

Neutrino reactions and restrictions on the $O(4) \times U(1)$ -gauge theory of weak and electromagnetic interactions

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Abstract. In this paper we have analysed and restricted the freedom of constructing gauge models of Pais type, based on the symmetry group $O(4) \times U(1)$ using the latest experimental data from neutrino reactions. The hadrons are made out of the fractionally charged Gell-Mann-Zweig quarks. The left-handed quarks and leptons are restricted to (a) 4-vector (b) 4-spinor and (c) adjoint representations of $O(4)$. The right-handed quarks and leptons are taken as scalars of $O(4)$. It is found that the model based on 4-spinor L -leptons and 4-spinor L -quarks agrees the best with the known phenomenology of weak interactions.

Keywords. $O(4) \times U(1)$ -gauge theory; weak currents; neutrinos; leptons; quarks.

1. Introduction

Pais (1973) has proposed a gauge theory with superweak CP violation based on the gauge group $O(4) \times U(1)$, in which the $\Delta s = 0$ and $\Delta s = 1$ semileptonic decays are mediated by distinct intermediate bosons. The Cabibbo angle θ is related to the masses of the two charged vector bosons and arises as a result of spontaneous symmetry breaking. The μ decay and the semileptonic $\Delta s = 0$ and $\Delta s = 1$ decays are in the ratio $1 : \cos^2 \theta : \sin^2 \theta$ only if the CP is maximally violated in the lepton sector. The scheme incorporates most of the known features of the phenomenology of weak interactions at low energy. The choice of representations for leptons and quarks of the gauge group $O(4) \times U(1)$ provide various alternative models, which have in common all the above-mentioned features but differ in their predictions for neutrino-induced reactions. In this paper we have analysed and restricted the freedom of constructing gauge models of this type using the latest experimental data from neutrino physics. The colour gauge group $SU(3)$ of the strong interactions is assumed to commute with the gauge group of the weak and electromagnetic interactions and the quarks are assumed to be fractionally charged. The choice of representations for L -leptons may be restricted to (a) 4-vectors, (b) 4-spinors, (c) tensor (adjoint representation) and for R -leptons to scalar of $O(4)$. The choice of representations for L -quarks is restricted to (a) 4-vector, (b) 4-spinor and R -quarks are taken as scalars under $O(4)$. It has been

shown by Pais (1974) that for L -leptons in the 4-vector representation of $O(4)$ the weak neutral currents are "inelastic" and therefore we have left out the 4-vector representation for L -leptons from further considerations.

2. Weak currents

We recall from Pais (1974) that the weak currents in these theories are*:

$$J_{\mu}^{(1)} = -\frac{ig}{2} \langle \gamma_{\mu} (t_{-} - \rho_{+}) \rangle, \quad (1)$$

$$J_{\mu}^{(2)} = -\frac{ig}{2} \langle \gamma_{\mu} (t_{+} + \rho_{-}) \rangle, \quad (2)$$

$$J_{\mu}^{(0)} = -\frac{ig}{\sqrt{2}} \langle \gamma_{\mu} (t_3 - \rho_3) \rangle, \quad (3)$$

$$J_{\mu}^{(V)} = -\frac{ig}{\sqrt{2} \cos \gamma} \langle \gamma_{\mu} (t_3 + \rho_3 - Q \sin^2 \gamma) \rangle \quad (4)$$

coupled respectively to the vector mesons W^1 , W^2 , Z and V (masses M_1 , M_2 , M_0 and M_V).

$$t_{\pm} = (t_1 + it_2), \quad \rho_{\pm} = (\rho_1 + i\rho_2), \quad Q = t_3 + \rho_3 + y,$$

$$g = e\sqrt{2}/\sin \gamma, \quad g' = e/\cos \gamma.$$

t , ρ and y commute with each other, and $t \times t = it$, $\rho \times \rho = i\rho$. The charged currents $J_{\mu}^{(1)}$ and $J_{\mu}^{(2)}$ respectively transmit $\Delta s = 0$ and $\Delta s = 1$ semileptonic reactions in the lowest order.

(A) 4-spinor L -leptons

We need two 4-spinors of $y = -\frac{1}{2}$. A possible choice is

$$M_L = \left(\nu_{\mu}, \frac{\mu + y^{-}}{\sqrt{2}}, y^0, \frac{\mu - y^{-}}{\sqrt{2}} \right),$$

$$E_L = \left(\frac{\nu_e + x^0}{\sqrt{2}}, \frac{\varepsilon e + x^{-}}{\sqrt{2}}, \frac{i(-\nu_e + x^0)}{\sqrt{2}}, \frac{\varepsilon e - x^{-}}{\sqrt{2}} \right),$$

$$t = \frac{1}{2} \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & \rho \end{pmatrix} \text{ and } \varepsilon = e^{i\pi/4}.$$

The contribution to weak currents of the L -leptons and R -leptons is easily found to be

$$J_{\alpha}^{(1)t} = \frac{-ig}{4\sqrt{2}} \{ \bar{\nu}_e \gamma_{\alpha} (1 + \gamma_5) e + \bar{\nu}_{\mu} \gamma_{\alpha} (1 + \gamma_5) \mu + \dots \}, \quad (5)$$

$$J_{\alpha}^{(2)t} = \frac{-ig}{4\sqrt{2}} \{ i \bar{\nu}_e \gamma_{\alpha} (1 + \gamma_5) e + \bar{\nu}_{\mu} \gamma_{\alpha} (1 + \gamma_5) \mu + \dots \}, \quad (6)$$

* The symbol $\langle \rangle$ stands for the expectation value of the group generators in the various lepton and quark representations of the internal group $O(4) \times U(1)$.

$$J_a^{(\nu)\mu} = \frac{-ig}{4\sqrt{2}} \{ \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu + \dots \} \tag{7}$$

$$J_a^{(\nu)\mu} = \frac{-ig}{4\sqrt{2} \cos \gamma} \{ \bar{\nu}_e \gamma_a (1 + \gamma_5) \nu_e + \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu - \bar{e} \gamma_a (1 + \gamma_5) e - \bar{\mu} \gamma_a (1 + \gamma_5) \mu + 4 \sin^2 \gamma [e \gamma_a e + \bar{\mu} \gamma_a \mu + \dots] + \dots \} \tag{8}$$

where ... denotes terms which involve heavy leptons.

(B) 4-spinor and 6-tensor L-leptons

Such a possibility was considered by Georgi and Pais (1974) in an attempt to make the desirable properties of $O(4) \times U(1)$ natural. The L-muonic leptons transform like a 4-spinor with $y = -\frac{1}{2}$

$$M^L = \left(\nu_\mu, \frac{\mu + \nu^-}{\sqrt{2}}, \nu^0, \frac{\mu - \nu^-}{\sqrt{2}} \right),$$

and the L-electronic leptons form two six-component adjoint representations

$$E_1^L = \frac{1}{\sqrt{2}} \left(g^+ + h^+, \frac{(\nu_e + N^0)}{\sqrt{2}} + x^0, ee + f^-, g^+ - h^+, i \left[-\frac{(\nu_e + N^0)}{\sqrt{2}} + x^0 \right], ee - f^- \right),$$

$$E_2^L = \frac{1}{\sqrt{2}} \left(G^+ + H^+, \frac{(\nu_e - N^0)}{\sqrt{2}} - x^0, E^- + F^-, G^+ - H^+, i \left[\frac{(\nu_e - N^0)}{\sqrt{2}} + x^0 \right], E^- - F^- \right).$$

$$t = \begin{bmatrix} T & 0 \\ 0 & 0 \end{bmatrix}, \quad \iota = \begin{bmatrix} 0 & 0 \\ 0 & T \end{bmatrix},$$

where T are spin-one angular momentum matrices in the spherical basis. The contributions to the weak currents are

$$J_a^{(e)\mu} = \frac{-ig}{4\sqrt{2}} \{ \bar{\nu}_e \gamma_a (1 + \gamma_5) e + \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \mu + \dots \}, \tag{9}$$

$$J_a^{(\mu)\mu} = \frac{-ig}{4\sqrt{2}} \{ i \bar{\nu}_e \gamma_a (1 + \gamma_5) e + \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \mu + \dots \}, \tag{10}$$

$$J_a^{(0)\mu} = \frac{-ig}{4\sqrt{2}} \{ \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu + \dots \}, \tag{11}$$

$$J_a^{(\nu)\mu} = \frac{-ig}{\sqrt{2} \cos \gamma} \{ \frac{1}{4} \bar{\nu}_\mu \gamma_a (1 + \gamma_5) \nu_\mu - \frac{1}{2} \bar{e} \gamma_a (1 + \gamma_5) e - \frac{1}{4} \bar{\mu} \gamma_a (1 + \gamma_5) \mu + \sin^2 \gamma [e \gamma_a e + \bar{\mu} \gamma_a \mu + \dots] + \dots \}. \tag{12}$$

(C) 4-vector L -quarks

We need two 4-vector representations for L -quarks with $y = -\frac{1}{3}$,

$$Q_1^L = [p, \frac{1}{2}(n + \lambda + q\sqrt{2}), -\frac{1}{2}(n - \lambda + r\sqrt{2}), -t],$$

$$Q_2^L = [c, \frac{1}{2}(n - \lambda - r\sqrt{2}), -\frac{1}{2}(n + \lambda - q\sqrt{2}), -g],$$

$$t = \frac{1}{2}1 \times \sigma, \quad p = \frac{1}{2}\sigma \times 1.$$

The charge assignments are

$$p, c (Q = \frac{2}{3});$$

$$n, \lambda, q, r (Q = -\frac{1}{3}); \quad t, g (Q = -\frac{4}{3}).$$

The weak currents are

$$J_a^{(1)\mu} = \frac{-ig}{4} [\bar{p}\gamma_a(1 + \gamma_5)n + \dots], \quad (13)$$

$$J_c^{(2)\mu} = \frac{-ig}{4} [\bar{p}\gamma_a(1 + \gamma_5)\lambda + \dots] \quad (14)$$

$$J_a^{(0)\mu} = \frac{-ig}{4} [(\bar{n} - \bar{\lambda})\gamma_a(1 + \gamma_5)r - (\bar{n} + \bar{\lambda})\gamma_a(1 + \gamma_5)q + \text{h.c.}], \quad (15)$$

$$J_a^{(\nu)\mu} = \frac{-ig}{\sqrt{2}\cos\gamma} \left[\frac{1}{2} \{ \bar{p}\gamma_a(1 + \gamma_5)p + \bar{c}\gamma_a(1 + \gamma_5)c \right. \\ \left. - \bar{t}\gamma_a(1 + \gamma_5)t - \bar{g}\gamma_a(1 + \gamma_5)g \} - \frac{\sin^2\gamma}{3} \{ 2\bar{p}\gamma_a p \right. \\ \left. + 2\bar{c}\gamma_a c - \bar{n}\gamma_a n - \bar{\lambda}\gamma_a \lambda - \bar{r}\gamma_a r - \bar{q}\gamma_a q \right. \\ \left. - 4\bar{t}\gamma_a t - 4\bar{g}\gamma_a g \} \right], \quad (16)$$

where ... denotes terms which involve "charmed" quarks.

(D) 4-spinor L -quarks

We need two 4-spinor representations for L -quarks with $y = \frac{1}{6}$:

$$Q_1^L = \frac{1}{\sqrt{2}} \left(p + c, \frac{n + \lambda}{\sqrt{2}} + q, p - c, \frac{(-n + \lambda)}{\sqrt{2}} - \gamma \right),$$

$$Q_2^L = \frac{1}{\sqrt{2}} \left(t + g, \frac{n - \lambda}{\sqrt{2}} - \gamma, t - g, \frac{-(n + \lambda)}{\sqrt{2}} + q \right).$$

The quarks p, c, t, g have charge $\frac{2}{3}$, and quarks λ, n, q, γ have charge $-\frac{1}{3}$. The contributions of these quarks to the weak currents are

$$J_a^{(1)\mu} = \frac{-ig}{4\sqrt{2}} [\bar{p}\gamma_a(1 + \gamma_5)n + \dots] \quad (17)$$

$$J_a^{(2)\mu} = \frac{-ig}{4\sqrt{2}} [\bar{p}\gamma_a(1 + \gamma_5)\lambda + \dots], \quad (18)$$

$$J_a^{(0)\mu} = \frac{-ig}{4\sqrt{2}} \left[\bar{c}\gamma_a(1+\gamma_5)p - \frac{1}{\sqrt{2}}(\bar{n}+\bar{\lambda})\gamma_a(1+\gamma_5)q \right. \\ \left. + \frac{1}{2}(\bar{n}-\bar{\lambda})\gamma_a(1+\gamma_5)r + \bar{t}\gamma_a(1+\gamma_5)g + \text{h.c.} \right], \quad (19)$$

$$J_a^{(\nu)\mu} = \frac{-ig}{4\sqrt{2}\cos\gamma} [\bar{p}\gamma_a(1+\gamma_5)p + \bar{c}\gamma_a(1+\gamma_5)c \\ + \bar{t}\gamma_a(1+\gamma_5)t + \bar{g}\gamma_a(1+\gamma_5)g - \bar{n}\gamma_a(1+\gamma_5)n \\ - \bar{\lambda}\gamma_a(1+\gamma_5)\lambda - \bar{q}\gamma_a(1+\gamma_5)q - \bar{r}\gamma_a(1+\gamma_5)r \\ - \frac{4}{3}\sin^2\gamma \{2\bar{p}\gamma_a p + 2\bar{c}\gamma_a c + 2\bar{t}\gamma_a t \\ + 2\bar{g}\gamma_a g - \bar{l}\gamma_a l - \bar{\lambda}\gamma_a \lambda - \bar{q}\gamma_a q - \bar{r}\gamma_a r\}]. \quad (20)$$

3. Models and their analysis

We can construct 4 different models by selecting one of the two representations (A), (B) for L -leptons and one of the two representations (C), (D) for L -quarks. By comparing the contributions of the leptons and quarks to the $\Delta_S=0$ and $\Delta_S=1$ currents it is easily seen that the universality for the μ and the semileptonic β and λ decays along with maximal CP violation cannot be realized by combining the vector representation (C) for quarks with the spinor and spinor and tensor representations (A) and (B) for the lepton. We are thus left with two alternatives (A) (D) and (B) (D). The condition of universality of weak interaction in the models (A) (D) and (B) (D) are met if

$$M_1 = \frac{26 \cdot 8}{(\sin^2 \gamma \cos \theta)^{1/2}} \text{ GeV}, \quad (21)$$

$$M_2 = \frac{26 \cdot 8}{(\sin^2 \gamma \sin \theta)^{1/2}} \text{ GeV}, \quad (22)$$

$$g = c\sqrt{2}/\sin \gamma, \quad (23)$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{32} \cdot \left(\frac{1}{M_1^4} + \frac{1}{M_2^4} \right)^{1/2}, \quad (24)$$

where the Cabibbo angle θ is related to the vector meson masses M_1 and M_2 ,

$$\tan \theta = \frac{M_1^2}{M_2^2}. \quad (25)$$

The masses of the leptons and quarks are generated in the standard manner by using the Higgs scalar mesons. The structure and the number of Higgses needed for appropriate mass generation in both models (A) (D) and (B) (D) is such that we can treat the mass M_ν along with the mixing parameter $\sin \gamma$ as undetermined quantities. We make the important observation that only the weak neutral current $J_a^{(\nu)}$ contributes to the elastic neutral effect and the deep inelastic neutrino scatterings.

The data on deep inelastic neutrino-nucleon scattering can be used to fix the parameters $\sin \gamma$ and M_V . In the following calculation of weak neutral current effect we shall neglect the contributions of the isoscalar $V-A$ strangeness and "charm" currents. These currents are normally assumed to couple only weakly to nonstrange low mass hadrons. The effective Lagrangian for ν_μ -nucleon interactions in both models (A) (D) and (B) (D) has the form*

$$\mathcal{L}_{\text{eff}}^N = \frac{-g^2}{8 \cos^2 \gamma} \cdot \frac{1}{M_V^2} \bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu \left\{ \left[\frac{1}{2} - \sin^2 \gamma \right] F_3^\lambda - \frac{\sin^2 \gamma}{\sqrt{3}} F_8^\lambda + \frac{1}{2} F_3^{5\lambda} \right\}, \quad (26)$$

$$F_j^\lambda = \bar{\psi} \gamma^\lambda \frac{\lambda_j}{2} \psi, \quad F_j^{5\lambda} = \bar{\psi} \gamma^\lambda \gamma_5 \frac{\lambda_j}{2} \psi,$$

$$\psi = \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix}.$$

If we parametrize M_V as

$$M_V^2 = \frac{4aM_1^2}{\cos^2 \gamma}, \quad (27)$$

equation (26) takes the form

$$\mathcal{L}_{\text{eff}}^N = \frac{-G}{\sqrt{2}} \bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu \{ g_{V3} F_3^\lambda + g_{V8} F_8^\lambda + g_{A3} F_3^{5\lambda} + g_{A8} F_8^{5\lambda} \}, \quad (28)$$

with

$$g_{V3} = \frac{1}{a} \left[\frac{1}{2} - \sin^2 \gamma \right], \quad g_{V8} = -\frac{\sin^2 \gamma}{a \sqrt{3}},$$

$$g_{A3} = \frac{1}{2a}.$$

The ratios R_ν and $R_{\bar{\nu}}$ (De Rujula *et al* 1974) can be calculated from Adler's relations (Adler 1975).

$$R_\nu = \frac{1}{18} \{ g_{V8}^2 + g_{A8}^2 + g_{V8} g_{A8} \} + \frac{1}{6} \{ g_{V3}^2 + g_{A3}^2 + g_{V3} g_{A3} \}, \quad (29)$$

$$R_{\bar{\nu}} = \frac{1}{6} \{ g_{V8}^2 + g_{A8}^2 - g_{V8} g_{A8} \} + \frac{1}{2} \{ g_{V3}^2 + g_{A3}^2 - g_{V3} - g_{A3} \}. \quad (30)$$

* The accuracy of the neutrino data is such that it cannot test the presence of the Cabibbo-suppressed part of the weak current in neutrino reactions. We use the approximation $\sin \theta = 0$

We get

$$a^2 R_\nu = \frac{1}{8} - \frac{1}{4} \sin^2 \gamma + \frac{5}{27} \sin^4 \gamma, \tag{31}$$

$$a^2 R_{\bar{\nu}} = \frac{1}{8} - \frac{1}{4} \sin^2 \gamma + \frac{5}{9} \sin^4 \gamma. \tag{32}$$

For $a = \frac{1}{2}$, eqs (31) and (32) reduce to the results of WS (Weinberg-Salam) model for R_ν and $R_{\bar{\nu}}$. We thus get $\sin^2 \gamma = 0.32$ corresponding to $R_\nu = 0.25$ and $R_{\bar{\nu}} = 0.40$. The vector meson masses M_1 , M_2 and M_V are now fixed to the values

$$M_1 = 48 \text{ GeV}, M_2 = 103 \text{ GeV}, M_V = 86 \text{ GeV}.$$

The recent $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ elastic scattering data (Gershtein 1976, Pandit 1976) can now be compared with the predictions of the models (A) (D) and (B) (D) to select between the two representations (A) and (B) for L -leptons. The effective Lagrangian for $\nu - e$ elastic scattering (Sehgal 1974) can be obtained from the currents $J_\mu^{(1)}$ and $J_\mu^{(\nu)}$ (the current $J_\mu^{(2)}$ is Cabibbo suppressed relative to $J_\mu^{(1)}$)

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{G}{\sqrt{2}} [(\bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \nu_\mu) (\bar{e} \gamma_\lambda (c_V + c_A \gamma_5) e) \\ & + (\bar{\nu}_e \gamma_\lambda (1 + \gamma_5) \nu_e) (\bar{e} \gamma_\lambda (c'_V + c'_A \gamma_5) e)]. \end{aligned}$$

For the case (A) (D), $c_V = -\frac{1}{2} + 2 \sin^2 \gamma$, $c_A = -\frac{1}{2}$,

$$c'_V = \frac{1}{2} + 2 \sin^2 \gamma, \quad c'_A = \frac{1}{2}.$$

For the case (B) (D), $c_V = -1 + 2 \sin^2 \gamma$, $c_A = -1$, $c'_V = 1$, $c'_A = 1$.

The expressions for elastic $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ scattering cross sections in units of $G^2 S/2\pi$ are

$$\sigma(\nu_\mu e) = \frac{1}{2} - 2 \sin^2 \gamma + \frac{8}{9} \sin^4 \gamma,$$

$$\sigma(\bar{\nu}_\mu e) = \frac{1}{6} - \frac{2}{3} \sin^2 \gamma + \frac{8}{9} \sin^4 \gamma,$$

for the (A) (D) model, and

$$\sigma(\nu_\mu e) = 2 - 4 \sin^2 \gamma + \frac{8}{9} \sin^4 \gamma,$$

$$\sigma(\bar{\nu}_\mu e) = \frac{2}{3} - \frac{4}{3} \sin^2 \gamma + \frac{8}{9} \sin^4 \gamma,$$

for the (B) (D) model. We note that the results of the (A) (D) model are identical to those of WS model, which is in agreement with the experimental data (Gershtein 1976). The cross sections for the (B) (D) model calculated with $\sin^2 \gamma = 0.32$ are bigger than the experimental limits and so this model can be ruled out unless it is modified with nonscalar representations for R -leptons.

4. Conclusions

We have found from our analysis that among the various $O(4) \times U(1)$ models which can be built with alternative choice of representations for L -leptons and quarks, the model (A) (D), spinor leptons and spinor quarks, agrees the best with the known phenomenology of weak interactions. This model which is identical to WS-GIM in its predictions of the weak neutral effects has distinct distinguishing

features such as $\Delta I = \frac{1}{2}$ rule for nonleptonic weak effects (follows from the structure of $J_\mu^{(0)}$ current) and the explanation for the origin of the Cabibbo angle which arises on spontaneous symmetry breaking of the gauge symmetry.

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