Pramāṇa, Vol. 8, No. 3, 1977, pp. 205-213. © Printed in India.

Velocity gradient driven flute instabilities in plasmas

M SINHA and A K SUNDARAM
Physical Research Laboratory, Ahmedabad 380009

MS received 9 August 1976; in revised form 27 November 1976

Abstract. The effect of velocity gradient across the magnetic field on the low frequency flute modes is examined in detail, using the normal mode analysis. It is shown that some new type of instabilities driven primarily by the velocity gradient arise and these excited modes eventually attain the convective saturation. The onset of plasma turbulence due to these instabilities may possibly be one of the major contributors for anomalous heating process and enhanced plasma resistivity.

Keywords. Velocity gradients; electrostatic instabilities; plasma density gradient.

1. Introduction

Spatially nonuniform particle drifts in many laboratory studies [for instance, in pulsed plasma heating (Wharton *et al* 1971) and rotating plasma devices (Lehnert 1971] are fairly well known. These drifts, arising basically due to such factors as inhomogeneous electric and magnetic fields or nonuniformity in the electron beam, etc., provide additional sources of energy for the onset of new instabilities or for the suppression of unstable modes excited otherwise in the system. The recent work of Hirose and Alexeff (1973) is an illustrative example in which a variety of high frequency ($\omega > \Omega_i$) electrostatic modes are excited in the presence of sheared velocity along and across the magnetic field. Although these high frequency instabilities are directly responsible for the enhancement of effective electron-electron (and electron-ion) collision frequency and the anomalous skin effect, they do not contribute to the dc anomalous resistivity as the ions are unaffected by the instabilities. On the other hand, the low frequency ($\omega < \Omega_i$) plasma instabilities do play an important role in the effective heating of ions. The present work will therefore be devoted to studying this later aspect of the problem in detail. Using the fluid equations, we investigate the excitation of long wavelength flute modes by the velocity gradient across the magnetic field in both convective and non-convective regimes.

Interfacial plasma instabilities (of Kelvin-Helmholtz type) with shear velocity parallel and perpendicular to the magnetic field have been thoroughly investigated by earlier workers (D'Angelo 1965; Jassby and Perkins 1970; Jassby 1972 and others). The investigation closet to our theory is that of Jassby and Perkins

(1970) who study numerically the growth of transverse Kelvin-Helmholtz mode (induced due to velocity shear) in a Q-machine plasma. At this step, it is pertinent to emphasize the important difference between Jassby's and our approach. Our work valid for a general situation involving electric and magnetic field gradients differs from that of Jassby in the basic excitation mechanism. It may be pointed out that Jassby's main physical process is the centrifugal effect which causes differential motion between electrons and ions leading to the charge separation and the consequent instability. In a slab geometry, however, the excitation mechanism manifests in terms of varying drift velocity (due to non-uniform electric field or other inhomogeneities) giving rise to spatially separated streams in the plasma, possibly responsible for driving the system unstable Further, in our work, exact analytical expressions for the growth rates and the normal mode structure for different eigen values are obtained. We find that the results derived here are governed by a dimensionless parameter, β (a measure of velocity gradient, defined in the text) and the growth rates for the low frequency modes vary directly as a function of β.

For the remaining part of our text, the plan is as follows. In the next section, the basic equations leading to the eigen value problem are derived. The nature of low frequency modes and their growth rates are discussed in sections (3) and (4) for local and convective regimes. Finally, the application of the results obtained in this work are briefly discussed in the last section.

2. Normal mode analysis

We consider an inhomogeneous plasma embedded in a nonuniform magnetic field, B_0, along z-axis of a slab geometry. The plasma density and the magnetic field vary along x-direction. We assume a shear drift velocity, $V_0(x)$ for the charged particles directed along y-axis. Since we deal with a general situation, the functional dependence and the nature of particle drift velocity will be kept arbitrary at this stage. We shall now study the stability characteristics of the above equilibrium configuration, using the fluid equations for both electrons and ions. Finite temperature effects (and the stabilizing influence of Larmor radius corrections) will be ignored throughout our analysis for mathematical simplicity. Seeking an electrostatic perturbation of the flute type $\sim \tilde{q}(x) \exp i(ky - \omega t)$ where $\tilde{q}$ represents a typical perturbed quantity, the particle drifts in the presence of small perturbed electric fields are given by

$$\tilde{V}_x = \frac{ie}{m}\left(\omega_0 \frac{d\phi}{dx} - k\phi\Omega\right) \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right] \tag{1}$$

$$\tilde{V}_y = \frac{e}{m}\left[\left(\Omega + \frac{dV_0}{dx}\right)\frac{d\phi}{dx} - k\omega_0\phi\right] \Big/ \left[\Omega\left(\Omega + \frac{dV_0}{dx}\right) - \omega_0^2\right] \tag{2}$$

where ϕ is the electrostatic potential, Ω, the cyclotron frequency and $\omega_0 (= \omega - kV_0)$ is the Doppler shifted frequency. The other symbols have their usual meanings. If n_0 denotes the equilibrium guiding-centre density and δn its perturbed value, then conservation of guiding-centre mass demands that

$$i\omega_0 \frac{\delta n}{n_0} = \kappa_n \tilde{V}_x + ik\tilde{V}_y + \frac{d\tilde{V}_x}{dx} \quad (3)$$

where K_n stands for $(d/dx) \ln n_0$. Substituting the values for $\tilde{V}_x$ and $\tilde{V}_y$ [eqs (1) and (2)] and using eq. (3) together with the Poisson's equation, $\nabla^2\phi = 4\pi e (\delta n_e - \delta_{n_i})$ we finally derive the differential equation for ϕ in the form

$$A \frac{d^2\phi}{dx^2} + B \frac{d\phi}{dx} + C\phi = 0 \quad (4)$$

where A, B and C are functions of x and other physical parameters of the problem and are given by the expressions

$$A = 1 + \sum \omega_p^2 \Big/ \left[\Omega \left(\Omega + \frac{dV_0}{dx} \right) - \omega_0^2 \right] \quad (5)$$

$$B = \sum \omega_p^2 \left[\kappa_n \left\{ \Omega \left(\Omega + \frac{dV_0}{dx} \right) - \omega_0^2 \right\} - 2k\omega_0 \frac{dV_0}{dx} \right.$$
$$\left. - \left(\Omega \frac{d^2 V_0}{dx} + 2\Omega \frac{d\Omega}{dx} + \frac{dV_0}{dx}\frac{d\Omega}{dx} \right) \right] \Big/ \left[\Omega \left(\Omega + \frac{dV_0}{dx} \right) - \omega_0^2 \right]^2 \quad (6)$$

$$C = -k^2 + \sum k\omega_p^2 \left[\kappa_n \Omega \left\{ \omega_0^2 - \Omega \left(\Omega + \frac{dV_0}{dx} \right) \right\} + (\omega_0^2 + \Omega^2) \frac{d\Omega}{dx} \right.$$
$$\left. + \Omega^2 \frac{d^2 V_0}{dx^2} + k\omega_0 \left\{ \omega_0^2 - \Omega \left(\Omega - \frac{dV_0}{dx} \right) \right\} \right] \Big/ \omega_0 \left\{ \Omega \left(\Omega + \frac{dV_0}{dx} \right) \right.$$
$$\left. - \omega_0^2 \right\}^2 \quad (7)$$

where the summation sign extends over the species and ω_p is the plasma frequency. We note from eqs (5) to (7) that the effect of velocity shear becomes important only if dV_0/dx is comparable to cyclotron frequency. Since in actual experiments, it is difficult to realize dV_0/dx of the same order as Ω_e, hence the discussion in the forthcoming sections will be confined to $dV_0/dx \lessgtr \Omega_i$. Further, the application of guiding-centre approximation limits our consideration only to low frequency waves with long wavelengths (greater than ion larmor radius) but still shorter than the plasma dimension, (κ_n^{-1}). Thus in the subsequent sections, the dispersive properties of eq. (4) will be discussed in detail for local and nonlocal regimes.

3. Local dispersion relation

To begin with, we shall derive the dispersion relation valid under 'local' approximation (Mikhailovskii 1973). For a typical density perturbation, we shall assume that the wavelengths, λ_y ($\sim 2\pi/k$) is small compared to the characteristic distance over which the equilibrium quantities such as density, velocity and magnetic field vary. Further we shall assume $\lambda_x \gg \lambda_y$, where λ_x is the wavelength along x-axis. Such situations can be realized in magnetospheric plasma sheet regions where the wavelengths along north-south direction could be large compared to the wavelengths along the dawn-dusk direction. Thus neglecting the terms involving the derivatives of ϕ and the electron velocity gradient (compared to Ω_e) in eq. (4), we get the local dispersion relation

$$\frac{-\kappa_n\,\alpha+\kappa_B}{\alpha^2\,\Omega_{oi}}+\frac{k\,\omega_{oi}^2}{\Omega_i^3\,\alpha^2}+\frac{\kappa_n-\kappa_B}{\omega_{oe}}-\frac{k}{\Omega_e}-\frac{kV_A{}^2}{\Omega_i c^2}=0, \tag{8}$$

where $\alpha = 1+\beta$ and $\beta = (dV_0/dx)/\Omega_i$. The quantities κ_B, V_A ($\ll c$) and c stand respectively for inverse scale length $(d\Omega/dx)/\Omega$, Alfven velocity and the light speed. The subscripts i and e identify the ion and electron species. In deriving the dispersion relation (8) it is assumed that (dV_0/dx) is constant (linear velocity profile) and $\beta \lesssim 1$. Equation (8) being a quartic in ω, can be solved in general for the roots using standard numerical methods. However we shall examine a particular case when $V_{oi} = V_{oe} = V_0$. The particle drift, V_0, in this instance can be visualized as a consequence of $\boldsymbol{E} \times \boldsymbol{B}$ motion, $\boldsymbol{E}$ and $\boldsymbol{B}$ being the externally applied nonuniform fields in the system. Furthermore, assuming $1 \gg (\omega_0/\Omega_i)^2 \gg (m/M,\ V_A{}^2/C^2)$ where m and M are the electron and ion masses, eq. (8) reduces to

$$(\omega_0/\Omega_i)^3 = -\beta\,\{\kappa_n\alpha-(\alpha+1)\,\kappa_B\}/k, \tag{9}$$

which clearly admits complex roots as solutions leading to either growing or decaying perturbations. Thus we find that the low frequency instability driven by the velocity gradient (either for positive or negative slope) arises in this limit and clearly the magnetic field shear contributes to the stabilization of this instability, provided its gradient keeps an unfavourable sign with the density gradient. Also we note that the growth rate varies directly as $\beta^{1/3}$ and inversely as $(kL_n)^{1/3}$, where $kL_n \gg 1$, being the scale length of the density gradient ($= 1/\kappa_n$). Lastly it must be remarked that the above instability does not arise in the limit $\beta \ll 1$. For this case, it may easily be verified that the second term [which originates from the expression for $\boldsymbol{C}$ in eq. (7)] in eq. (8) drops out and consequently the relation (8) becomes a quadratic in ω. This situation will be considered next.

For the limit when $(\omega_0/\Omega_i)^2 \ll m/M$ (or $V_A{}^2/c^2$), eq. (8) can be revised in the form

$$\frac{K_B-K_n\alpha}{\alpha^2\,\omega_{oi}}+\frac{K_n-K_B}{\omega_{oe}}-\frac{k}{\Omega_i}\left(2-\alpha+\alpha^2\frac{m}{M}+\frac{V_A{}^2}{c^2}\,\alpha^2\right)=0. \tag{10}$$

For $V_{oi} = V_{oe}$, we note that there exists no instability while for $V_{oi} \neq V_{oe}$ eq. (10) can be solved to give the roots

$$\omega = k\,(V_{oe}+V_{oe})+\Omega_i(p+q)\pm\{k^2\,(V_{oe}-V_{oe})^2 + \Omega_i^2\,(p+q)^2+2k\Omega_i\,(V_{oi}-V_{oe})\,(p-q)\}^{1/2}, \tag{11}$$

where the quantities p and q are given by the expressions

$$p = (\kappa_B-\kappa_n\alpha)\Big/\left[k\left\{2-\alpha+\alpha^2\left(\frac{m}{M}+\frac{V_A{}^2}{c^2}\right)\right\}\right]$$

$$q = (\kappa_n - \kappa_B)\, a^2 \Big/ \left[k \left\{ 2 - a + a^2 \left(\frac{m}{M} + \frac{V_A^2}{c^2} \right) \right\} \right].$$

Noting that $|p+q| < |p-q|$ for $\beta < 1$ and assuming that $V_{oi} > V_{oe}$ we find there exists a low frequency instability, in the absence of magnetic field shear provided $\kappa_n > 0$. In the opposite limit ($\kappa_n < 0$) the low frequency mode becomes stable. In the former case, the ranges of wavelengths for which the low frequency unstable modes occur are defined by

$$p - q > k(V_{oi} - V_{oe})/\Omega_i > p + q. \tag{12}$$

The physical mechanism responsible for this instability does not depend on the velocity gradient. The instability arises due to relative streaming between electrons and ions in an inhomogeneous plasma. As such this instability will be the analogue of Kelvin-Helmholtz instability for transverse streaming. Similarly the same conclusion will hold good for the case when $\beta \ll 1$.

4. Nonlocal effects

We shall now investigate the convective effects on the low frequency unstable modes discussed in the previous section.

At this juncture, it must be remarked that the results derived in eqs (9) and (11) clearly belong to two distinct classes. Particularly we note that a finite velocity gradient is quite essential for the existence of plasma instability defined by eq. (9). Since we have assumed a linear variation for the drift velocity, dV_0/dx is a non-zero constant and the Doppler-shifted frequency, ω_0, should not become zero (that is, ω is complex). However for marginally stable state, taking the limit, $dV_0/dx \to 0$ and $\omega_0 \to 0$, we find that eq. (7) reduces to a finite value $-k^2(1 + \Sigma\, \omega_p^2/\Omega^2)$ which is free from any singularity that can arise due to vanishins ω_0. Thus the solutions of differential eq. (4) are regular and well-behaved functiong at infinity. However, practical situations do arise wherein the drift velocity profiles depart considerably from the linearity. In such cases, if ω is real, ω_0 can vanish as the velocity shear will cause to happen. Equation (7) in this case reduces to $-k^2(1 + \Sigma\, \omega_p^2/\Omega^2) + \Sigma\, \omega_p^2 (d^2 V_0/dx^2)/\Omega^2 \omega_0$ in the limit when $\omega \to kV_0$ and $dV_0/dx \to 0$ (at the points where the velocity attains stationary values). Since the second term for $d^2 V_0/dx^2 \neq 0$ becomes singular, the well-behaved solutions for eq. (4) can be computed using the standard methods outlined in the works of Chandrasekhar (1961) and Drazin and Howard (1962). Since the main purpose of our analysis is aimed at highlighting the effect of finite velocity gradient, further discussion of complicated velocity profiles falls outside the scope of this text. Similarly for the unstable mode of Kelvin-Helmholtz type where the relative streaming between electrons and ions (however small it may be) becomes important, the velocity shear will cause the vanishing of either $\omega - kV_{oe}$ or $\omega - kV_{oi}$, for real ω. In either of these cases, bounded solutions can be obtained by a suitable transformation of independent variable in eq. (4) (Chandrasekhar 1961). Since there are well entrenched theories for Kelvin-Helmholtz instability problems, we have not attempted these aspects in our paper

In the next section the nonlocal effects of the velocity gradient driven instability will be discussed. In an inhomogeneous plasma, wave propagation develops in the direction of varying density and as a result of this property, the dispersion relation is significantly modified. Therefore, in order to account this effect properly, we must solve eq. (4) for its solution with the appropriate boundary conditions. As considered earlier, we shall treat dV_0/dx as constant and β to be less than unity. We shall assume that the scale length of the magnetic field inhomogeneity is much longer than L_n (the density scale length). With these approximations, the quantities, A, B and C defined by eqs (5) to (7), simplify to

$$A \simeq \frac{1}{\alpha}\frac{\omega_{pi}^2}{\Omega_i^2}, \qquad B \simeq \frac{\omega_{pi}^2}{\alpha\Omega_i^2}\left(\kappa_n - \frac{2k\omega_{oi}}{\alpha\Omega_i^2}\frac{dV_0}{dx}\right),$$

$$C \simeq -k^2 + k\frac{\omega_{pi}^2}{\Omega_i}\left[\kappa_n\left(\frac{1}{\omega_{oe}} - \frac{1}{\alpha\omega_{oi}}\right) + \frac{k}{\Omega_i\alpha^2}(\beta - 1 + \omega_{oi}^2/\Omega_i^2)\right]. \quad (13)$$

Making a transformation, $\phi = \psi \exp[- \int (B/2A)\, dx]$, eq. (4) can be reduced to the normal form

$$\frac{d^2\psi}{dx^2} + Q^2(x)\psi = 0 \quad (14)$$

where

$$Q^2 = (C/A) - (B^2/4A^2) - \frac{d}{dx}\left(\frac{B}{2A}\right).$$

It is clear that the solutions for ψ will depend on the nature and behaviour of function, Q. For our purposes, to illustrate the principal effect of the velocity gradient, we shall choose the density and velocity profiles in the form

$$n_0 = \bar{n}_0 \exp(-x/L_n), \quad V_0 = \bar{V}_0(1 + x/L_V), \quad (15)$$

where $\bar{n}_0$ and $\bar{V}_0$ are constant, L_V is the scale length of the velocity gradient and $(x/L_V) < 1$. With the above choice for the equilibrium parameters, after making suitable transformations the differential eq. (14) becomes a parabolic equation of Weber's type

$$\frac{d^2\psi}{dx'^2} + [(a^2\alpha/k^2\beta^2) - x'^2]\psi = 0, \quad (14\,a)$$

where β can now be redefined as $\bar{V}_{oi}/L_V\,\Omega_i$ and the other quantities such as x' and a are given by

$$x' = \frac{k\beta}{\sqrt{\alpha}}\left[x - \frac{\alpha}{(k\beta)^2}\left\{\frac{1}{2L_n} + \frac{k}{\alpha}\beta\frac{\bar{\omega}_{oi}}{\Omega_i} + \frac{\alpha}{2L_n}\frac{\Omega_i^2}{\beta\bar{V}_{oi}} \cdot \left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2} - \frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)\right\}\right], \quad \bar{\omega}_0 = \omega - k\bar{V}_0, \quad (16)$$

$$a^2 = \frac{k^2}{\alpha^2}\left(\beta - 1 + \frac{\bar{\omega}_{oi}^2}{\Omega_i^2}\right) - \frac{k\Omega_i}{L_n}\left(\frac{1}{\bar{\omega}_{oe}} - \frac{1}{\alpha\bar{\omega}_{oi}}\right) - \frac{k^2\beta^2}{\alpha}$$

$$+\frac{a^2}{4L_n^2\beta^2}\frac{\Omega_i^4}{\bar{V}_{oi}^2}\left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2}-\frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)^2+\frac{\alpha}{L_n}\frac{\Omega_i^2}{\beta\bar{V}_{oi}}$$

$$\cdot\left(\frac{\bar{V}_{oi}}{\alpha\bar{\omega}_{oi}^2}-\frac{\bar{V}_{oe}}{\bar{\omega}_{oe}^2}\right)\left(\frac{1}{2L_n}+\frac{k}{\alpha}\beta\bar{\omega}_{oi}/\Omega_i\right). \tag{17}$$

From eq. (14 *a*), the solution for the potential ϕ can be in terms of the Weber function, $D_n(x')$ where n is any eigen value. The condition that eq. (14 *a*) represents the Weber's equation identically demands that

$$a^2=\frac{2n+1}{\alpha}k^2\beta^2. \tag{18}$$

It may be remarked that the relation (18) can also be derived directly from equation (14 *a*) by using the quantization condition in WBK method. Hence eq. (18) defines the dispersion relation which takes into account the convective effects arising due to the changing density. We find that eq. (18) is a polynomial in ω with real coefficients and in general, the roots (real or complex) can be evaluated numerically using the standard methods. Such a procedure helps possibly in delineating the stable and unstable regions for different values of β and kL_n. However this procedure will not be adopted here. Instead, we shall discuss some special cases in which the dispersion relation (18) yields simple analytical solutions revealing the essential characteristic role of the velocity gradient. In the first instance, we shall examine the case when $\bar{V}_{oi}=\bar{V}_{oe}=\bar{V}_0$. With this condition, eq. (18) becomes a sixth degree polynomial in δ, namely,

$$\delta^6-2\alpha\beta\delta^3/kL_n-\alpha^2\delta^2/2k^2L_n^2+\alpha^2/4k^2L_n^2$$
$$=2(n+1)\alpha\beta^2\delta^4, \tag{18 a}$$

where $\delta=\bar{\omega}_0/\Omega_i$. Making use of the fact that $\delta\ll 1$, $\beta\lesssim 1$ and $kL_n\gg 1$, this equation can be reduced to a quartic approximately by dropping the terms involving δ^6 and δ^2 (being small compared to the terms retained) in eq. (18 *a*). Further, writing $\delta=\delta_R+i\delta_I$. where δ_R and δ_I, are the real and imaginary parts of δ and separately equating to zero the real and imaginary contributions of eq. (18 *a*) we get the coupled equation for δ_R and δ_I in the form

$$\delta_R(\delta_R^2-3\delta_I^2)\left[\delta_R+\frac{1}{(n+1)\beta kL_n}\right]-\delta_I^2(3\delta_R^2-\delta_I^2)$$
$$-\alpha/[8(n+1)(\beta kL_n)^2]=0, \tag{19}$$

$$\delta_I\left[4\delta_R(\delta_R^2-\delta_I^2)+\frac{1}{(n+1)\beta kL_n}(3\delta_R^2-\delta_I^2)\right]=0. \tag{20}$$

For $\delta_I\neq 0$ and $\delta_I\gg\delta_R$, eqs (19) and (20) can be solved exactly for δ_R and δ_I which can be expressed as

$$\delta_R=-1/[4(n+1)\beta kL_n],$$

$$\delta_I=\pm\frac{3}{4\sqrt{2}}\left[-1+\left\{1+\frac{128}{81}\alpha(n+1)^3\beta^2k^2L_n^2\right\}^{1/2}\right]^{\frac{1}{2}}$$
$$\cdot[(n+1)\beta kL_n]^{-1}, \tag{21}$$

where the positive sign before the radical within the square parenthesis is chosen so that δ_I remains a real quantity. In eq. (21), the growth rate expression can be further approximated to give $\delta_I \sim (n+1)^{-1/4} (\beta k L_n)^{-1/2}$ ($\delta_I < 1$ for $\beta k L_n \gg 1$). It may be mentioned here that the excitation of low frequency mode with the same growth rate could occur even for smaller β (unlike the nonconvective results) provided $\beta k L_n > 1$ and n is large (such that $\delta_I < 1$ is satisfied consistently). Hence for the convective case we find that there exists a growing mode driven purely by the velocity gradient. In contrast to the results derived in the earlier section, we observe that the growth rate for the nonlocal case varies inversely as $(\beta k L_n)^{1/2}$. Thus it enables us to conclude that the convective saturation of the low frequency modes occurs with a lesser growth rate than its counterpart in the nonconvective case.

Finally we shall examine the situation in which the ions drift with a velocity $\bar{V}_{oi}$, the electrons being stationary ($\bar{V}_{oe} = 0$). Dropping the bar over the quantities the dispersion relation (18), for small β, takes the form

$$\frac{1}{(2kL_n\beta)^2}\left(\frac{\Omega_i}{\omega_{oi}}\right)^4 + \frac{1}{kL_n}\left(\frac{\Omega_i}{\omega_{oi}}\right) + \frac{\Omega_i}{kL_n}\frac{kV_0}{\omega_0\omega_{oi}} = \beta_1, \tag{22}$$

where $\beta_1 = 1 + 2(n+1)\beta^2$. Instead of discussing the general solution of eq. (22), we shall deal with the particular case when ω is close to kV_0. Therefore, writing $\omega = kV_0 + \triangle$, where $|\triangle| \ll kV_0$, eq. (22) transforms into a quartic in $\triangle$ given by

$$\left(\frac{\triangle}{\Omega_i}\right)^3\left(\frac{\triangle}{\Omega_i} - \frac{2}{\beta_1 kL_n}\right) - \frac{1}{4\beta_1 (kL_n\beta)^2} = 0. \tag{23}$$

Following the same procedure as outlined before, the real and imaginary parts of $\triangle$ ($= \triangle_R + i\triangle_I$, where $\triangle_R \ll \triangle_I$) can be written in the form

$$\triangle_R = \Omega_i/2\beta_1 kL_n$$

$$\triangle_I = \pm\frac{3\Omega_i}{kL_n}\left[-1 + \left\{1 + \frac{16}{81}\beta_1^3 k^2 L_n^2/\beta^2\right\}^{1/2}\right]^{1/2} \Big/ 2\sqrt{2}\,\beta_1. \tag{24}$$

The expression for $\triangle_I$ can be simplified further to yield a value proportional to $1/(\beta k L_n)^{1/2}$ and this result is similar to the one discussed previously. It follows therefore that in either of the cases described here, the low frequency instabilities with growth rates given by eqs (21) or (24) can arise due to the nonlocal effects and small velocity gradient ($\beta < 1$).

5. Results and application

We have investigated in general the effect of velocity shear on the low frequency waves in a two-component plasma. We find that some new unstable modes driven primarily by the velocity gradient occur near $\omega_R = kV_0$ for both convective and nonconvective regimes. The growth rates of these modes are larger in the nonconvective limit and they attain the convective saturation with a value proportional to $\Omega_i/(\beta k L_n)^{1/2}$. Of course it must be mentioned that the velocity gradient driven instability in the nonconvective limit triggered only for $\beta \lesssim 1$ and range of frequencies such that $1 \gg (\omega_0/\Omega_i)^2 \gg (m/M, V_A^2/c^2)$. On the other hand, for a frequency range, $(\omega_0/\Omega_i)^2 \ll m/M$ or V_A^2/c^2 transverse Kelvin-Helmholtz mode is excited and the effect of velocity shear ($\beta \lesssim 1$) causes a slight modification of its mode structure leading essentially to the localization of Kelvin-Helmholtz mode. This latter result is in conformity with an earlier investigation by Rosenbluth and Simon (1965) who examine the influence of nonuniform electric fields on Kelvin-Helmholtz and Rayleigh-Taylor modes employing FLR time ordering (that is, $\omega/\Omega_i \ll (\rho_i/L)^2 \ll 1$ where ρ_i is the ion larmor radius and L is the typical characteristic length). Thus we conclude that the velocity gradient driven mode becomes operative only for the frequency range satisfying the inequality

$$1 \gg (\omega_0/\Omega_i)^2 \gg (m/M, \quad V_A^2/c^2).$$

Perhaps the results of our analysis will be more important to the magnetospheric plasma sheet regions (where the condition $\beta \lesssim 1$ is easily met) during the pre- and post-growth periods of the magnetic substorms. In such conditions, it is conceivable that strong velocity gradients could co-exist in the plasma sheet because of the sudden variations in the magnetic field components. Some of the observed features through satellite studies (Akasofu *et al* 1973) seem to indicate (i) the presence of highly energetic protons and electrons and (ii) thinning and thickening of the plasma sheet during and after the substorm periods. In fact, a recent theoretical study (Kan 1973) directly correlates the presence of velocity shear to the plasma sheet thickness (showing that increasing velocity shear thickens the sheet). All these features seem to support our theory to the extent that the velocity gradient driven instabilities may offer as a potential candidate for the proton heating and the anomalous crossfield conductivity (for enhanced rate of particle scattering does affect the conductivity across the field). Of course, in the neutral sheet region, the guiding-centre approach is invalid since adiabatic approximations do not hold good. However, our results do give the qualitative account of a possible mechanism for plasma heating. A detailed study incorporating the kinetic effects to reveal the features for shorter wavelengths (comparable to larmor radius of ions) will be the topic for future work.

References

Akasofu S I, Perrault P D, Yashuhara F and Meng C I 1973 *J. Geophys. Res.* **78** 7490
Chandrasekhar S 1961 *Hydrodynamic and Hydromagnetic Stability* (Oxford; Clarendon Press)
D'Angelo N 1965 *Phys. Fluids* **8** 1748
Drazin P G and Howard L N 1962 *J. Fluid Mech.* **14** 257
Hirose A and Alexeff I 1973 *Phys. Fluids* **16** 1087
Jassby D L and Perkins F W 1970 *Phys. Rev. Lett.* **24** 256
Jassby D L 1972 *Phys. Fluids* **15** 1590
Kan J R 1973 *J. Geophys. Res.* **78** 3773
Lehnert B 1971 *Nucl. Fusion* **11** 485
Mikhailovskii A B 1973 *Theory of Plasma Instabilities* (Consultants Bureau) Vols 1 and 2
Rosenbluth M N and Simon A 1965 *Phys. Fluids* **8** 1300
Wharton C B, Korn P and Robertson S 1971 *Phys. Rev. Lett.* **27** 499