Pramāṇa, Vol. 8, No. 4, 1977, pp. 309-314. © Printed in India

Decays of the mesons and the singlet photon

JATINDER K BAJAJ and M P KHANNA
Department of Physics, Panjab University, Chandigarh 160014

MS received 30 August 1976; in revised form 16 December 1976

Abstract. In an attempt to explain the recent measurements on the radiative decays of the vector-mesons ($V \to P\gamma$), we study the consequences of introducing a small admixture of SU(3) singlet piece in the electromagnetic current. We find that this leads to an excellent fit of the theory with the new measurements on the $V \to P\gamma$ decays. However, this addition adversely affects the fit of the leptonic decays of the vector mesons ($V \to e^+ e^-$) and of the radiative decay of the pion ($\pi \to 2\gamma$). We conclude that the overall fit to the available data does not favour a large ($> 10\%$) admixture of the SU(3) singlet. The decay rates have been calculated in the vector-meson dominance model. At the hadronic vertex (VVP), we assume asymptotic nonet symmetry. The electromagnetic couplings ($V - \gamma$) are the ones appropriate to vector-mixing.

Keywords. Vector-meson dominance; radiative decays.

1. Introduction

It has long been realized that the vector meson dominance (VMD) model of Gell-Mann, Sharp and Wagner (Gell-Mann *et al* 1962) fails to explain the radiative decays of mesons if one insists on SU(3) symmetry at the hadronic (VVP) vertices (Brown *et al* 1968). Nevertheless, it was found possible, till recently, to understand these decays within the VMD scheme by introducing some sort of SU(3) symmetry breaking at the VVP vertices (Brown *et al* 1968, Chan *et al* 1969). A reasonably good explanation of these decays was also available in the quark model picture (Anisovich *et al* 1965, Becchi and Morpurgo 1965, Dar and Weisskopf 1968, van Royen and Weisskopf 1967, Soloviev 1965, Thirring 1965). In fact, it was found that the two approaches—*viz.*, the quark-model picture and the vector meson dominance model—could be put in empirical correspondence with each other (van Royen and Weisskopf 1967). However, the recently reported new data about these decays does not corroborate the earlier understanding of the problem (Bemporad 1975). New numbers are now available for the decays $\rho^- \to \pi^-\gamma$ (Gobbi *et al* 1974), $K^{*0} \to K^0\gamma$ (Carithers *et al* 1975), $\phi \to \pi\gamma, \phi \to \eta\gamma$ (Bemporad 1975), and $\eta \to 2\gamma$ (Browman *et al* 1974 *a*). The problem posed by these new measurements is typified by the situation regarding the ratio $\Gamma(\rho^- \to \pi^-\gamma)/\Gamma(\omega \to \pi\gamma)$. On the basis of both the VMD model and the quark model one expects this ratio to be around 1/9. The new experimental value is nearer 1/25—

i.e., three times smaller than theory. It is interesting to note that within the VMD scheme no SU(3) breaking helps to improve the theoretical value*, because the same hadronic couplings are involved in the two processes being compared. As such it should be instructive to have a fresh look at the other couplings in the model—namely, the vector-meson-photon ($V\gamma$) couplings. The strength of $V\gamma$ couplings depends crucially on the choice of the electromagnetic current. In this paper, we explore the consequences of adding an SU(3) singlet piece to the usual electromagnetic current. We find that this addition alters the $V\gamma$ couplings in the right direction. Keeping the singlet mixing at about 10% level, we are able to make a reasonable fit to the available data. At the hadronic vertices we have assumed asymptotic nonet symmetry (Chan *et al* 1969).

2. The couplings and decay widths

2.1. *V-γ vertex*

We assume the electromagnetic current to be of the form:

$$\mathcal{J}_\mu^{em}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}} V_\mu^8(x) + \epsilon V_\mu^0(x) \tag{1}$$

where V's are the SU(3) components of the hadronic current. Here ϵ is a parameter to be fixed from experiment. The electromagnetic current of the form (1) has earlier been suggested in the context of leptonic decays of the vector-mesons (Mathur and Okubo 1969, Wienke 1973, Khanna 1974). More recently, Khanna (1975) has conjectured that the effects of the singlet component may not be visible in the uncharmed world. However, the experimental and theoretical uncertainties are such that a small admixture of the singlet cannot be definitely ruled out†.

We define the various $V\gamma$ couplings through the equations:

$$\langle 0 \mid V_\mu^i(x) \mid v(k)\rangle = G_v \, . \, \epsilon_\mu^v(k) \, . \, (2k_0)^{-1/2} \qquad \text{for } i = 1, \ldots 8.$$

$$\langle 0 \mid V_\mu^0(x) \mid v(k)\rangle = \sigma_v \, . \, \epsilon_\mu^v(k) \, . \, (2k_0)^{-1/2}$$

and

$$\langle 0 \mid \mathcal{J}_\mu^{em}(x) \mid v(k)\rangle = f_v \, . \, \epsilon_\mu^v(k) \, . \, (2k_0)^{-1/2}. \tag{2}$$

In terms of the above definitions we write:

$$f_\rho = G_\rho$$

$$f_\omega = \frac{1}{\sqrt{3}} G_\omega + \epsilon\sigma_\omega$$

$$f_\phi = \frac{1}{\sqrt{3}} G_\phi + \epsilon\sigma_\phi \tag{3}$$

ω and ϕ as used in eq. (3) are the physical particles; and thus have non-zero vacuum to particle transition probabilities *via* the singlet current.

* This remark seems to have more general validity. For example in the recent paper by Edwards *et al* 1976, different types of symmetry breaking at the effective $V\Gamma\gamma$ vertex are tried. But the ratio $\Gamma(\rho^- \to \pi^-\gamma)/\Gamma(\omega \to \pi\gamma)$ persists around 1/9.

† For example, in Ueda 1975, the analysis is similar to that in Khanna 1976, but the electromagnetic current does show a mild singlet dependence without any serious experimental consequences.

Our next problem is to relate the various V-γ coupling constants (f_v's). The simplest method to achieve this is through the Weinberg's sum rules (*e.g.*, in Khanna 1974). Unfortunately, there is a lot of ambiguity as to the various states to be included in saturating the sum rules. In what follows, we use only asymptotic nonet symmetry, and well established current-algebra results.

Using asymptotic nonet symmetry (Chan *et al* 1969) or current mixing formalism (Kroll *et al* 1967) one derives field current identities that lead to the octet-parts of the electromagnetic couplings being related as:

$$G_\omega / G_\rho = (m_\omega / m_\rho) \sin \theta$$

$$G_\phi / G_\rho = (m_\phi / m_\rho) \cos \theta \qquad (4)$$

Here θ is ω-ϕ mixing angle.

From the commutator:

$$[V_\mu^0 (0), \quad F_5^{4+i5}] = 0$$

one gets (Khanna 1974):

$$\frac{\sigma_\omega \, g_{\omega KK}}{m_\omega^2} + \frac{\sigma_\phi \, g_{\phi KK}}{m_\phi^2} = 0 \qquad (5)$$

and from the soft-kaon approach (Wada 1966, Khanna *et al* 1967):

$$\frac{g_{\phi KK}}{g_{\omega KK}} = \frac{G_\phi}{G_\omega} \qquad (6)$$

From eqs (4), (5) and (6):

$$\frac{\sigma_\omega}{\sigma_\phi} = - \frac{m_\omega}{m_\phi} \cot \theta. \qquad (7)$$

Since σ_ω and G_ω both have the same dimensions, one can safely assume these to be linearly related. We put

$$\sigma_\omega = a G_\omega \qquad (8)$$

Then from eq. (7)

$$\sigma_\phi = - a G_\phi \tan^2 \theta. \qquad (9)$$

Using eqs (3), (4), (8) and (9), we get:

$$\frac{f_\omega}{m_\omega} = \frac{f_\rho}{m_\rho} \frac{\sin \theta}{\sqrt{3}} (1 + \sqrt{3}\, a\epsilon)$$

$$\frac{f_\phi}{m_\phi} = \frac{f_\rho}{m_\rho} \frac{\cos \theta}{\sqrt{3}} (1 - \sqrt{3}\, a\epsilon \, \tan^2 \theta). \qquad (10)$$

We remark that if we had used Weinberg's sum rules we would have still got eq. (10); but with $a = \sqrt{2}$ and θ equal to the canonical angle, 35·26°. Thus the assumption of eq. (8) is justified, *a posteriori*. We do not attempt to fix 'a', and use '$a\epsilon$' as a measure of the singlet mixing.

2.2. *VVP-vertex*

We assume asymptotic nonet symmetry at this vertex. This seems to be aesthetically simplest symmetry breaking for the VVP-couplings, and is consistent with the treatment of the V-γ coupling in the preceding sub-section. What is more, this type of breaking leads to the nearest fit with experimental information. We follow the treatment of Chan *et al* 1969 for this vertex. The various couplings are expressed in terms of two parameters—the mixing angle and an overall constant $h_0 m_0^2$. The coupling constants g_{VVP} are listed in Chan *et al* 1969 (eqs 86 of that reference). For the η-η' mixing angle γ we take the conventional value of $-10°$.

With the definitions of VVP and $V\gamma$ couplings above, the decay widths can be written as:

$$\Gamma(V \to V' P \to P\gamma) = \frac{\alpha}{24}\left(\frac{g_{VV'P} f_{V'}}{m_{V'}^2}\right)^2 \left(\frac{m_V^2 - m_P^2}{m_V}\right)^3$$

$$\Gamma(P \to VV' \to V\gamma) = \frac{\alpha}{8}\left(\frac{g_{VV'P} f_{V'}}{m_{V'}^2}\right)^2 \left(\frac{m_P^2 - m_V^2}{m_P}\right)^3$$

$$\Gamma(V \to e^+ e^-) = \frac{4\pi}{3} \alpha^2 \frac{f_V^2}{m_V^3} \tag{11}$$

3. Results and conclusion

For ideal $\omega - \phi$ mixing the ratio $\Gamma(\rho^- \to \pi^- \gamma)/\Gamma(\omega \to \pi\gamma)$ is directly proportional to the square of the ratio f_ω/f_ρ. This latter ratio depends upon '$a\epsilon$'; and from eqs (10) and (11) it is easy to observe that $\Gamma(\rho^- \to \pi^- \gamma)/\Gamma(\omega \to \pi\gamma)$ can be brought near the experimental number $\sim 1/25$ if one assumes $a\epsilon \simeq -0{\cdot}20$. For $a = \sqrt{2}$, this amounts to $\sim 15\%$ admixture of the singlet in the electromagnetic current. With this value of $a\epsilon$, the ratio $\Gamma(\phi \to \eta\gamma)/\Gamma(\omega \to \pi\gamma)$ becomes $\sim 1/15$—very close to the experimental value. Thus, the major problems posed by the new measurements on the radiative decays of vector mesons seem to be solvable if the singlet admixture is kept at around 15% level. However, there are obvious snags in this procedure. First, the ratios of the leptonic decay rates depend upon the ratios f_ω/f_ρ and f_ϕ/f_ρ. With $a\epsilon \simeq -0{\cdot}20$, $\Gamma(\omega \to e^+ e^-)$ becomes too low. Second, at this level of singlet admixture, $\Gamma(\pi \to 2\gamma)/\Gamma(\omega \to \pi\gamma)$, which also depends on f_ω/f_ρ, gets lowered. The decay width $\Gamma(\pi \to 2\gamma)$ has recently been measured again, *via* the Primakoff effect (Browman *et al* 1974*b*). Contrary to expectations the new value is not lower than the earlier experimental value of $(7{\cdot}75 \pm 0{\cdot}93) \times 10^{-3}$ keV (Chaloupka *et al* 1974).

In view of this complicated interdependence of all the numbers, we discard the possibility of a large—*i.e.*, large enough to bring the ratio $\Gamma(\rho^- \to \pi^- \gamma)/\Gamma(\omega \to \pi\gamma)$ near 1/25—singlet admixture. Instead, we take all the data available on the radiative decays and try the best possible fit. The three fits reported in table 1 are obtained by making least square fits to the first 10 numbers, properly weighted. There are four parameters to be fixed. These are: ω—ϕ mixing angle θ, the strength of the singlet admixture '$a\epsilon$', and the two parameters denoting the strengths of VVP and $V\gamma$ couplings. The solution 1 in table 1 is for '$a\epsilon$' $= 0$—*i.e.*, no singlet admixture. In solution 2, the singlet component is allowed, but the mixing angle is constrained to the ideal mixing value of $35{\cdot}26°$. This value for θ seems to be disfavoured both by the leptonic decay rates and by the non-vanishing value of $\Gamma(\phi \to \pi\gamma)$. In solution 3 all parameters are varied. We get the best fit for

Table 1. Radiative decays of mesons.*

Sl. No.	Process	1 $a\epsilon = 0$, $\theta = 39{\cdot}4°$	2 $a\epsilon = -0{\cdot}096$ $\theta = 35{\cdot}3°$	3 $a\epsilon = -0{\cdot}101$ $\theta = 39{\cdot}7°$	Experimental	Reference
1.	$\rho \to e^+ e^-$	5·32	5·93	6·06	6·43±0·81	Chaloupka *et al* 1974
2.	$\omega \to e^+ e^-$	0·70	0·46	0·55	0·76±0·17	do.
3.	$\phi \to e^+ e^-$	0·80	1·18	1·13	1·34±0·16	do.
4.	$\omega \to \pi\gamma$	712	758	744	870±80	do.
5.	$\phi \to \pi\gamma$	5·0	0·0	6·1	5·9±2·1	Bemporad 1975
6.	$\rho^- \to \pi^-\gamma$	78	54	54	35±10	Gobbi *et al* 1974
7.	$K^{*0} \to K^0\gamma$	82	101	103	75±35	Bemporad 1975
8.	$\phi \to \eta\gamma$	32	33	39	65±15	Carithers *et al* 1975
9.	$\pi \to 2\gamma$	$7{\cdot}96\times10^{-3}$	$6{\cdot}06\times10^{-3}$	$6{\cdot}23\times10^{-3}$	$(7{\cdot}75\pm0{\cdot}93)\times10^{-3}$	Chaloupka *et al* 1974 and Browman *et al* 1974 *b*
10.	$\eta \to 2\gamma$	0·604	0·644	0·635	0·352±0·135	Browman *et al* 1974 *a*
11.	$\eta' \to 2\gamma/\eta' \to \rho\gamma$	0·0482	0·0511	0·0534	0·0693±0·0120	Chaloupka *et al* 1974
12.	$K^{*+} \to K^+\gamma$	66	58	55	< 80	do.
13.	$\omega \to \eta\gamma$	4·9	4·0	3·1	< 50	do.
14.	$\phi \to \eta'\gamma$	0·12	0·15	0·15	?	..
15.	$\eta' \to \gamma\gamma$	4·7	5·3	5·4	< 19	Chaloupka *et al* 1974
16.	$\eta' \to \omega\gamma$	10	6	8	< 80	do.
17.	$\rho \to \eta\gamma$	46	49	48	?	..

* All numbers are in keV.

$\theta = 39{\cdot}7°$ and $a\epsilon \simeq -0{\cdot}10$. The other two parameters are such as to give $\Gamma(\omega \to 3\pi) \simeq 7$ MeV, which is near the experimental value of 9 MeV. This fit is distinctly better than the fit 1 with no singlet admixture. In fact, the χ^2-value for the three fits to the 10 numbers are 109, 99 and 63 respectively. The last one, admittedly, is not an excellent fit. But our contention that a small admixture of the singlet in the electromagnetic current can improve the situation is clearly borne out. Moreover, to appreciate the quality of fit 3, it must be realized that this is obtained with very simple assumptions and fares much better than other fits (*e.g.*

Edwards and Kamal 1976, Boal *et al* 1976, O'Donnell 1976, Ono 1976), in many of which much larger number of symmetry breaking parameters are used.

We conclude that a small admixture of the singlet in the electromagnetic current is favoured by the present data. However, this may not be the whole story. Some other effect seems to be definitely required in order to further lower the $\Gamma(\rho^- \to \pi^- \gamma)$ and raise $\Gamma(\phi \to \eta\gamma)$. A farfetched possibility is that the ψ-resonances that are largely believed to be vector-bosons mediate these decays like ρ^0, ω^0, ϕ^0. However, present data about ψ-particles do not justify assigning these particles a contribution of the order required. Another possibility is that contributions are arising from radially excited vector-meson resonances. We intend to explore this possibility in detail.

References

Anisovich V V *et al* 1965 *Phys. Lett.* **16** 194

Becchi C and Morpurgo G 1965 *Phys. Rev.* **B140** 687

Bemporad C 1975 *Proc. Int. Symp. Photon and Lepton Interactions at High Energies* (Stanford: California)

Boal D H, Graham R H and Moffat J W 1976 *Phys. Rev. Lett.* **36** 714

Browman A *et al* 1974 *a Phys. Rev. Lett.* **32** 1067

Browman A *et al* 1974 *b Phys. Rev. Lett.* **33** 1400

Brown L M, Munczck H and Singer P 1968 *Phys. Rev. Lett.* **21** 707

Carithers W C *et al* 1975 *Phys. Rev. Lett.* **35** 349

Chaloupka V *et al* 1974 *Phys. Lett.* **B50** 1

Chan L H, Clavelli L and Torgerson R 1969 *Phys. Rev.* **185** 1754

Dar A and Weisskopf V F 1968 *Phys. Lett.* **B26** 670

Edwards B J and Kamal A N 1976 *Phys. Rev. Lett.* **36** 241

Gell-Mann M, Sharp D and Wagner W G 1962 *Phys. Rev. Lett.* **8** 261

Gobbi B *et al* 1974 *Phys. Rev. Lett.* **33** 1450

Khanna M P and Vaidya A 1967 *Nuovo Cim.* **49** 341

Khanna M P 1974 *Lett. Al Nuovo Cim.* **9** 277

Khanna M P 1975 *Phys. Rev.* **D12** 1512

Kroll N, Lee T D and Zumino B 1967 *Phys. Rev.* **157** 1376

Mathur V S and Okubo S 1969 *Phys. Rev.* **181** 2148

O'Donnell P 1976 *Phys. Rev. Lett.* **36** 177

Ono S 1975 *Nuovo Cim. Lett.* **14** 569

Soloviev C 1965 *Phys. Lett.* **16** 345

Thirring W 1965 *Phys. Lett.* **16** 335

Ueda Y 1975 ICTP Preprint IC/75/165

van Royen R and Weisskopf V F 1967 *Nuovo Cim.* **A50** 617

Wada W W 1966 *Phys. Rev. Lett.* **16** 956

Wienke B R 1973 *Phys. Rev.* **D7** 2253