Pramâṇa, Vol. 8, No. 5, 1977, pp. 462–470. © Printed in India.

Magnetic moments of baryons in higher symmetry schemes including charm

RAMESH C VERMA and M P KHANNA
Department of Physics, Panjab University, Chandigarh 160014

MS received 3 December 1976; in revised form 12 February 1977

Abstract. Relations among the magnetic moments of charmed and uncharmed baryons are derived in the framework of SU(4) and SU(8) symmetries. The SU(3) result $\mu(\Sigma^0) = -\mu(\Lambda)$ is not present in SU(4), but is obtained in SU(8). Higher order effects are further considered to improve the situation.

Keywords. Magnetic moments; higher symmetries; baryons and baryon resonances.

List of Symbols

B	Tensor representing baryon multiplets
$\bar{B}$	Tensor for antibaryons
B_{ABC}	Tensor for 120 representation of SU(8); [SU(8) indices A, B, C run from 1 to 8]
$D_{\alpha\beta\gamma}$	SU(4) tensor for 20 multiplet } [SU(4) indices $\alpha, \beta, \gamma = 1, 2, 3, 4$]
$B_\gamma^{[\alpha,\beta]}$, $N_\gamma^{[\alpha,\beta]}$	SU(4) tensors for 20′ multiplet } [SU(4) indices $\alpha, \beta, \gamma = 1, 2, 3, 4$]
$(J_\mu)^\alpha_\beta$	Baryonic current
$M^C_{C'}$	SU(8) magnetic moment operator
(m, n)	SU(4) $\times$ SU(2) component of SU(8) representation.
$Q^\gamma_{\gamma'}$	SU(4) charge operator
γ_μ	Dirac matrices
$(\vec{\sigma})$	Pauli matrices
ϵ_{ij}, $\epsilon_{\alpha\beta\rho\delta}$	SU(2) and SU(4) Levi-Civita symbols
χ_{ijk}	Spin 3/2 functions } [SU(2) indices $i, j, k = 1, 2$]
χ_i	Spin 1/2 functions } [SU(2) indices $i, j, k = 1, 2$]
ψ_α	Quark function
$[\alpha, \beta]$	Antisymmetrization between α and β.

1. Introduction

The SU(4) charm scheme predicts a rich spectrum of additional particles (Gaillard *et al* 1975) carrying non-zero charm quantum number. Experimental searches (Cazzoli *et al* 1975; Knapp *et al* 1976; Goldhaber 1976) have given some evidence for these particles to exist. Study of the properties of charmed particles in the higher symmetry frameworks is useful, since it provides tests for the validity of these symmetries.

In this note we wish to derive relations among the magnetic moments of the $J^P = 1/2^+$ and $3/2^+$ baryons in SU(4) and SU(8) symmetries. Magnetic moments of hyperons have been calculated earlier in the SU(3) symmetry scheme (Coleman and Glashow 1961; Okubo 1962) assuming certain transformation properties of the charge operator. In the SU(4) considerations we have assumed magnetic moment operator to be proportional to the charge operator that transforms like $T_1^1 + T_4^4 - 1/3\, T_a^a$ member of $15 \oplus 1$ of SU(4). With this transformation property, the SU(3) result $\mu(\Sigma^0) = -\mu(\Lambda)$ is not obtained.

Correct magnetic moment ratio $\mu(p)/\mu(N) = -3/2$ was obtained (Beg *et al* 1964; Pais 1964) in SU(6) symmetry. In these considerations magnetic moment operator was assumed to transform like (8, 3) component of 35 representation. Similar results were also obtained in quark models (Thirring 1965; Franklin 1969) by vectorially adding the magnetic moments of the constituent quarks of the particles.

Basing on the success of SU(6) symmetry, we study the magnetic moments of uncharmed and charmed baryons in SU(8), which incorporates the intrinsic spin of the particles. In SU(8) considerations, magnetic moments of all the forty baryons and the transition moments are expressible in terms of only one parameter. The SU(3) result $\mu(\Sigma^0) = -\mu(\Lambda)$ reappears. We have further considered the second order effects to remove any discrepancies which may exist in the relations.

2. Preliminaries

In SU(4), we take $J^P = 1/2^+$ and $3/2^+$ baryons to belong to mixed symmetric 20′ and totally symmetric 20 representations respectively. Their SU(3) contents are:

$$\begin{aligned} 20' :\ & 8_0\,(N\,\Lambda\,\Sigma\,\Xi) \\ & 6_1\,(\Sigma_1\,\Xi_1\,\Omega_1) \\ & 3_1^*\,(\Lambda'_1\,\Xi'_1) \\ & 3_2\,(\Xi_2\,\Omega_2) \end{aligned} \tag{1}$$

$$\begin{aligned} 20 :\ & 10_0\,(\triangle\Sigma^*\,\Xi^*\Omega) \\ & 6_1\,(\Sigma_1^*\,\Xi_1^*\,\Omega_1^*) \\ & 3_2\,(\Xi_2^*\,\Omega_2^*) \\ & 1_3\,(\Omega_3^*) \end{aligned} \tag{2}$$

where subscript denotes the charm of the SU(3) multiplets.

Charge operator Q can be written as

$$Q = e/3 \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = e \left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - 1/3 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right]$$

Then the electromagnetic (em) current is

$$J_\mu = ie\,(\bar{\psi}^1 \gamma_\mu \psi_1 + \bar{\psi}^4 \gamma_\mu \psi_4 - 1/3\,\langle \bar{\psi}\, \gamma_\mu\, \psi \rangle) \tag{3}$$

which is a $T_1{}^1 + T_4{}^4 - 1/3\, T_a{}^a$ component of the 15-plet tensor current:

$$(J_\mu)_\beta{}^a = ie\,\bar{\psi}^a\, \gamma_\mu\, \psi_\beta. \tag{4}$$

The magnetic moments of the baryons to lowest order are obtained by taking the expectation value of this current operator between the baryon states.

In second order effects, representations which can contribute to (Sakita 1966) the magnetic moments are present in the direct product.

$$15 \otimes 15 = 1 \oplus 2\,(15) \oplus 20'' \oplus 45 \oplus 45^* \oplus 84. \tag{5}$$

Here $15 \oplus 1$ gives the same contributions as in first order. Other representations give different second order contributions. Second order magnetic moment operator (Okubo 1963) is assumed to transform like $a_1\, T^{11}_{11} + a_2\, T^{44}_{44} + a_3\, T^{14}_{14}$ member of these representations.

Thus magnetic moments of the baryons can be obtained from the contraction $\bar{B}BQ$ in various possible combinations. Here B is tensor representing baryon multiplets and Q is the charge operator. Similarly second order effects are considered in $\bar{B}BQQ$.

In SU(8) classifications $J^P = 1/2^+$ and $3/2^+$ baryons may be put in the totally symmetric representation 120, where $120 = (20', 2) \oplus (20, 4)$. All the magnetic moments in the first order can be calculated from the trace

$$\bar{B}^{ABC'}\, B_{ABC}\, M^{C}_{C'} \tag{6}$$

where B_{ABC} is a tensor representing 120 multiplet and has the SU(4) $\times$ SU(2) structure:

$$B_{ABC} = \chi_{ijk}\, D_{\alpha\beta\gamma} + 1/6\sqrt{2}\ [\epsilon_{ij}\, \chi_k\, \epsilon_{\alpha\beta\rho\delta}\, N^{[\rho,\delta]}_{\gamma} + \text{cyclic terms}] \tag{7}$$

$$(i, j, k = 1, 2) \text{ and } (\alpha, \beta, \gamma, \rho, \delta = 1, 2, 3, 4).$$

Magnetic moment operator $M^C_{C'}$ transforms like $(15 \oplus 1, 3)$ component of 63 representation.

$$M^C_{C'} = (\bar{\sigma})^k_{k'}\, Q^{\gamma}_{\gamma'} \tag{8}$$

$(\bar{\sigma})^k_{k'}$ are the Pauli matrices.

In SU(8), D/F ratio becomes unique because 63 can be coupled to $120^* \otimes 120$ in only one way.

$$120^* \otimes 120 = 1 \oplus 63 \oplus 1232 \oplus 13104. \tag{9}$$

Second order contributions are obtained from the trace, $\bar{B}^{AB'C'}\, B_{ABC}\, M^B_{B'}\, M^C_{C'}$ where magnetic moment operator corresponds to 1232.

3. Magnetic moments in SU(4)

In this section we have listed various relations among the magnetic moments of baryons.

3.1. 20′ *multiplet*

For $J^P = \frac{1}{2}^+$ baryons we have the following two types of $\bar{B}BQ$ coupling (F and D type) in first order.

$$\left(1/2\, \bar{B}^{\alpha}_{[\rho,\delta]}\, B^{[\rho,\delta]}_{\beta} \mp \bar{B}^{\delta}_{[\rho,\beta]}\, B^{[\rho,\alpha]}_{\delta}\right) Q_{\alpha}{}^{\beta}. \tag{10}$$

Following relations are obtained:

$$\mu(\Omega_2{}^+) = \mu(\Xi_2{}^+) = \mu(\Sigma^+) = \mu(P) \tag{11}$$

$$\mu(\Omega_1{}^0) = \mu(\Xi_1{}^0) = \mu(\Sigma_1{}^0) = -\, 1/2\, \mu(\Xi_2{}^{++}) = -\, 1/2\, \mu(\Sigma_1{}^{++})$$
$$= \mu(\Xi^0) = \mu(N) \tag{12}$$

$$\mu(\Xi_1{}^+) = \mu(\Sigma_1{}^+) = -\, 1/2\, \mu(N) \tag{13}$$

$$\mu(\Xi^-) = \mu(\Sigma^-) = \mu(P) + 3\mu(N) \tag{14}$$

$$\mu(\Sigma^0) = 1/2\, \big(\mu(\Sigma^+) + \mu(\Sigma^-)\big) = 3\mu(\Lambda) - 2\mu(N) \tag{15}$$

$$\mu(\Xi_1{}'^+) = \mu(\Lambda_1{}'^+) = 2/3\, \mu(P) - 1/6\, \mu(N) \tag{16}$$

$$\mu(\Xi_1{}'^0) = 4/3\mu(P) + 5/3\, \mu(N) \tag{17}$$

$$\langle \Lambda_1{}'^+ \mid \mu \mid \Sigma_1{}^+ \rangle = -\, \langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^+ \rangle = \langle \Lambda \mid \mu \mid \Sigma^0 \rangle = \sqrt{3}\, \big(\mu(\Lambda) - \mu(N)\big) \tag{18}$$

$$\langle \Xi_1{}'^0 \mid \mu \mid \Xi_1{}^0 \rangle = 0 \tag{19}$$

Notice that $\mu(\Sigma^0) \neq -\mu(\Lambda)$. Except this all other results of SU(3) are reproduced. Gupta and Kögerler have obtained our relations for uncharmed baryons in SU(3) by adding singlet contribution to em current. At the SU(4) level second order contributions may come from 20″, 84, 45 and 45* representations, as these representations are common in the direct product (5) and

$$20^{*\prime} \otimes 20' = 1 \oplus 15 \oplus 20'' \oplus 45 \oplus 45^* \oplus 84 \oplus 175 \tag{20}$$

84 dominance for the second order effects gives the following results:

(1) Relations (11), (12), (15) and (19) are maintained.

(2) In addition:

$$\mu(\Xi^-) = \mu(\Sigma^-) \tag{21}$$

$$\mu(\Xi_1{}^+) = \mu(\Sigma_1{}^+) = 6\mu(\Lambda) - \mu(\Sigma^-) - \mu(P) - 9/2\, \mu(N) \tag{22}$$

$$\mu(\Xi_1{}'^+) = \mu(\Lambda_1{}'^+) = \mu(\Xi_1{}'^0) + \mu(\Lambda) + 1/2\, \big(\mu(\Sigma^-) - 3\mu(P) + 9\mu(N)\big) \tag{23}$$

(3) Relation (18) breaks up to give:

$$\langle \Lambda \mid \mu \mid \Sigma^0 \rangle = \sqrt{3}\, \big(\mu(\Lambda) - \mu(N)\big) \tag{24}$$

$$\langle \Lambda_1{}'^+ \mid \mu \mid \Sigma_1{}^+ \rangle = -\, \langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^+ \rangle \tag{25}$$

Inclusion of 20″ contributions does not disturb the relations much. Relation (23) is reduced to

$$\mu(\Xi_1{}'^+) = \mu(\Lambda_1{}'^+). \tag{26}$$

On the other hand, if magnetic moment contributions are calculated from all the representations 20″, 45, 45* and 84, we get

$$\mu(\Xi_2^+) = \mu(\Omega_2^+) \tag{27}$$

$$\mu(\Sigma^+) = \mu(P). \tag{28}$$

And the relations (12), (15), (19), (21), (22), (24), (25), and (26) are also given. Note that contributions from 45 and 45* have to be equal and opposite in order to yield $\langle x \mid \mu \mid y \rangle = \langle y \mid \mu \mid x \rangle$, where x, y are the states mixed due to SU(2) and SU(3) breaking.

3.2. 20 *multiplet*:

In SU(4), magnetic moments of $J^P = 3/2^+$ baryons are obtained from the trace:

$$\bar{D}^{\alpha\beta\gamma'} D_{\alpha\beta\gamma} Q^{\gamma}_{\gamma'} \tag{29}$$

We get the following relations,

$$\mu(\triangle^{++}) = \mu(\Sigma_1^{*++}) = \mu(\Xi_2^{*++}) = \mu(\Omega_3^{*++}) = 2\mu(\triangle^+) \tag{30}$$

$$\mu(\Sigma^{*+}) = \mu(\Sigma_1^{*+}) = \mu(\Xi_1^{*+}) = \mu(\Xi_2^{*+}) = \mu(\Omega_2^{*+}) = \mu(\triangle^+) \tag{31}$$

$$\mu(\triangle^0) = \mu(\Sigma^{*0}) = \mu(\Xi^{*0}) = \mu(\Sigma_1^{*0}) = \mu(\Xi_1^{*0}) = \mu(\Omega_1^{*0}) = 0 \tag{32}$$

$$\mu(\triangle^-) = \mu(\Sigma^{*-}) = \mu(\Xi^{*-}) = \mu(\Omega^-) = -\mu(\triangle^+) \tag{33}$$

One can easily see that magnetic moments satisfy the relation

$$\mu(x) = Q_s\,\mu(\triangle^+). \tag{34}$$

Here the second order contributions come only from the 84 representation, which maintain (32) and (33) and further yields:

$$\mu(\Sigma^{*+}) = \mu(\triangle^+);\quad \mu(\Xi_1^{*+}) = \mu(\Sigma_1^{*+});\ \mu(\Omega_2^{*+}) = \mu(\Xi_2^{*+}) \tag{35}$$

$$3\mu(\triangle^+) + \mu(\triangle^-) = \mu(\triangle^{++}) \tag{36}$$

$$3\mu(\Omega_2^{*+}) + \mu(\Omega^-) = \mu(\Omega_3^{*++}) \tag{37}$$

$$\mu(\Xi_2^{*+}) + \mu(\Xi_1^{*+}) = \mu(\Xi_2^{*++}) \tag{38}$$

$$\mu(\Sigma_1^{*+}) + \mu(\Sigma^{*+}) = \mu(\Sigma_1^{*++}) \tag{39}$$

3.3. *Transition moments between* 20 *and* 20′ *multiplets*:

SU(4) symmetry expresses all the transition moments among 20 and 20′ in terms of one parameter. This is clear from the direct product

$$20^* \otimes 20' = 15 \oplus 45^* \oplus 84 \oplus 256^* \tag{40}$$

where 15 occurs only once. Because singlet representation is not present in the direct product, only 15 part of charge operator contributes. We denote the transition moment between the state A and B by $\langle A \mid \mu \mid B \rangle$.

The trace,

$$\epsilon_{\alpha\gamma\rho\delta}\, \bar{D}^{\alpha\beta\gamma'} B^{[\rho,\delta]}_{\beta} Q^{\gamma}_{\gamma'} \tag{41}$$

then gives:

$$\langle P \mid \mu \mid \Delta^+\rangle = \langle N \mid \mu \mid \Delta^0\rangle = -\langle \Sigma^+ \mid \mu \mid \Sigma^{*+}\rangle = -\langle \Xi^0 \mid \mu \mid \Xi^{*0}\rangle$$
$$= \langle \Sigma_1{}^0 \mid \mu \mid \Sigma_1{}^{*0}\rangle = \langle \Xi_1{}^0 \mid \mu \mid \Xi_1{}^{*0}\rangle = \langle \Omega_1{}^0 \mid \mu \mid \Omega_1{}^{*0}\rangle = \langle \Xi_2{}^+ \mid \mu \mid \Xi_2{}^{*+}\rangle$$
$$= \langle \Omega_2{}^+ \mid \mu \mid \Omega_2{}^{*+}\rangle = 2\,\langle \Sigma^0 \mid \mu \mid \Sigma^{*0}\rangle = 2\,\langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = 2\,\langle \Xi_1{}^+ \mid \mu \mid \Xi_1{}^{*+}$$
$$= 2/\sqrt{3}\,\langle \Lambda \mid \mu \mid \Sigma^{*0}\rangle = 2/\sqrt{3}\,\langle \Lambda_1{}'^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = -2/\sqrt{3}\,\langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^{*+}\rangle \tag{42}$$

$$\langle \Sigma^- \mid \mu \mid \Sigma^{*-}\rangle = \langle \Xi^- \mid \mu \mid \Xi^{*-}\rangle = \langle \Xi_1{}'^0 \mid \mu \mid \Xi_1{}^{*0}\rangle = \langle \Sigma_1{}^{++} \mid \mu \mid \Sigma_1{}^{*++}\rangle$$
$$= \langle \Xi_2{}^{++} \mid \mu \mid \Xi_2{}^{*++}\rangle = 0. \tag{43}$$

It is interesting to see that in the first order limit, $\langle \Xi_2{}^{++} \mid \mu \mid \Xi_2{}^{*++}\rangle$ and $\langle \Sigma_1{}^{++} \mid \mu \mid \Sigma_1{}^{*++}\rangle$ vanish. But these transition moments are non-zero in the second order limit. At the SU(4) level, second order magnetic moment contributions may come from 45* and 84. The 84 dominance gives the following relations.

$$\langle N \mid \mu \mid \Delta^0\rangle = 2/\sqrt{3}\,\langle \Lambda \mid \mu \mid \Sigma^{*0}\rangle = 2\,\langle \Sigma^0 \mid \mu \mid \Sigma^{*0}\rangle = \langle \Xi^0 \mid \mu \mid \Xi^{*0}\rangle \tag{44}$$
$$= \langle \Sigma_1{}^0 \mid \mu \mid \Sigma_1{}^{*0}\rangle = \langle \Xi_1{}^0 \mid \mu \mid \Xi_1{}^{*0}\rangle = \langle \Omega_1{}^0 \mid \mu \mid \Omega_1{}^{*0}\rangle$$

$$\langle \Xi^- \mid \mu \mid \Xi^{*-}\rangle = \langle \Sigma^- \mid \mu \mid \Sigma^{*-}\rangle = \langle \Xi_1{}'^0 \mid \mu \mid \Xi_1{}^{*0}\rangle = 0 \tag{45}$$

$$\langle \Sigma^+ \mid \mu \mid \Sigma^{*+}\rangle = \langle P \mid \mu \mid \Delta^+\rangle \tag{46}$$

$$\langle \Xi_2{}^+ \mid \mu \mid \Xi_2{}^{*+}\rangle = \langle \Omega_2{}^+ \mid \mu \mid \Omega_2{}^{*+}\rangle \tag{47}$$

$$\langle \Lambda_1{}'^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = -\langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^{*+}\rangle = \sqrt{3}\,\langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = \sqrt{3}\,\langle \Xi_1{}^+ \mid \mu \mid \Xi_1{}^{*+}\rangle \tag{48}$$

$$\langle \Sigma_1{}^{++} \mid \mu \mid \Sigma_1{}^{*++}\rangle = \langle P \mid \mu \mid \Delta^+\rangle - 2\,\langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle \tag{49}$$

$$\langle \Xi_2{}^{++} \mid \mu \mid \Xi_2{}^{*++}\rangle = 2\,\langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle - \langle \Xi_2{}^+ \mid \mu \mid \Xi_2{}^{*+}\rangle. \tag{50}$$

Thus we expect decays $\Sigma_1{}^{*++} \to \Sigma_1{}^{++} + \gamma$ and $\Xi_2{}^{*++} \to \Xi_2{}^{++} + \gamma$ to occur only through the higher order interactions. Inclusion of 45* representation does not disturb the relations much. Relations (48) and (50) are modified in the following manner:

$$\langle \Lambda_1{}'^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = -\langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^{*+}\rangle \tag{51}$$

$$\langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle = \langle \Xi_1{}^+ \mid \mu \mid \Xi_1{}^{*+}\rangle \tag{52}$$

$$\langle \Xi_2{}^{++} \mid \mu \mid \Xi_2{}^{*++}\rangle = -\langle \Xi_2{}^+ \mid \mu \mid \Xi_2{}^{*+}\rangle - \langle \Sigma_1{}^+ \mid \mu \mid \Sigma_1{}^{*+}\rangle$$
$$- \sqrt{3}\,\langle \Xi_1{}'^+ \mid \mu \mid \Xi_1{}^{*+}\rangle \tag{53}$$

4. Magnetic moments in SU(8)

In SU(8), we take magnetic moment operator to transform as spin triplet under SU(2) subgroup of SU(8). This structure is indicated by the presence of Pauli matrices in (8).

4.1. 20′ *multiplet*

Projecting (15 ⊕ 1, 3) component of 63 representation in (6) we obtain

$$\mu(\Omega_2^+) = \mu(\Xi_2^+) = \mu(\Sigma^+) = \mu(P) \tag{54}$$

$$-\mu(\Xi_2^{++}) = -\mu(\Sigma_1^{++}) = \mu(N) = \mu(\Xi^0) = 2\mu(\Sigma^-) = 2\mu(\Xi^-)$$
$$= \mu(\Omega_1^0) = \mu(\Xi_1^0) = \mu(\Sigma_1^0) = -2/3\,\mu(P) \tag{55}$$

$$\mu(\Xi_1'^+) = \mu(\Xi_1'^0) = \mu(\Lambda_1'^+) = 2/3\,\mu(P) \tag{56}$$

$$\mu(\Sigma^0) = -\mu(\Lambda) = 1/3\mu(P) \tag{57}$$

$$\mu(\Xi_1^+) = \mu(\Sigma_1^+) = 0 \tag{58}$$

$$\langle \Lambda_1'^+ \mid \mu \mid \Sigma_1^+\rangle = -\langle \Xi_1'^+ \mid \mu \mid \Xi_1^+\rangle = \langle \Lambda \mid \mu \mid \Sigma^0\rangle = \lambda/3\,\mu(P) \tag{59}$$

$$\langle \Xi_1'^0 \mid \mu \mid \Xi_1^0\rangle = 0. \tag{60}$$

The well known result, $\mu(P)/\mu(N) = -3/2$, of SU(6) is gotten. SU(3) result $\mu(\Sigma^0) = -\mu(\Lambda)$ also appears. Because D/F ratio is fixed, the magnetic moments of all the baryons are expressed in terms of one parameter.

Second order effects are studied in 1232 representation. Here 20″ does not contribute at SU(4) level, since (20″, 3) component is not present in 1232. Inclusion of (84, 3) component gives the following results:

(1) (54) and (55) and (25) are reproduced;

(2) other relations are modified to:

$$\mu(\Sigma^0) = 3\mu(\Lambda) - 2\mu(N) \tag{61}$$

$$\mu(\Xi_1^+) = \mu(\Sigma_1^+) \tag{62}$$

$$\mu(\Xi_1'^+) = \mu(\Lambda_1'^+) = 2\,\mu(\Xi_1'^0) - 3\mu(N) - 1/3\mu(\Sigma_1^+) - 8/3\mu(P). \tag{63}$$

4.2. 20 *multiplet*

In the first order limit (15 ⊕ 1, 3) component of 63 gives the same relations (30–34) for $J_P = 3/2^+$ baryons. In addition SU(8) relates the magnetic moment of two multiplets 20′ and 20.

$$i.e.,\ \mu(\triangle^+) = \mu(P). \tag{64}$$

Second order effects also give the similar modifications as in (35)–(39).

4.3. *Transition moments between* 20 *and* 20′ *multiplets*

In addition to all the results (42) and (43) obtained in SU(4) framework, SU(8) gives

$$\langle P \mid \mu \mid \triangle^+\rangle = 2\sqrt{2}/3\,\mu(P) \tag{65}$$

which has already been obtained in SU(6) (Beg *et al* 1964). This result has been compared with experiment (Gourdin and Salin 1963; Geshkenbein 1965) and discussed (Pais 1966). Higher order effects seem to improve the situation.

In SU(8) framework, we have calculated second order effects, assuming the magnetic moment operator to transform as (45*, 3) and (84, 3) components of 1232 representation. The results of SU(4) are reproduced.

5. Conclusion

We have first calculated the magnetic moments of baryons in SU(4) assuming magnetic moment operator to be proportional to the charge operator $Q^{\gamma}_{\gamma'}$. All the results of SU(3) are obtained except $\mu(\Sigma^0) = -\mu(\Lambda)$. This is because SU(4) charge operator transforms like $15 \oplus 1$, so that in addition to SU(3) contributions SU(4) gives a constant contribution to all the baryons. But this relation is regained in SU(8). It is due to the fact that in SU(8), the magnetic moment operator belongs to a traceless tensor representation. In SU(4), magnetic moments of 20′ multiplet are expressed in terms of two parameters while those of 20 in terms of one. We expect that like Coleman-Glashow relation, our relations would be valid for both total and anomalous magnetic moments.

We have incorporated intrinsic spin of the particles by handling the problem in SU(8) framework. All the magnetic moments are expressed in terms of only one parameter. The relations among uncharmed baryons already obtained in SU(6) (Beg *et al* 1964) are repeated in SU(8).

Second order effects are also considered. Modified relations are given in sections 3 and 4. These relations are expected to remove any discrepancies present in the first order relations. We hope that our relations obtained in SU(4) and SU(8) framework would be useful to test the validity of these symmetries.

In SU(8) framework if magnetic moment operator is taken up to all orders, we have the following three baryon contractions:

$$\bar{B}^{ABC'} B_{ABC} M^{C}_{C'} \; ; \; \bar{B}^{AB'C'} B_{ABC} M^{B}_{B'} M^{C}_{C'} ; \; \bar{B}^{A'B'C'} B_{ABC} M^{A}_{A'} M^{B}_{B'} M^{C}_{C'} . \quad (66)$$

These correspond to 63, 1232, and 13104 representations present in the direct product.

$$120^* \otimes 120 = 1 \oplus 63 \oplus 1232 \oplus 13104. \quad (67)$$

We have not taken into account the contributions of 13104 in calculating the magnetic moments, since it will lead to the introduction of a number of parameters. However, if we assume magnetic moment operator to be of the current $\otimes$ current form (Sakita 1964) then the magnetic moment operator transforms like:

$$63 \otimes 63 = 1 \oplus 63_S \oplus 63_A \oplus 720 \oplus 945 \oplus 945^* \oplus 1232. \quad (68)$$

Only representations common in the direct products (67) and (68) will then contribute to the magnetic moments. In this case 13104 is neglected. On the same grounds 175, 300 and 256 representations can be neglected in the SU(4) framework.

Acknowledgement

One of us (RCV) gratefully acknowledges the financial support given by the Council of Scientific and Industrial Research, New Delhi.

References

Becchi C and Morpurgo G 1965 *Phys. Lett.* **17** 352
Beg M A B, Lee B W and Pais A 1964 *Phys. Rev. Lett.* **13** 514
Cazzoli E G *et al* 1975 *Phys. Rev. Lett.* **34** 1125
Coleman S and Glashow S L 1961 *Phys. Rev. Lett.* 6 423
Franklin J 1969 *Phys. Rev.* **182** 1607
Gaillard M K, Lee B W and Rosner J L 1975 *Rev. Mod. Phys.* **47** 227
Geshkenbein B V 1965 *Phys. Lett.* **16** 323
Geshkenbein B V 1965 *JETP Lett.* **1** 127
Goldhaber A S and Goldhaber M 1975 *Phys. Rev. Lett.* **34** 36
Gourdin M and Salin P 1963 *Nuovo Cimento* **27** 193
Gupta V and Kögerler R 1975 CERN—TH 1986
Knapp B *et al* 1976 *Phys. Rev. Lett.* **37** 882
Okubo S 1962 *Prog. Theor. Phys.* **27** 949
Okubo S 1963 *Phys. Lett.* **4** 14
Pais A 1964 *Phys. Rev. Lett.* **13** 175
Pais A 1966 *Rev. Mod. Phys.* **38** 215
Sakita B 1964 *Phys. Rev. Lett.* **13** 643
Sakita B 1966 in *Advances in Particle Physics* ed. R L Cool and R E Marshak Vol. 1.
Thirring W 1965 *Acta Phys. Austriaca Suppl.* **2** 205