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Spectral features of radiation from Nordström and Kerr–Newman white holes

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Abstract. Unlike the Schwarzschild white hole, Nordström and Kerr-Newman white holes cannot explode right down from the space time singularity $R = 0$. For example a charged white hole has to commence explosion (*i.e.*, comes into existence) with a radius $R_0 = R_c(2 - R_c/R_b)^{-1}$ where R_c is the 'classical radius' and R_b is the final radius attained when the stationary state is reached. That means charged and rotating black holes also cannot hit the singularity $R = 0$ and perish.

Here the explosion is decelerated by the presence of charge and rotation and hence the radiation emitted would be not as energetic as in the Schwarzschild case where its energy is infinitely large for emission from $R = 0$.

Keywords. General relativity; radiation; white holes.

1. Introduction

Recently (Narlikar *et al* 1974) attention was drawn to white holes, so far neglected by theoreticians as compared to their time reversed versions-black holes. The authors (Narlikar *et al* 1974) have considered radiation emitted from the surface of Schwarzschild's white holes and have made out a case for white holes being possible candidates of sources of high energy radiation. Their astrophysical applications have been suggested.

A white hole is essentially an object exploding from a highly dense or singular state when it was originally well inside its black hole. It consists of outward moving particles, and as is well known (Faulkner *et al* 1964) these particles, and the outward moving light quanta can emerge out of black hole barrier with appreciable energy.

A white hole may be taken as a delayed big bang (Neeman 1965) in a Friedman universe or a collapsing object reversing implosion to explosion. For the latter possibility we are led to unusual equations of state (Novikov and Zeldovich 1973) or negative energy fields (Narlikar 1974) if the theoretical framework is that of general relativity (Hawking and Penrose 1970).

In this paper we shall consider spectral features of radiation emitted from the surface of charged and charged and rotating (Nordström and Kerr-Newman) white holes. The relation for spectral shift is obtained in the next section to be followed by the section discussing spectral features and inferences drawn from them.

2. Calculation of spectral shift

We shall take the white hole to be an object exploding from a singularity and obeying Einstein's equations of gravitation subsequent to the singular event. For convenience and simplicity we shall assume the following:

(*a*) The white hole emerges from the singularity as a Nordström or a Kerr-Newman particle.

(*b*) The light emitted by the white hole is monochromatic and is being emitted radially outwards from the surface at a uniform rate.

(*c*) The space time exterior to the object is described by the Nordström or the Kerr-Newman line element. Here we have assumed that the non-static character of the field would not appreciably change the exterior geometry.

(*d*) Here explosion is considered as an outward ' free fall '. That is, the surface of exploding object follows a geodesic in the exterior geometry. There may however occur lateral motions on the surface retaining the over all symmetry. So long as we consider propagation of radiation in the equatorial plane and along the axis of symmetry, we can ignore these motions.

Let us, first of all, note that we shall employ T as the exterior time coordinate and t as the comoving proper time coordinate, measured by an observer attached to the outward freely falling surface of the white hole.

The Nordström white hole

In this case though the exterior space time is non empty, the field outside is essentially static (Zeldovich and Novikov 1971) as is well known in the classical electrodynamics that a radially oscillating charged sphere does not radiate out electromagnetic energy (Panofsky and Phillips 1962). Hence we are quite justified in taking the Nordström line element for the exterior space time.

The space time exterior to the white hole is described by

$$\left.\begin{aligned} ds^2 &= \psi c^2\, dT^2 - \psi^{-1}\, dR^2 - R^2\, (d\theta^2 + \sin^2\theta\, d\phi^2) \\ \psi &= 1 - 2Gm/c^2 R + e^2\, G/c^4\, R^2 \end{aligned}\right\} \tag{1}$$

where G and c are respectively the gravitational constant and the velocity of light.

We shall write

$$R = R_b S(t), \quad S_0 \leqslant S(t) \leqslant 1 \tag{2}$$

where $S(t)$ is the white hole expansion factor. S_0 would be soon specified.

In the exterior coordinates (1) we have for a freely falling particle on the surface of the white hole,

$$c\frac{dT}{ds} = \gamma\psi^{-1} \tag{3}$$

while in the comoving coordinates

$$c\frac{dt}{ds} = 1. \tag{4}$$

Hence we can write from (1), (3) and (4)

$$\left(\frac{dR}{ds}\right)^2 = \frac{\dot{R}^2}{c^2} = \gamma^2 - \psi, \quad \dot{R} = \frac{dR}{dt}. \tag{5}$$

γ could be specified by noting $\dot{R} = 0$ when $R = R_b$ at $S = 1$ and so we obtain

$$\dot{R}^2 = c^2(\psi_b - \psi) \tag{6}$$

where

$$\psi_b = (\psi)_{R=R_b}. \tag{7}$$

This gives

$$\dot{S}^2 = \alpha c^2 \left[\left(\frac{1}{S} - 1\right) - \beta\left(\frac{1}{S^2} - 1\right)\right] \tag{8}$$

where

$$\alpha c^2 = 2Gm/R_b{}^3, \quad \beta = R_e/2R_b. \tag{9}$$

$R_e = e^2/mc^2$ is the 'classical radius'. The corresponding Schwarzschild's relation follows on putting $e = 0$.

Now reality of $\dot{R}$ determines S_0 as

$$S_0 = \beta/(1 - \beta). \tag{10}$$

Thus $S(t)$ could only range between $S = S_0$ and $S = 1$, that means explosion should always commence from $S \geqslant S_0$ (not from $S = 0$). Taking zero of the proper time t (for comoving observer) for $S = S_0$ and on integrating (6) from $S = S_0$ to $S(t = t_0) = 1$ we obtain

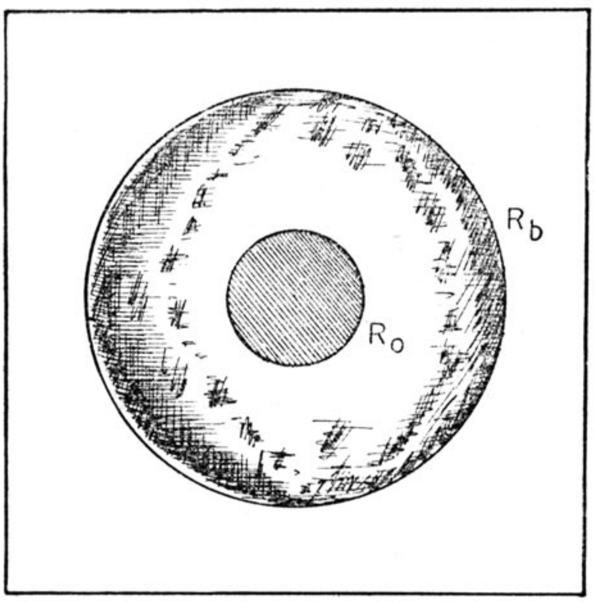

Figure 1. Nordström white hole.

$$t_0 = \frac{\pi}{2\sqrt{ac^2}}(1-\beta)^{-3/2}. \tag{11}$$

It measures the duration of explosion in the comoving coordinates and it agrees with t_0 in (Narlikar *et al* 1974) when $e = 0$.

Since we have been able to determine $\dot{R}$ we can calculate the spectral shift even though the interior solution is not known. Suppose two successive light signals are sent out radially from the surface at instants t and $t + dt$ in the comoving t-coordinate (which correspond to T and $T + dT$ in the exterior T-coordinate) and are received by the distant observer $R = R_1 \gg R_+$ (R_+ and R_- are the two roots of $\psi = 0$) at instants T_1 and $T_1 + dT_1$. Now a simple calculation of null geodesics yields

$$c\,[(T_1 + dT_1 - T - dT) - (T_1 - T)]$$
$$= \int_{R+dR}^{R_1} \frac{dR}{\psi} - \int_{R}^{R_1} \frac{dR}{\psi} = -\frac{dR}{\psi}.$$

Hence

$$\frac{dT_1}{dt} = \frac{\partial T}{\partial t} - \frac{\dot{R}}{c\psi}. \tag{12}$$

$\partial T/\partial t$ is evaluated at the outward moving surface at the time of emission and so are the functions $S(t)$ and $\dot{S}(t)$.
On substituting (3) and (6) in (12) we get

$$\frac{dT}{dt} = (\sqrt{\psi_b} - \sqrt{\psi_b - \psi})/\psi. \tag{13}$$

This means that a light signal of frequency ν_0 is emitted from the surface of the white hole, it will be received by the distant observer with frequency

$$\nu = \nu_0(\sqrt{\psi_b} + \sqrt{\psi_b - \psi}) \tag{14}$$

where a small correction needed to change from T to the receiver's proper time has been neglected. In terms of the spectral shift, it reads as

$$1 + z = (\sqrt{\psi_b} + \sqrt{\psi_b - \psi})^{-1}. \tag{15}$$

The Kerr-Newman white hole

The space time exterior to the white hole is described by the Kerr-Newman line-element (Misner *et al* 1973),

$$ds^2 = -\rho^2 \triangle^{-1} dR^2 - \rho^2 d\theta^2 - \rho^{-2} \sin^2\theta\,[a dT - (R^2 + a^2)\,d\phi]^2$$
$$+ \rho^{-2} \triangle\,(dT - a\sin^2\theta\, d\phi)^2 \tag{16}$$

where

$$\rho^2 = R^2 + a^2\cos^2\theta, \quad \triangle = R^2 - 2mR + a^2 + e^2. \tag{17}$$

Here G and c are suppressed and a is the rotation parameter.

Here we shall consider the two cases of radiation emitted by a particle falling with the surface (i) in the equatorial plane $\theta = \pi/2$ and (ii) along the axis $\theta = 0$.

Following the above procedure we obtain

$$\dot{R}^2 = \triangle \left[\frac{(\triangle_b - a^2)}{R_b^2(\triangle - a^2)} - \frac{1}{R^2}\right], \quad \theta = \pi/2 \tag{18}$$

$$= \frac{\triangle_b}{\rho_b^2} - \frac{\triangle}{\rho^2}, \quad \theta = 0 \tag{19}$$

which gives

$$S_0 = \frac{\beta}{1-\beta}, \quad \theta = \pi/2 \tag{20}$$

$$= (\beta + a^2/R_b^2)(1-\beta)^{-1}, \quad \theta = 0. \tag{21}$$

For expansion in the equatorial plane, S_0 is independent of the rotation parameter a and hence the expansion could start from $R = 0$ when $\beta = 0$ for the Kerr white hole as in the Schwarzschild's case.

It is quite difficult to integrate (18) to find the proper duration of expansion The analogues of (14) would read as

$$\frac{v}{v_0} = \sqrt{\frac{\triangle_b - a^2}{R_b^2}} + \sqrt{\left(\frac{\triangle_b - a^2}{R_b^2} - \frac{\triangle - a^2}{R^2}\right)}, \quad \theta = \pi/2 \tag{22}$$

$$= \sqrt{\frac{\triangle_b}{\rho_b^2}} + \sqrt{\left(\frac{\triangle_b}{\rho_b^2} - \frac{\triangle}{\rho^2}\right)} \quad \theta = 0. \tag{23}$$

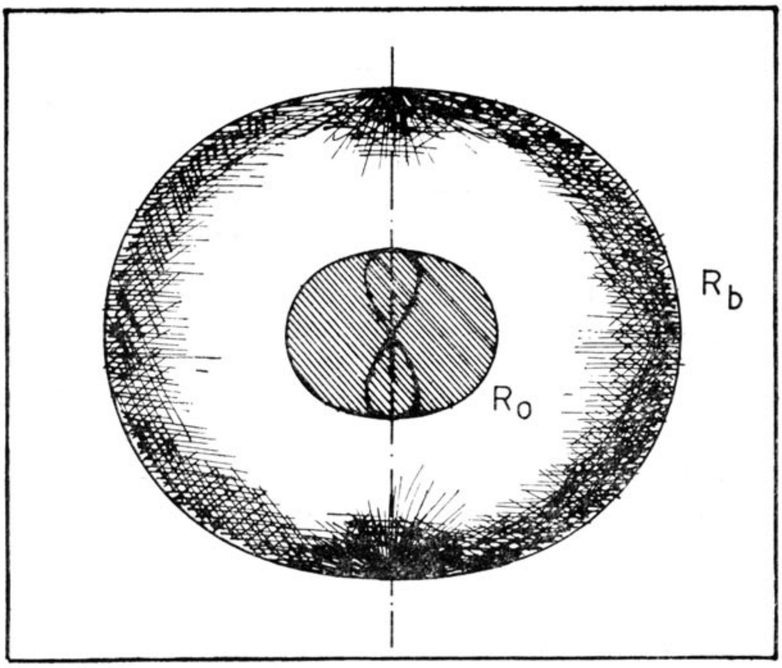

Figure 2. Kerr-Newman white hole.

3. Discussion

(i) Unlike the Schwarzschild white hole, the Nordström and the Kerr-Newman white holes cannot emerge from the singularity $R = 0$ instead they have always to come into existence with a finite radius $R \geqslant R_0$. The expansion in the equatorial plane $\theta = \pi/2$ for the Kerr white hole can however start from $R = 0$.

For the Nordström white hole we write from (2) and (10)

$$R_0 = R_b S_0 = R_c (2 - R_c/R_b)^{-1} \tag{24}$$

or

$$R_c/R_0 + R_c/R_b = 2 \tag{25}$$

which implies $R_0 < R_c$ if $R_b > R_c$, $R_0 = R_c/2$ for $R_b = \infty$ and $R_0 = R_c$ for $R_b = R_c$. Hence R_0 always lies between $R_c/2$ and R_c.

It is quite interesting to note that the electron has just the right dimensions in the 'classical radius'. It could not have been any smaller.

For the Kerr-Newman white hole R_0 is given by

$$R_0 = \frac{R_c}{2 - R_c/R_b}, \qquad \theta = \pi/2 \tag{26}$$

$$= \frac{R_c}{2 - R_c/R_b} + \frac{2a^2}{2R_b - R_c}, \qquad \theta = 0 \tag{27}$$

which will, for the Kerr white hole, reduce to

$$R_0 = 0, \qquad \theta = \pi/2 \tag{28}$$

$$= \frac{a^2}{R_b}, \qquad \theta = 0. \tag{29}$$

The rotation on the particle helps to increase the initial radius R_0 for $\theta \neq \pi/2$. In this case R_0 should lie between the limits $R_c/2$ and $\frac{1}{2}[R_c + (R_c^2 + 4a^2)^{1/2}]$. The upper limit, however, lies above R_c due to the presence of rotation.

Since a black hole is a time reversed white hole and hence it has also to cease collapsing at R_0 which is greater than half the 'classical radius' for a charged object. Nordström and Kerr-Newman black holes eventually settle down to a stationary state above R_0 and the region below R_0 is inaccessible. A Kerr black hole can collapse down to $R = 0$ in the equatorial plane while for $\theta \neq \pi/2$ the collapse stops at R_0 and so it may finally take a dumble-like shape.

(ii) From (14) we observe for the Nordström white hole

$$(\nu/\nu_0)_{R=R_0} = (\nu/\nu_0)_{R=R_b} = \tfrac{1}{2}(\nu/\nu_0)_{R=R_+} = \sqrt{\psi_b} \tag{30}$$

and so is true for the Kerr-Newman white hole.

It should be noted that the frequency change for signals emitted at initial and final instants of expansion is the same. This is because in (6) $\dot{R} = 0$ for $R = R_0$ and for $R = R_b$ of which the former lies below the anti-event horizon ($R = R_-$) and the latter lies above the event horizon ($R = R_+$). This is the characteristic feature of the Nordström metric. Thus at these two positions, the Doppler shifts vanish due to $\dot{R} = 0$ and the purely gravitational shifts have the same value

$$1 + z = \frac{1}{\sqrt{\psi_b}} = \left(1 - \frac{2Gm}{c^2 R_b} + \frac{Ge^2}{c^4 R_b^2}\right)^{-1/2} = \left(1 - \frac{2Gm}{c^2 R_0} + \frac{Ge^2}{c^4 R_0^2}\right)^{-1/2} \tag{31}$$

although $R_0 \neq R_b$. In the Schwarzschild's case signals emitted at the initial instant are received with infinitely large frequency, for explosion could set in right down from $R = 0$, the same is true for the Kerr white hole with $\theta = \pi/2$. Hence the signals emitted at the initial instant in the equatorial plane by the Kerr white hole are infinitely blue shifted.

In the Nordström case, ν/ν_0 is maximum for signals emanating from $R = R_e$ and for the Kerr-Newman case, those emanating from

$$R = \tfrac{1}{2}[R_e + (R_e^2 + 4a^2 \cos^2\theta)^{1/2}] \tag{32}$$

$$= R_e, \qquad \theta = \pi/2 \tag{33}$$

$$= \tfrac{1}{2}[R_e + (R_e^2 + 4a^2)^{1/2}], \qquad \theta = 0 \tag{34}$$

are maximally blue shifted. Incidentally at this R the field changes sign; attraction changes over to repulsion.

(iii) The white hole crosses the event horizon with blue shift if

$$R_b > \frac{4m}{3}\left[1 + \left(1 - \frac{3}{16m^2}(3a^2 + 4e^2)\right)^{1/2}\right]. \tag{35}$$

If the signals emitted at the event horizon are blue shifted then so would be those emitted at the anti event horizon R_-. We shall have blue shifted signals from all through the singular region from the anti event horizon to the event horizon, the maximum blue shift corresponding to R given by (33) and (34). If (35) is not true, the blue shifted signals would only come from the small region around R_e [or R given by (34)]. We can readily see

$$R_e/2 < R_0 < R_- \leqslant R_e \leqslant R_+. \tag{36}$$

(iv) It is evident from (6), (18) and (19) that rate of expansion is slower than that of the Schwarzschild case. That means charge and rotation tend to decelerate expansion and consequently tend to decrease energy of the outgoing photons.

The presence of charge weakens the gravitational field and so does rotation, since charge and rotation parameters, as could be easily seen, counteract the influence of mass. In the region $R_0 \leqslant R < R_e$ the field is repulsive and it is attractive for $R > R_e$. In view of (30) it appears that $R = R_e$ (which corresponds to the maximum spectral shift) is symmetrically placed with respect to R_0 and R_b. The photons emitted at $R = R_e$ are maximally blue shifted since the field is (weakest) zero there and hence their kinetic energy is least diminished by gravitation

This is not true for signals emitted by the Kerr white hole in the equatorial plane, $\theta = \pi/2$, because they could originate right down at $R = 0$ and would be infinitely blue shifted. The kinetic energy imparted to the photons is not large owing to slow rate of expansion hence charged and rotating white holes cannot be sources of very high energy radiations as are uncharged and non rotating white holes.

In view of the above observations it seems that half the ' classical radius ' could be taken as the ' hard core ' radius for a charged particle. A charged particle ought to have radius always greater than this.

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