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Gravity induced magnetic instability

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Abstract. In the presence of a gravitational field the stability of a magnetoplasma is studied against electromagnetic perturbations. We have shown that a pinching type of instability can be triggered with a sizable growth rate affecting the equilibrium configuration of the confining magnetic field. This might have a profound effect on the magnetic fields of astrophysical bodies.

Keywords. Magnetoplasmas; EM instabilities; gravitational field.

1. Introduction

It is well known that a gravitational field can give rise to an electrostatic instability, the so called Rayleigh-Taylor instability in a magneto-plasma in presence of density inhomogeneities. In laboratory plasmas, the time scales of interest in general are very small and electromagnetic instabilities play a secondary role because of their smaller growth rates. In astrophysical problems the time scales are indeed very large and magnetic instabilities do play a significant role. We have shown here that even in the absence of any inhomogeneity of the density, a gravitation field can give rise to a magnetic instability in a magneto-plasma which eventually may decide the magnetic field configuration in the plasma surrounding the gravitational body.

2. Basic equations and the solution

We consider a homogeneous plasma with an uniform magnetic field $\boldsymbol{B}$ in the z direction. The gravitation field is taken perpendicular to $\boldsymbol{B}$, and along x direction. The basic equations are the Vlasov-Maxwell set given by

$$\frac{\partial f}{\partial t} + \boldsymbol{V}.\nabla f + \frac{q_j}{m_j}\left(\frac{m_j}{q_j} g + \frac{\boldsymbol{V}\times\boldsymbol{B}}{c}\right).\nabla_V f = 0 \tag{1}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}\,;\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{J} = \sum_j n_j\, q_j \int f\, \boldsymbol{V}\, d\boldsymbol{V} \tag{2}$$

where the symbols have their usual meanings with j denoting the particle species. In equilibrium we have

$$E_0 = 0 \ ; \ B_0 = B_0 \hat{e}_z \ ; \ g = g\hat{e}_x. \tag{3}$$

The presence of the crossed gravitational field and magnetic field give rise to particle drifts which for the j-th species is,

$$V_{gj} = (g/\Omega_j)\ \hat{e}_y \tag{4}$$

where $\Omega_j = eB_0/m_j c$.

The equilibrium distribution function for the j-th species is hence given by

$$f_{0j} = (a_{j/\pi})^{3/2} \exp\left[- a_j (V_x^2 + V_z^2) - a_j (V_y - V_{gj})^2\right] \tag{5}$$

where $a_j = m_j / 2kT_j$; T_j is the temperature of the j-th species.

Linearising eq. (1) and integrating over the unperturbed orbits, (Krall 1973) $V_x' = V_\perp \cos(\phi - \Omega_j \tau)$, $V_y' = V_\perp \sin(\phi - \Omega_j \tau) + V_{gj}$, $V_z' = V_z$ and $Z' = V_z \tau + z$ along with eq. (5) one gets the perturbed distribution function $\tilde{f}_{jk}$ as

$$\tilde{f}_{jk} = F_{1j}(V)\,\tilde{E}_{1x} + F_{2j}(V)\,\tilde{E}_{1y} + F_{3j}(V)\,\tilde{E}_{1z} \tag{6}$$

where

$$F_{1j}(V) = \frac{a_j q_j f_{0j}}{i m_j}\, V_\perp \left(\frac{e^{i\phi}}{\gamma_-} + \frac{e^{-i\phi}}{\gamma_+}\right)$$

$$F_{2j}(V) = \frac{a_j q_j f_{0j}}{m_j}\left[V_\perp\left(\frac{e^{-i\phi}}{\gamma_+} - \frac{e^{i\phi}}{\gamma_-}\right) - \frac{2kV_z V_g}{i\omega(\omega - kV_z)}\right] \tag{7}$$

$$F_{3j}(V) = 2a_j\, q_j f_{0j} / i m_j\,(kv_z - \omega)$$

where $\gamma_- = kV_z - \omega - \Omega_j$; $\gamma_+ = kV_z - \omega + \Omega_j$

$(\tilde{E}_{1x}, \tilde{E}_{1y}, \tilde{E}_{1z})$ is the perturbed electric field where $\tilde{E} = \tilde{E}_k \exp(i\bar{k}\cdot\bar{r} - i\omega t)$. Now we consider a mode with $\omega^2 \ll \Omega_i^2$ and under this approximation the right polarized wave uncouple from the left polarized one. Eliminating $\tilde{E}_{1y}$ from (6) and (2) we finally arrive at the dispersion relation in the form

$$\omega^4 \left(1 + \sum_j \omega_{pj}^2/\Omega_j^2\right) - k^2 c^2 \omega^2 - \Sigma\, \omega_{pj}^2\ k^2 V_{gj}^2 = 0 \tag{8}$$

Since $V_{gj}^2 = g^2/\Omega_j^2$, in the term $(k^2 \sum_1 \omega_{pj}^2 V_{gj}^2)$ the ion term is larger than the electron term by a ratio m_i/m_e and therefore we can neglect the electron contribution from this term and retain only the ion term $k^2\, \omega_{pi}^2\, V_{gi}^2$. In deriving eq. (8) we have made the approximation $\Omega_i/k \gg |\,\omega/k\,| \gg V_{thi}$ (*i.e.*, phase velocity of the wave much higher than the ion thermal velocity along $\boldsymbol{B}$ and also 'ω' smaller than the ion gyrofrequency Ω_i. This approximation also corresponds to the case $kR_i \ll 1$ where ($R_i = V_{thi}/\Omega_i$, is the ion gyroradius). The roots of eq. (8) are

$$\omega^2 = \frac{k^2 V_A^2}{2(1 + V_A^2/C^2)}\left\{1 \pm \left[1 + \frac{4\omega_{pi}^2}{k^2 C^2}\frac{V_g^2}{V_A^2}(1 + V_A^2/C^2)\right]^{\frac{1}{2}}\right\}. \tag{9}$$

The positive sign corresponds to the familiar Alfven mode with

$$\omega^2 = k^2 V_A^2/(1 + V_A^2/C^2)$$

where $V_A^2 = B_0^2/4\pi nm_i$, is the Alfven velocity. The negative sign corresponds to an absolutely growing mode ($\omega_r = 0$, ω_r being the real part of the frequency) and the imaginary part given by

$$\omega_I^2 \simeq -\omega_A^2 (\omega_{pi}^2 V_g^2/k^2 V_A^2 C^2) = -\omega_{pi}^2 (V_g^2/C^2) \quad (10)$$

where $\omega_A^2 = k^2 V_A^2$ with $V_g^2/C^2 \ll 1$ and $V_A^2/C^2 \ll 1$.

We therefore note that our initial assumption of $|\omega/k| \gg V_{thi}$ can always be satisfied for wavelengths of the order of [from eq. (10)]

$$\left|\frac{\omega}{k}\right| = \left\{\frac{\omega_{pi}^2 (V_g^2/C^2)}{k^2}\right\}^{\frac{1}{2}} > V_{thi}, \ i.e. \ \lambda > (C/V_g)\lambda_D \quad (11)$$

where λ_D is the Debye length with $T_e \simeq T_i$. The instability therefore is a long wavelength instability since from eq. (11) $k\lambda_D \simeq Vg/C \ll 1$.

The growth rate will be given by [from eq. (11)]

$$\gamma = \left(\frac{\omega_{pi}}{c} \cdot g/\Omega_i\right). \quad (12)$$

3. Conclusion

We have shown that a new instability is triggered with a sizable growth rate in the presence of a crossed gravitational field and a magnetic field. It is essentially a pinching type of instability which bends the confining magnetic field. This increase in magnetic energy is at the expense of the gravitational energy. This instability will try to align the confining magnetic field along the direction of the gravitational acceleration. A similar type of instability is invoked for the reconnection process of the neutral sheet in our magnetosphere. Even though the energy source for this instability is not a gravitational field, the basic mechanism is exactly similar (Sinha and Sundaram 1976). However, a nonlinear analysis is required to find the degree of distortion of the magnetic field due to the presence of the gravitational field.

References

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