later on turned out to be so, the Weinberg-Salam theory being the most outstanding example.

In spite of these problems, it seems fair to conclude that the basic attempt of this paper, that of intrinsically and consistently generating the Cabibbo suppression, has been largely successful.

Acknowledgements

This work has benefited greatly from many clarifying discussions I have had with colleagues, notably, Profs Tulsi Dass, H S Mani, N Mukunda, L K Pandit, G Rajasekaran, R Ramachandran and Virendra Singh, over the past two years. Warm thanks are given to all of them.

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