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The U_3(W)-gauge theory III : Atomic physics parity-violation

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Abstract. It is shown how a slight natural generalization of the mechanism of the spontaneous gauge symmetry breaking in the U_3(W) gauge theory (Pandit 1976) can accommodate the degree of parity-violation in atomic physics suggested by some recent experiments, along with the neutral current processes involving the neutrinos.

Keywords. U_3(W)-gauge theory; parity-violation; atomic physics; spontaneous symmetry breaking; neutrinos.

1. Introduction

Some time ago we had proposed the unified U_3(W)-gauge theory of weak and electromagnetic interactions (Pandit 1976, referred to hereon as I). In a subsequent publication (Pandit 1977, referred to hereon as II) we had examined the implications of this theory for some typical phenomena resulting from its weak neutral currents. It was found that the results for the neutral current processes involving the neutrinos fared quite as well as the popular standard WS–GIM theory, when compared with the available experimental information. It was further noted that important differences between the two theories arose in some other effects of interest. One of these is the predicted violation of parity in atomic physics. Our result for this effect was a good bit smaller in magnitude than that of the WS–GIM model; and further, what is even more important, of the opposite sign.

In a recent joint letter to *Nature*, two experimental groups have reported their interim results of independent measurements of optical rotation in atomic bismuth, carried out at the University of Oxford (Baird *et al* 1976) and at the University of Washington (Fortson *et al* 1976). For the relevant parameter characterising parity violation they quote the results:

$$R(\lambda = 648\,\text{nm}) = (+\,10 \pm 8) \times 10^{-8}\ \text{(Oxford)}, \tag{1}$$

$$R(\lambda = 876\,\text{nm}) = (-\,8 \pm 3) \times 10^{-8}\ \text{(Washington)}. \tag{2}$$

The errors quoted are two standard deviations and the systematic errors (though not well understood yet) are believed by the experimenters not to exceed $\pm\,10 \times 10^{-8}$. It is clear that the experiments stand in need of further refinement, with a firmer understanding of the systematic errors, before a definite sign and magnitudes of the R values can be considered as established. All the same, even at this preliminary stage, the values predicted by the standard WS–GIM model, $R(\lambda = 648\ \text{nm}) \simeq -40 \times 10^{-8}$ and $R(\lambda = 876\ \text{nm}) \simeq -30 \times 10^{-8}$, seem already in severe difficulty.

The purpose of the present paper is to show that the $U_3(W)$-gauge theory is perfectly capable of coping with the emerging situation with regard to the question of parity-violation in atomic physics. In our theory there are two neutral intermediate weak interaction vector bosons, Z_1 and Z_2. Only the weak neutral current coupled to Z_1 involves the neutrinos. On the other hand, both the currents coupled to Z_1 as well as to Z_2 involve the electron, so that both contribute to the parity-violating electron-nucleus weak potential relevant for atomic physics. In paper I we had adopted a particularly simple and elegant mechanism for the spontaneous breaking of the gauge symmetry leading to specific relations between the masses and the couplings of Z_1 and Z_2. This gave rise to a parity-violating potential of sign opposite to that of the WS–GIM theory, and of a magnitude around 1·6 times smaller. This result cannot be ruled out by the Oxford result (1), though it would disagree in sign with the Washington result (2).

In the present paper we show that the mechanism of the spontaneous gauge symmetry breaking of paper I may be generalized quite naturally in such a way that the resulting masses and couplings strengths of Z_1 and Z_2 conspire to give much smaller strengths for the parity-violating potential. The eventual outcome of the relevant experiments will then be able to fix more precisely the nature of the generalized gauge symmetry breaking.

In section 2 we briefly describe the proposed generalization of our spontaneous $U_3(W)$-gauge symmetry breaking and indicate the ensuing changes in the vector boson masses and coupling strengths. The question of the resulting parity-violation in atomic physics is then discussed on this basis in section 3. In section 4 we show that the reasonable agreement of our theory with experiments on the neutral currents processes involving the neutrinos is still very well maintained.

2. Mechanism of the spontaneous breaking of the gauge symmetry

To implement the spontaneous breaking of the $U_3(W)$-gauge symmetry, as described in detail in paper I, we introduce three $SU_3(W)$ triplets of Higgs scalar fields $\phi^{(i)} \equiv (\phi_1^{(i)}, \phi_2^{(i)}, \phi_3^{(i)})$, $i = 1, 2, 3$, having for the $U_1(W)$ generator G_0 the values $G_0 = -\frac{2}{3}$ for $\phi^{(1)}$ and $G_0 = +\frac{1}{3}$ for $\phi^{(2)}$ and $\phi^{(3)}$. The 'vacuum' is then arranged to be such (by a suitable choice of the potential function in the Lagrangian) that $\phi_1^{(1)}$, $\phi_2^{(2)}$ and $\phi_3^{(3)}$ attain non-zero vacuum expectation values: $\langle\phi_1^{(1)}\rangle_0 = \langle\phi_2^{(2)}\rangle_0 = \langle\phi_3^{(3)}\rangle_0 = \eta \neq 0$; all the other scalar fields having zero vacuum expectation values. This simple and elegant choice ensures that the stringent requirement (necessary in any gauge theory seeking to unify weak and electromagnetic interactions), that only the gauge vector boson A_μ coupled to the electromagnetic current turns out to be mass-less while all the other eight (weak) vector bosons become massive, is successfully met. This requirement is still satisfied even if we adopt the somewhat more general choice $\langle\phi_1^{(1)}\rangle_0 = \eta' \neq 0$; $\langle\phi_2^{(2)}\rangle_0 = \langle\phi_3^{(3)}\rangle_0 = \eta \neq 0$ and $\eta' \neq \eta$. It is imperative, however, that the vacuum expectation values of $\phi_2^{(2)}$ and $\phi_3^{(3)}$ remain still equal.

In the present paper we shall indicate some phenomenological consequences of adopting the above more general spontaneous gauge symmetry breaking mechanism, taking η' and η defined above to be, in general, different. For $\eta' = \eta$, of course, we shall get back to the old results of I and II.

The most important modification of phenomenological consequence is in the values of the (weak) intermediate gauge vector boson masses. We now find the results [to be compared with eq. (3.31) of I]:

$$m^2(W^\pm) = m^2(R^\pm) = f^2\eta^2(1 + \tfrac{1}{2}\delta),$$

$$m^2(Z_2) = m^2(H, \bar{H}) = f^2\eta^2,$$

$$m^2(Z_1) = (1 + \sigma^2)f^2\eta^2(1 + \tfrac{3}{4}\delta), \tag{3}$$

where we have defined

$$(\eta'/\eta)^2 \equiv 1 + \delta, \ (\delta \geq -1). \tag{4}$$

For $\eta' = \eta$, we have $\delta = 0$ and the results (3) go over to those of I. The $SU_3(W)$ gauge coupling constant is denoted by f, the $U_1(W)$-gauge coupling constant by f' and we use the parameters (introduced in I and II)

$$\tan \chi \equiv \sigma \equiv (2f'/\sqrt{3}f). \tag{5}$$

The expressions for the gauge vector fields of definite mass after symmetry breaking, $W_\mu{}^\pm$, $R_\mu{}^\pm$, H_μ, $H_\mu{}^*$, $Z_{1\mu}$, $Z_{2\mu}$ and A_μ, in terms of the hermitian gauge fields, $W_{\mu a}$, $a = 1, \ldots, 8$, and B_μ, remain the same as before [see eq. (3.30) of I] as also the form of their interactions with the leptons and quarks [see eq. (5.4) of I and eqs (6) and (7) of II]. The effect of the changes in the vector boson masses, eq. (3), if we generalize to $\delta \neq 0$, will be seen in the effective four-fermion Lagrangians mediated by these vector bosons. Keeping to the fixed normalization:

$$G = \frac{f^2}{4\sqrt{2}\, m^2\,(W^\pm)}, \quad f = \frac{2e}{\sqrt{3}\sin\chi}, \tag{6}$$

where G is the standard Fermi coupling constant for the $W^\pm$-mediated weak interaction, we now have to make the following replacements in the effective four-fermion interactions of I and II:

$$G \to G, \text{ in } R^\pm\text{-mediated int.,} \tag{7}$$

$$G \to G\,\delta_1, \text{ in } Z_1\text{-mediated int.,} \tag{8}$$

$$G \to G\,\delta_2, \text{ in } Z_2\text{-mediated int.,} \tag{9}$$

$$G \to G\,\delta_2, \text{ in } (H, \bar{H})\text{-mediated int.,} \tag{10}$$

where we have introduced the abbreviations:

$$\delta_1 \equiv 1 - \frac{\delta}{6 + 4\delta}, \quad \delta_2 \equiv 1 + \tfrac{1}{2}\delta. \tag{11}$$

If we take $\eta' = \eta$, *i.e.*, $\delta = 0$, we have $\delta_1 = \delta_2 = 1$ and we get back to the old results of I and II.

3. Parity-violation in atomic physics

The effective parity-violating electron-nucleus potential (*see* section 9 of II), arising from the Z_1 and the Z_2 exchanges, is proportional to the parameter $Q(Z, N)$ given by:

$$Q(Z, N) = (\delta_2 - \delta_1) N + 2(\delta_2 - \delta_1 \sin^2 \chi) Z, \tag{12}$$

where N is the neutron number and Z the proton number of the nucleus in question. For $\delta = 0$, *i.e.*, for $\delta_1 = \delta_2 = 1$, we get back the expression, derived in II, depending only on Z and $\sin^2 \chi$. In the standard WS–GIM theory, where only one neutral weak vector boson occurs, the expression is

$$Q(Z, N) = (1 - 4 \sin^2 \theta_W) Z - N, \text{ (WS–GIM theory).} \tag{13}$$

As discussed in II, the neutral current processes involving the neutrinos are reasonably well described by $\delta \simeq 0$ and $\sin^2 \chi \simeq \frac{3}{8}$ in our theory, as well as in the WS–GIM theory with $\sin^2 \theta_W \simeq \frac{3}{8}$. The approximations in the theoretical models used (chiefly that of using the valence quark parton model) and the uncertainties in the experimental results, however, still allow us the freedom in our theory to admit small non-zero values for δ.

To probe the effect of taking small non-zero values of δ in the atomic physics parity-violation, let us fix $\sin^2 \chi \simeq \frac{3}{8}$, and consider the example of the experimentally interesting case of Bi ($Z = 83$, $N = 126$).

We then find

$$Q(\text{Bi}) \simeq \begin{cases} +7, \text{ for } \delta = -\frac{1}{2}, \\ +58, \text{ for } \delta = -\frac{1}{4}, \\ +104, \text{ for } \delta = 0, \\ +148, \text{ for } \delta = +\frac{1}{4}. \end{cases} \tag{14}$$

In the WS–GIM theory, for $\sin^2 \theta_W \simeq \frac{3}{8}$, $Q(\text{Bi}) \simeq -168$. Thus the preliminary experimental results quoted in eqs (1) and (2), while in almost certain disagreement with the WS–GIM theory, can yet be very well accommodated in our theory with a suitable choice of δ. More definitive future experiments should be able to determine the value of δ. Coupling the results (14) with the discussion of the next section on neutrino neutral current processes, suggests that values of $\delta \simeq -\frac{1}{4}$ up to 0 are rather likely.

4. Neutral current processes of neutrinos

The value of δ is also important for the neutral current processes involving the neutrinos, since the mass of the neutral vector boson Z_1, mediating them also now depends on it according to eq. (3). According to eq. (8), the Z_1-mediated effective Lagrangians, relevant for these processes, given in paper II must then be multiplied by a factor δ_1 defined in eq. (11).

As a first example of incorporating this change, let us consider the deep inelastic inclusive scatterings of ν_μ and $\bar{\nu}_\mu$ off the isospin averaged nucleon ($\mathcal{N}$) (*see* section 5 of II). The *ratio* of the cross-sections for these processes, of course, does not depend on δ but only on the value of $\sin^2\chi$. Taking $\sin^2\chi \simeq \frac{3}{8}$, we have ($X$ standing for "anything"):

$$[\sigma(\bar{\nu}_\mu + \mathcal{N} \to \bar{\nu}_\mu + X)/\sigma(\nu_\mu + \mathcal{N} \to \nu_\mu + X)] \simeq 0 \cdot 55, \tag{15}$$

which agrees quite well with the value $0 \cdot 59 \pm 0 \cdot 14$ of the Gargamelle experiment (Blietschau *et al* 1976). The values for the parameters

$$R_\nu \equiv [\sigma(\nu_\mu + \mathcal{N} \to \nu_\mu + X)/\sigma(\nu_\mu + \mathcal{N} \to \mu^- + X)], \tag{16}$$

$$R_{\bar{\nu}} \equiv [\sigma(\bar{\nu}_\mu + \mathcal{N} \to \bar{\nu}_\mu + X)/\sigma(\bar{\nu}_\mu + \mathcal{N} \to \mu^+ + X)], \tag{17}$$

however, must now be multiplied by δ_1^2. The values of R_ν and $R_{\bar{\nu}}$, obtained in the *approximation* of the valence quark parton model, are given in table 1 for various values of δ, along with the values of the Gargamelle experiment (Blietschau *et al* 1976).

From table 1, we see that the agreement with the experiment is quite reasonable for $\delta \gtrsim -\frac{1}{4}$. In judging the theoretical values we must, of course, remember that the approximation of the valence quark parton model has been used. The same model gives for the well established charged current interaction process the result: $[\sigma(\bar{\nu}_\mu + \mathcal{N} \to \mu^+ + X)/\sigma(\nu_\mu + \mathcal{N} \to \mu^- + X)] \simeq 0 \cdot 33$ (independently of $\sin^2\chi$ and δ), whereas the value of the Gargamelle experiment (Blietschau *et al* 1976) is $0 \cdot 38 \pm 0 \cdot 02$.

The above result coupled with the results of eq. (14), relevant to the parity-violation in atomic physics, leads us to suggest that values of $\delta \simeq -\frac{1}{4}$ up to 0 would be rather likely.

As another example of the Z_1-mediated processes, we consider the $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ scattering cross-sections (*see* section 3 of II). In the commonly employed notation:

$$\sigma(\nu_\mu e) \equiv C(\nu_\mu e) \times 10^{-41} (E_\nu/\text{GeV}) \text{ cm}^2,$$

$$\sigma(\bar{\nu}_\mu e) \equiv C(\bar{\nu}_\mu e) \times 10^{-41} (E_\nu/\text{GeV}) \text{ cm}^2, \tag{18}$$

we obtain, with $\delta \simeq -\frac{1}{4}$, $\sin^2\chi \simeq \frac{3}{8}$, the values:

$$C(\nu_\mu e) \simeq 0 \cdot 09, \quad C(\bar{\nu}_\mu e) \simeq 0 \cdot 27. \qquad (19$$

These are quite reasonable in view of the rather uncertain experimental situation. Thus the Gargamelle experiment (Musset 1976) gives $C(\bar{\nu}_\mu e) = 0 \cdot 11^{+0 \cdot 21}_{-0 \cdot 09}$, $C(\nu_\mu e) < 0 \cdot 26$; and the Aachen-Padova experiment (Faissner *et al* 1976) gives $C(\bar{\nu}_\mu e) = 0 \cdot 54 \pm 0 \cdot 17$, $C(\nu_\mu e) = 0 \cdot 24 \pm 0 \cdot 12$.

Table 1. (R_ν and $R_{\bar{\nu}}$ for $\sin^2\chi \simeq \frac{3}{8}$.)

	$\delta = -\frac{1}{2}$	$\delta = -\frac{1}{4}$	$\delta = 0$	$\delta = +\frac{1}{4}$	Experiment (Gargamelle)
R_ν	0·42	0·36	0·33	0·31	0·26±0·04
$R_{\bar{\nu}}$	0·67	0·58	0·53	0·49	0·39±0·06

All the Z_1 and Z_2 mediated interactions, discussed in paper II, may be similarly modified if $\delta \neq 0$. The changes are obvious and may be read off using the replacements indicated in eqs (7) to (10). Thus for the weak interaction effects in the process $e^+e^- \rightarrow \mu^+\mu^-$, the parameters of section 8 of II, must now be changed to $h_{VV} = \frac{1}{2}\delta_2 + \frac{2}{3}\ \delta_1\ (\frac{1}{2} - 3\sin^2\chi)^2$, $h_{AA} = \frac{1}{2}\delta_2 + \frac{1}{6}\ \delta_1$, $h_{VA} = \frac{1}{2}\delta_2 + \frac{1}{3}\delta_1(\frac{1}{2} - 3\sin^2\chi)$. These particular changes, however, will be of interest in the experiments of a more distant future and are given here for completeness.

We hope that more definitive experimental results on the atomic physics parity violation, as well as on neutrino neutral current processes, will be available in the near future to enable a full scale testing of our theory.

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