Pramâna, Vol. 8, No. 5, 1977, pp. 420–426. © Printed in India.

Stroboscopic holographic interferometry of vibration with space-variant phase

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MS received 13 November 1976

Abstract. A method employing pulsed illumination for the holographic analysis of vibration with space-variant phase is described. Superimposition of two stroboscopically recorded interference patterns to get the amplitude and the phase of vibration at the intersections of fringes is suggested. Contour maps of these two quantities over the entire object surface can then be prepared by interpolation. Detailed solutions are presented for single and double frequency vibrations.

Keywords. Holography interferometry; vibrations; phase information.

1. Introduction

Levitt and Stetson (1976) have presented a method to generate a phase map of object undergoing sinusoidal vibration by superimposing irradiances of two reconstructions, with different phase modulations of the reference beam (Aleksoff 1969) during the recording process. The method is basically the one described by Neumann *et al* (1970) but the number of recordings performed in the Levitt and Stetson method is extremely reduced and hence it should be very useful in actual practice.

This method is with the continuous exposure (time-average) technique applicable to single-frequency vibration only. Secondly, the time-average method results in rapidly falling intensity in the reconstruction making a very limited measurement range. Stroboscopic or pulsed illumination for the recording does not only provide a rather unlimited measurement range (Archbold and Ennos 1968, Shajanko and Johnson 1968, Vikram 1972, Watrasiewiez and Spicer 1968) but also independent patterns due to different modes if the vibration is the superposition of two or more time functions (Vikram 1974). Therefore, in this paper we suggest superimposition of reconstructed irradiances from different stroboscopically recorded holograms in order to attain advantages of Levitt and Stetson method of requiring less number of experiments, and also that of stroboscopic holographic interferometry. The detailed analysis is presented for single and double frequency sinusoidal vibrations.

2. Single frequency vibrations

The optical phase variation of the object beam at the hologram plane in this situation is

$$\Omega_0 \sin(\omega t + \phi_0), \tag{1}$$

where Ω_0 is the usual fringe locus function considered to be positive, ω is the angular frequency of vibration and $\phi_0 = \phi_0(\boldsymbol{R}_0)$ is phase of the vibration at the point in space $\boldsymbol{R}_0$. In terms of the displacement vector $\boldsymbol{L}(\boldsymbol{R}_0)$ of the object surface and the sensitivity vector $\boldsymbol{K}$, the phase variation given by (1) can be written as $\boldsymbol{K} \cdot \boldsymbol{L}(\boldsymbol{R}_0)$. Now we shall discuss the analysis of Ω_0 and ϕ_0 by superimposition of reconstructed irradiances of stroboscopically recorded holograms.

2.1. *Any two positions of the pulsed recordings*

Suppose half of the total exposure is provided by the pulsed illumination with $\omega t = 2(p-1)\pi + \theta_1$ and the remaining half with $\omega t = 2(p-1)\pi + \theta'_1$, p being a positive integer, then the normalized intensity in the reconstructed pattern is

$$I_1 = \tfrac{1}{4} \left| \exp[i\Omega_0 \sin(\theta_1 + \phi_0)] + \exp[i\Omega_0 \sin(\theta'_1 + \phi_0)] \right|^2$$
$$= \cos^2 \left[\Omega_0 \cos\left(\frac{\theta_1 + \theta'_1}{2} + \phi_0\right) \sin\left(\frac{\theta_1 - \theta_1'}{2}\right)\right]. \tag{2}$$

Obviously, the difference between θ_1 and θ'_1 should not be 0, 2π, 4π, 6π, etc., in order to avoid uniform intensity distribution given by (2). The position of the m-th (bright or dark) fringe of the distribution (2) is given by

$$\left| \Omega_0 \cos\left(\frac{\theta_1 + \theta'_1}{2} + \phi_0\right) \sin\left(\frac{\theta_1 - \theta'_1}{2}\right)\right| = x\pi, \tag{3}$$

where $x = m$ or $m - \frac{1}{2}$ depending whether the fringe is bright or dark respectively. The bright fringe corresponding to $\Omega_0 = 0$ is labelled as zeroth order

Similarly, another recording with $\omega t = 2(p-1)\pi + \theta_2$ and $\omega t = 2(p-1)\pi + \theta'_2$ will give

$$I_2 = \cos^2 \left[\Omega_0 \cos\left(\frac{\theta_2 + \theta'_2}{2} + \phi_0\right) \sin\left(\frac{\theta_2 - \theta'_2}{2}\right)\right]. \tag{4}$$

Here also, $|\theta_2 - \theta'_2|$ should not be 0, 2π, 4π, 6π, 8π, etc. Also, the difference between $\theta_2 + \theta'_2$ and $\theta_1 + \theta'_1$ should not be integral multiple of 2π in order to have different values of $|\cos[(\theta_1 + \theta'_1)/2 + \phi_0]|$ and $|\cos[(\theta_2 + \theta'_2)/2 + \phi_0]|$.

The position of the n-th fringe of the distribution (4) is given by

$$\left| \Omega_0 \cos\left(\frac{\theta_2 + \theta'_2}{2} + \phi_0\right) \sin\left(\frac{\theta_2 - \theta'_2}{2}\right)\right| = y\pi, \tag{5}$$

where $y = n$ or $n - \frac{1}{2}$ if the fringe is bright or dark respectively. The zeroth order bright fringe is considered that corresponding to $\Omega_0 = 0$.

Now suppose the irradiance distributions given by (2) and (4) are superimposed. If the m-th fringe due to (2) meet at some point with the n-th fringe due to (4), (3) and (5) can be solved, at that point, to result in

$$\left| \frac{\cos\left(\frac{\theta_1 + \theta'_1}{2} + \phi_0\right) \sin\left(\frac{\theta_1 - \theta'_1}{2}\right)}{\cos\left(\frac{\theta_2 + \theta'_2}{2} + \phi_0\right) \sin\left(\frac{\theta_2 - \theta'_2}{2}\right)} \right| = \frac{x}{y}, \tag{6}$$

from which ϕ_0 can be determined and then Ω_0 can be calculated out using (3) or (5).

To simplify the process, we can set $\theta_1 = \theta_2 = 0$ or integral multiple of 2π and $\theta'_2 = \pi + \theta'_1$, where (3) and (5) lead to

$$\left| \cot\left(\frac{\theta'_1}{2} + \phi_0\right) \right| = \frac{x}{y} \left| \cot\left(\frac{\theta'_1}{2}\right) \right| \tag{7}$$

and

$$\Omega_0 = \pi\,[x^2 \operatorname{cosec}^2(\theta'_1/2) + y^2 \sec^2(\theta'_1/2)]^{1/2}. \tag{8}$$

In this way, by knowing values of ϕ_0 and Ω_0 at every point of intersection, we can prepare contour maps of ϕ_0 and Ω_0 over the entire object surface.

2.2. *One static and one other position for the pulsed recording*

If half of the total exposure is given with the static object and the remaining half with the pulsed illumination such that $\omega t = 2(p-1)\pi + \theta_1$, p being positive integer, then the normalized reconstructed intensity is

$$\begin{aligned} I_1 &= \tfrac{1}{4}\,|\,1 + \exp[i\Omega_0 \sin(\theta_1 + \phi_0)]\,|^2 \\ &= \cos^2[\tfrac{1}{2}\,\Omega_0 \sin(\theta_1 + \phi_0)]. \end{aligned} \tag{9}$$

Another such experiment but with $\omega t = 2(p-1)\pi + \theta_2$, taking $\theta_2 = \pi/2 + \theta_1$ for simplicity, would give

$$I_2 = \cos^2[\tfrac{1}{2}\,\Omega_0 \cos(\theta_1 + \phi_0)]. \tag{10}$$

Fringe positions due to (9) and (10) are given as

$$|\,\tfrac{1}{2}\,\Omega_0 \sin(\theta_1 + \phi_0)\,| = x\pi \tag{11}$$

and

$$|\,\tfrac{1}{2}\,\Omega_0 \cos(\theta_1 + \phi_0)\,| = y\pi \tag{12}$$

respectively, where x and y are already defined in section 2.1. At points of intersections, (11) and (12) gives

$$|\tan(\theta_1 + \phi_0)| = x/y \tag{13}$$

and

$$\Omega_0 = 2\pi\,(x^2 + y^2)^{1/2}. \tag{14}$$

Thus, contour maps for ϕ_0 and Ω_0 over the entire object surface can be prepared using (13) and (14) respectively.

2.3. *Triple-exposure stroboscopy*

Let half of the total exposure be provided with the static object and the remaining half with two sets of pulses per vibration cycle described by $\omega t = 2p\pi + \theta_1$ and $(2p+1)\pi + \theta_1$, p being a positive integer. The normalized irradiance in reconstruction is

$$\begin{aligned} I_1 &= |\,\tfrac{1}{2} + \tfrac{1}{4}\exp[i\Omega_0 \sin(\theta_1 + \phi_0)] + \tfrac{1}{4}\exp[-\,i\Omega_0 \sin(\theta_1 + \phi_0)]\,|^2 \\ &= \cos^4[\tfrac{1}{2}\,\Omega_0 \sin(\theta_1 + \phi_0)]. \end{aligned} \tag{15}$$

Another such experiment with $\omega t = 2p\pi + \theta_2$ and $(2p + 1)\pi + \theta_2$, taking $\theta_2 = \theta_1 + \pi/2$ for simplicity, would give

$$I_2 = \cos^4 [\tfrac{1}{2} \Omega_0 \cos (\theta_1 + \phi_0)]. \tag{16}$$

The maxima and minima positions of (15) and (16) are similar to those due to (9) and (10) respectively and (13) and (14) can therefore be used here also for the preparation of the contour maps of ϕ_0 and Ω_0 respectively. The point to be mentioned here is that due to $\cos^4$ distribution here, the bright fringes are sharper and intersections of bright fringes would give more accurate location of the fringe positions.

3. Vibrations with two frequencies

The optical phase variation of the object beam at the hologram plane in this case is

$$\Omega_{01} \sin (\omega_1 t + \phi_{01}) + \Omega_{02} \sin (\omega_2 t + \phi_{02}), \tag{17}$$

where Ω_{01}, Ω_{02} are the usual fringe locus functions and ϕ_{01}, ϕ_{02} are the vibration phases associated with the angular vibration frequencies ω_1, ω_2 respectively. The purpose here is to analyse Ω_{01}, Ω_{02}, ϕ_{01}, ϕ_{02} over the object surface.

Suppose we perform the recording such that each time the condition

$$\omega_1 t = 2(p - 1)\pi + \theta, \quad p \text{ being a positive integer} \tag{18}$$

is satisfied. Then value of $\sin (\omega_1 t + \phi_{01})$ is each time unaltered and its effect will not come in the reconstructed irradiance. Thus, the analysis for ϕ_{02} and Ω_{02} can be done without the disturbance of the motion associated with the frequency ω_1.

Choose two recording times given by (18) such that $\omega_2 t$ comes out to be θ_1 and θ'_1. Thus, (17) leads to the normalized irradiance in the reconstruction to be

$$\begin{aligned} I_1 &= \tfrac{1}{4} \,|\, \exp [i\Omega_{02} \sin (\theta_1 + \phi_{02})] + \exp [i\Omega_{02} \sin (\theta'_1 + \phi_{02})] \,|^2 \\ &= \cos^4 \left[\Omega_{02} \cos \left(\frac{\theta_1 + \theta'_1}{2} + \phi_{02} \right) \sin \left(\frac{\theta_1 - \theta'_1}{2} \right) \right]. \end{aligned} \tag{19}$$

$|\theta_1 - \theta'_1|$ should not be 0, 2π, 4π, 6π, 8π, etc., in order to avoid uniform reconstruction. Another such experiment with two values of $\omega_2 t$ given by θ_2 and θ'_2 will give

$$I_2 = \cos^2 \left[\Omega_{02} \cos \left(\frac{\theta_2 + \theta'_2}{2} + \phi_{02} \right) \sin \left(\frac{\theta_2 - \theta'_2}{2} \right) \right]. \tag{20}$$

Here also, the difference between θ_2 and θ'_2 should not be 0, 2π, 4π, 6π, etc., and also, the difference between $|\theta_2 + \theta'_2|$ and $|\theta_1 + \theta'_1|$ should not be an integral multiple of 2π so that we get different patterns due to (19) and (20). If we look back, we see the distributions (19) and (20) are nothing but (2) and (4) respectively in nature. With special circumstance of $\theta_1 = \theta_2 = 0$ or integral multiple of 2π and $\theta'_2 = \pi + \theta'_1$, (7) and (8) give in this case

$$\left| \cot \left(\frac{\theta'_1}{2} + \phi_{02} \right) \right| = \frac{x}{y} \left| \cot \left(\frac{\theta'_1}{2} \right) \right| \tag{21}$$

and

$$\Omega_{02} = \pi \left[x^2 \operatorname{cosec}^2 (\theta'_1/2) + y^2 \sec^2 (\theta'_1/2)\right]^{1/2}. \tag{22}$$

(21) and (22) can be used to prepare contour maps of ϕ_{02} and Ω_{02} respectively over the object surface.

Extension for the analysis of Ω_{01} and ϕ_{01} is straightforward. We can repeat the above procedure but the recording times have to be selected such as to satisfy the condition

$$\omega_2 t = 2\,(p - 1)\,\pi + \theta, \quad p \text{ being a positive integer.} \tag{23}$$

This will result in no effect of the motion associated with the frequency ω_2 on the reconstructed irradiance.

There are specific situations when the above method is not applicable. Suppose that $\omega_2 > \omega_1$. If $\omega_2/\omega_1 = 2$, then the minimum difference between two values of $\omega_1 t$ to have the same value of $\sin(\omega_2 t + \phi_{02})$ is π. This means to say that only two values of $\sin(\omega_1 t + \phi_{01})$ are possible here and only one double exposure hologram can be obtained. In this situation, the analysis of Ω_{01} and ϕ_{01} is described in appendix A.

Another situation where the method fails is for the analysis of ϕ_{02} and Ω_{02} if ω_2/ω_1 is a positive integer. Here, each value of t to satisfy (18) will give the same value of $\sin(\omega_2 t + \phi_{02})$ and hence there is no useful information in the reconstruction. This type of situation is dealt with in appendix B.

We are not considering here the case $\omega_1 = \omega_2$ because this is effectively one component vibration as proved by Levitt and Stetson (1976) for their experimental set-up.

4. Discussion

The suggested method is capable of determining maps of the amplitude and the phase of vibration over the entire object surface just by using a small number of interferograms, as that in Levitt and Stetson method. On the other hand, the advantages of unit contrast and uniform intensity fringes are present to have a rather unlimited measurement range. We have presented detailed solutions for single and double frequency vibrations only but on similar lines, more number of modes can be dealt with, although with increased complications.

Dallas and Lohmann (1975) have suggested novel schemes for the analysis of such motions but the experimental realization is much complicated including the requirement of design and fabrication of special gratings and filters, sophisticated fourier transforming arrangements, etc. On the other hand, our method can do well with the usual arrangements for stroboscopic holography.

The method due to Takai *et al* (1976), which is applicable to single frequency vibrations only, is rather unfavourable because it further reduces the already falling intensity of J_0^2 fringes in the reconstruction.

Appendix A

Suppose, in the case of $\omega_1/\omega_2 = \frac{1}{2}$, we perform the recording such that

$$\omega_2 t = 2\,(p - 1)\,\pi + \theta, \tag{A.1}$$

where p is a positive integer. Then

$$\sin(\omega_1 t + \phi_{01}) = \pm \sin(\tfrac{1}{2}\theta + \phi_{01}), \tag{A.2}$$

depending whether p is odd or even. Thus, if we record a multiple exposure hologram by increasing the value of p by unity each time, eq. (17) leads to the normalized irradiance in the reconstruction as

$$\begin{aligned} I_1 &= \tfrac{1}{4} \,|\, \exp[i\Omega_{01} \sin(\tfrac{1}{2}\theta + \phi_{01})] + \exp[-i\Omega_{01}\sin(\tfrac{1}{2}\theta + \phi_{01})] \,|^2 \\ &= \cos^2 [\Omega_{01} \sin(\tfrac{1}{2}\theta + \phi_{01})]. \end{aligned} \tag{A.3}$$

Double exposure hologram, one with even value and the other with odd value of p in (A.1) will give the same distribution (A.3). Another such experiment with

$$\omega_2 t = 2(p-1)\pi + \theta' \tag{A.4}$$

and keeping $\theta' = \pi + \theta$ for simplicity, would give

$$I_2 = \cos^2 [\Omega_{01} \cos(\tfrac{1}{2}\theta + \phi_{01})]. \tag{A.5}$$

Fringe positions of (A.3) and (A.5) are given as

$$|\, \Omega_{01} \sin(\tfrac{1}{2}\theta + \phi_{01}) \,| = x\pi \tag{A.6}$$

and

$$|\, \Omega_{01} \cos(\tfrac{1}{2}\theta + \phi_{01}) \,| = y\pi. \tag{A.7}$$

At points of intersections, (A.6) and (A.7) give

$$|\, \tan(\tfrac{1}{2}\theta + \phi_{01}) \,| = x/y \tag{A.8}$$

and

$$\Omega_{01} = (x^2 + y^2)^{1/2}. \tag{A.9}$$

Appendix B

Let a double exposure hologram be recorded with static and $\omega_1 t = \theta$ positions, then (17) leads to

$$\begin{aligned} I_1 &= \tfrac{1}{4} \,|\, 1 + \exp[i\Omega_{01} \sin(\theta + \phi_{01}) + i\Omega_{02} \sin(q\theta + \phi_{02})] \,|^2 \\ &= \cos^2 [\tfrac{1}{2}\Omega_{01} \sin(\theta + \phi_{01}) + \tfrac{1}{2}\Omega_{02} \sin(q\theta + \phi_{02})], \end{aligned} \tag{B.1}$$

where $q = \omega_2/\omega_1$. Another such experiment with $\omega_1 t = \theta'$ such that $q\theta' = q\theta + (2p - 3/2)\pi$, p being a positive integer, would give the normalized reconstructed irradiance

$$I_2 = \cos^2 [\tfrac{1}{2}\Omega_{01} \sin(\theta' + \phi_{01}) + \tfrac{1}{2}\Omega_{02} \cos(q\theta + \phi_{02})], \tag{B.2}$$

Now suppose we observe the pattern given by (B.1) at points where the value of $\sin(\theta + \phi_{01})$ is zero. Notice that we are already familiar with ϕ_{01} and Ω_{01} distributions. Thus, if m is the fringe order (may be fractional) of the distribution (B.1) at that point, then

$$|\, \Omega_{02} \sin(q\theta + \phi_{02}) \,| = 2m\pi. \tag{B.3}$$

Therefore, at every point where $\sin(\theta + \phi_{01})$ is zero, the value of $|\, \Omega_{02} \sin(q\theta + \phi_{02}) \,|$ is determined and a contour map of the $|\, \Omega_{02} \sin(q\theta + \phi_{02}) \,|$ distribution over the object surface can be prepared.

In a similar way, by observing the pattern given by (B.2) at the points where $\sin(\theta' + \phi_{01})$ is zero, the contour map for $|\, \Omega_{02} \cos(q\theta + \phi_{02}) \,|$ distribution can be prepared over the object surface.

If at any point of the object, the values of $|\Omega_{02} \sin(q\theta + \phi_{02})|$ and $|\Omega_{02} \cos(q\theta + \phi_{02})|$ are x and y respectively, we get at that point

$$|\tan(q\theta + \phi_{02})| = x/y \tag{B.4}$$

and

$$\Omega_{02} = (x^2 + y^2)^{1/2}, \tag{B.5}$$

which help in preparing the contour maps for ϕ_{02} and Ω_{02} distributions over the object.

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